Fuzzy E.O.Q Model for Deteriorating Items, with Constant Demand, Shortages, and Fully Backlogging

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Abstract
In this paper analyzes fuzzy inventory system for deteriorating items with constant demand. Shortages are allowed under fully backlogged here. Fixed cost, deterioration cost, shortages cost, holding cost are the cost considered in this model. In this model first time we have considered a special condition that the demand falls to zero in a time interval \((t_0 \leq t \leq t_e)\) for an unexpected condition (flood, strike, earthquake, etc.) and considered three cases. Fuzziness is applying by allowing the cost components (holding cost, deterioration, shortage cost, etc). In fuzzy environment it considered all required parameter to be triangular fuzzy numbers. Here we use nearest interval approximation method to convert a triangular fuzzy number to an interval number and we have transformed this interval number to a parametric interval-valued functional form. Several numerical examples are given to verify optimal solutions. The purpose of the model is to minimize total cost function.

Keywords: Inventory, Fuzzy number, Shortages, Interval-valued function, Triangular fuzzy number.
Mathematics Subject Classification: 90B05, 90C70.
1 Introduction

An inventory deals with a decision that minimize cost function or maximize profit function. For this purpose the task is to construct a mathematical model of the real life Inventory system, such a mathematical model is based on various assumption and approximation. Deterioration play an important role of an Inventory model. Deterioration is defined as decay or damage in the quality of the inventory. Foods, Drugs, pharmaceuticals etc are deteriorating items. During inventory there have some losses of these deteriorating items, consequently this loss must be taken into account when analyzing the system. Shortage is also very important condition.

In ordinary inventory model it considered all parameter like shortage cost, holding cost, deteriorating cost, as a fixed. But in real life situation it will have some little fluctuations. So consideration of fuzzy variables is more realistic.


In this paper we first considered crisp inventory model with account a special condition (flood, strike, earthquake, etc.) while demand falls to zero for a time interval \((t_0 \leq t \leq t_\theta)\) and considered three cases-

1) Demand falls to zero, before the deterioration set in.

2) Demand falls to zero, after the deterioriation set in.

3) With-out accounts any special condition.

There after we transformed crisp inventory models to fuzzy inventory models, and it given several numerical examples and compared, it seen that \(\text{Tac}_3(t_1^*) \leq \text{Tac}_1(t_1^*) \leq \text{Tac}_2(t_1^*)\), i.e., the total average cost function maximum when demand falls to zero after deterioration start, it minimum when no such unexpected condition arise.
2 Preliminaries: Fuzzy numbers and its nearest interval approximation

2.1. Fuzzy number

A real number $A$ described as fuzzy subset on the real line $\mathcal{R}$ whose membership function $\mu_A(x)$ has the following characteristics with $-\infty < a_1 \leq a_2 \leq a_3 < \infty$

$$\mu_A(x) = \begin{cases} 
\mu_A^L(x) & \text{if } a_1 \leq x \leq a_2, \\
\mu_A^R(x) & \text{if } a_2 \leq x \leq a_3, \\
0 & \text{otherwise}.
\end{cases}$$

Where $\mu_A^L(x): [a_1, a_2] \rightarrow [0,1]$ is continuous and strictly increasing and $\mu_A^R(x): [a_2, a_3] \rightarrow [0,1]$ is continuous and strictly decreasing.

**α-level set:** The α-level of a fuzzy number is defined as a crisp set where $A(\alpha) = \{x: \mu_A(x) \geq \alpha, x \in \mathcal{R}\}$ where $\alpha \in [0,1]$. $A(\alpha)$ is a non empty bounded closed interval contained in $\mathcal{R}$ and it can be denoted by $A(\alpha) = [L(\alpha), R(\alpha)]$. $L(\alpha)$ and $R(\alpha)$ are the lower and upper bounds of the closed interval, respectively.

2.2. Interval number

An interval number $A$ is defined by an ordered pair of real numbers as follows $A = [a, b] = \{x: a \leq x \leq b, x \in \mathcal{R}\}$ where $a$ and $b$ are the left and right bounds of an interval $A$, respectively. The interval $A$ is also defined by center ($\alpha$) and half-width ($\omega$) as follows $A = (\alpha, \omega) = \{x: a - \omega \leq x \leq a + \omega, x \in \mathcal{R}\}$ where $\alpha = \frac{a + b}{2}$ is the center and $\omega = \frac{b - a}{2}$ is the half-width of $A$.

2.3. Nearest interval approximation

Here we want to approximate a fuzzy number by a crisp model. Suppose $\tilde{A}$ and $\tilde{B}$ are two fuzzy numbers with $\alpha$-cuts as $[A_L(\alpha), A_R(\alpha)]$ and $[B_L(\alpha), B_R(\alpha)]$, respectively. Then the distance between $\tilde{A}$ and $\tilde{B}$ is

$$d(\tilde{A}, \tilde{B}) = \sqrt{\int_0^1 (A_L(\alpha) - B_L(\alpha))^2 + \int_0^1 (A_R(\alpha) - B_R(\alpha))^2}d\alpha.$$ 

Given a fuzzy number $\tilde{A}$, we have to find a closed interval $C_D(\tilde{A})$, which is closest to $\tilde{A}$ with respect to some metric. We can do it since each interval is also a fuzzy number with constant $\alpha$-cut for all $\alpha \in [0,1]$. Hence $(C_D(\tilde{A}))\alpha = [C_L, C_R]$. Now we have to minimize

$$d(\tilde{A}, C_D(\tilde{A})) = \sqrt{\int_0^1 (A_L(\alpha) - C_L(\alpha))^2 + \int_0^1 (A_R(\alpha) - C_R(\alpha))^2}d\alpha$$

with respect to $C_L$ and $C_R$.

In order to minimize $d(\tilde{A}, C_D(\tilde{A}))$, it is sufficient to minimize the function $D(C_L, C_R) = (d^2(\tilde{A}, C_D(\tilde{A})))$. The first partial derivatives are

$$\frac{\partial}{\partial C_L} D(C_L, C_R) = -2 \int_0^1 A_L(\alpha)d\alpha + 2 C_L.$$ 

and

$$\frac{\partial}{\partial C_R} D(C_L, C_R) = -2 \int_0^1 A_R(\alpha)d\alpha + 2 C_R.$$ 

Solving $\frac{\partial}{\partial C_L} D(C_L, C_R) = 0$ and $\frac{\partial}{\partial C_R} D(C_L, C_R) = 0$, we get

$$C_L = \int_0^1 A_L(\alpha)d\alpha \text{ and } C_R = \int_0^1 A_R(\alpha)d\alpha.$$ 

Again since $\frac{\partial^2}{\partial C_L^2} (D(C_L^*, C_R^*)) = 2 > 0$ and $\frac{\partial^2}{\partial C_R^2} (D(C_L^*, C_R^*)) = 2 > 0$ and
\[ H(C_L^*, C_R^*) = \frac{a^2}{\partial C_L} (D(C_L^*, C_R^*)) + \frac{a^2}{\partial C_R} (D(C_L^*, C_R^*)) - \left( \frac{a^2}{\partial C_L} \right) (D(C_L^*, C_R^*))^2 = 4 > 0. \]

So \( D(C_L^*, C_R^*) \) i.e. \( d(\tilde{A}, C_D(\tilde{A})) \) is global minimum. Therefore, the interval

\[ C_d(\tilde{A}) = [\int_0^1 A_L(\alpha) d\alpha, \int_0^1 A_R(\alpha) d\alpha] \]

is the nearest interval approximation of fuzzy number \( \tilde{A} \) with respect to the metric \( d \).

Let \( \tilde{A} = (a_1, a_2, a_3) \) be a triangular fuzzy number. The \( \alpha \)-cut interval of \( \tilde{A} \) is defined as

\[ A_\alpha = [A_L(\alpha), A_R(\alpha)] \]

where \( A_L(\alpha) = a_1 + \alpha(a_2 - a_1) \) and \( A_R(\alpha) = a_3 - \alpha(a_3 - a_2) \). By nearest

interval approximation method the lower limit of the interval is

\[ C_L = \int_0^1 A_L(\alpha) d\alpha = \int_0^1 [a_1 + \alpha(a_2 - a_1)] d\alpha = \frac{a_1 + a_2}{2} \]

and the upper limit of the interval is

\[ C_R = \int_0^1 A_R(\alpha) d\alpha = \int_0^1 [a_3 - \alpha(a_3 - a_2)] d\alpha = \frac{a_3 + a_2}{2}. \]

Therefore, the interval number corresponding \( \tilde{A} \) is \([a_1 + a_2, a_3 + a_2]\) = \([m, n]\). In the centre and half –width form the interval number of \( \tilde{A} \) is defined as \( \left( \frac{1}{4}(a_1 + 2a_2 + a_3), \frac{1}{4}(a_3 - a_1) \right) \).

### 2.4. Parametric Interval-valued function:

Let \([m, n]\) be an interval, where \( m > 0, n > 0 \). From analytical geometry point of view, any real number can be represented on a line. Similarly, we can express an interval by a function. The parametric interval-valued function for the interval \([m, n]\) can be taken as \( g(s) = m^{1-s}n^s \) for \( s \in [0, 1] \), which is a strictly monotone, continuous function and its inverse exits. Let \( \psi \) be the inverse of \( g(s) \), then

\[ s = \frac{\log n - \log m}{\log n - \log m}. \]

### 3 Methods

#### 3.1. Notation

\( I(t) \) = Inventory level at any time, \( t \geq 0. \)

\( T \): Cycle of length.

\( t_0 \): Time point, when demand falls to zero (unexpected conditions arise).

\( t_c \): Time point, when demand again start.

\( t_e \): Time point, when the deterioration set in.

\( t_i \): Time point, when inventory level falls to zero.

\( c_i \): Inventory fixed cost.

\( c_s \): Shortages cost per unit, per unit time.

\( c_d \): Deteriorating cost per unit, per unit time.

\( c_h \): Holding cost per unit, per unit time.

\( Q \): stock level at the beginning of the cycle.

\( D \): Constant demand rate.

\( Tac_i(t_i) \): Total average cost per unit, \( i=1,2,3. \)

\( \tilde{c}_i \): Fuzzy fixed cost.

\( \tilde{c}_s \): Fuzzy shortage cost per unit, per unit time.

\( \tilde{c}_d \): Fuzzy deteriorating cost per unit, per unit time.

\( \tilde{c}_h \): Fuzzy holding cost per unit, per unit time.

\( Tac_i(t_i) \): Fuzzy total average cost per unit, \( i=1,2,3. \)
3.2. Assumption
a: The inventory system involves only one item.
b: The replenishment occur instantaneously at infinite rate.
c: The lead time is negligible.
d: Demand rate is constant.
e: $2\theta t$ is deterioration rate per unit time per cycle, $\theta$ is constant.

3.3. Crisp model
Case -1 (Here demand falls to zero, before the deterioration set in).
Let $I(t)$ be the inventory level at any time $t$, ($0 \leq t \leq T$). During $0 \leq t \leq t_0$ the inventory level $I(t)$ decrease due to customer demand only, in $t_0 \leq t \leq t_e$ the inventory level $I(t)$ remain same, again in $t_e \leq t \leq t_d$ the inventory level decrease due to customer demand only, and finally in $t_d \leq t \leq t_1$, the inventory level decrease due to customer demand and deterioration, and reaches to zero at $t=t_1$. During the time interval $t_1 \leq t \leq T$, the shortages with fully backlogged continued.

So the differential equation describing as follows,

$$\frac{dI(t)}{dt} = \begin{cases} 
-D, & 0 \leq t \leq t_0, \\
0, & t_0 \leq t \leq t_e, \\
-D, & t_e \leq t \leq t_d, \\
-D - 2\theta t I(t), & t_d \leq t \leq t_1, \\
-D, & t_1 \leq t \leq T.
\end{cases}$$

(3.1)

With boundary conditions $I(t) = Q$ and $I(t_1) = 0$.

And solutions of the above differential equations after applying the boundary condition are,

$$I(t) = \begin{cases} 
Q - Dt, & 0 \leq t \leq t_0, \\
Q - D(t_0 - t), & t_0 \leq t \leq t_e, \\
Q + D(t_e - t_0) - Dt, & t_e \leq t \leq t_d, \\
D(t_1 - t)(1 - \theta t^2) + \frac{D\theta}{3}(t_1^3 - t^3), & t_d \leq t \leq t_1, \\
D(t_1 - t), & t_1 \leq t \leq T.
\end{cases}$$

(3.2)

Now the holding cost per cycle is,

$$He = c_h \left[ \int_0^{t_0} I(t) dt + \int_{t_0}^{t_e} I(t) dt + \int_{t_e}^{t_d} I(t) dt + \int_{t_d}^{t_1} I(t) dt \right]$$

$$= c_h \left[ (Q t_0 - \frac{Dt_0^2}{2}) + (Q - D t_0)(t_e - t_0) + Q(t_d - t_e) + D(t_e - t_0)(t_d - t_e) - \frac{D}{2}(t_d^2 - t_e^2) + \\
D \left\{ t_1(t_1 - t_d) - \frac{1}{2}(t_1^2 - t_d^2) - \frac{Qt_1}{3}(t_1^3 - t_d^3) + \frac{\theta}{4}(t_1^4 - t_d^4) \right\} + \frac{D\theta}{3}\{t_1^3(t_1 - t_d) - \frac{1}{4}(t_1^4 - t_d^4)\} \right].$$
Deteriorating cost per cycle is,
\[
\text{Dc} = c_d \int_{t_d}^{t_1} D \left( t(t_1 - t)(1 - \theta t^2) + \frac{\theta t}{3} (t_1^3 - t^3) \right) dt
\]
\[
= c_d D \left[ t_1(t_1 - t_d) - \frac{1}{4}(t_1^2 - t_d^2) - \frac{\theta t_1}{3}(t_1^3 - t_d^3) + \frac{\theta}{4}(t_1^4 - t_d^4) \right] + \frac{D\theta}{3} \left[ t_1^3(t_1 - t_d) - \frac{1}{4}(t_1^4 - t_d^4) \right].
\]
Shortages cost per cycle is,
\[
\text{Sc} = -c_s \int_{t_1}^{T} D(t_1 - t) dt
\]
\[
= -c_s D \left( t_1(T - t_1) - \frac{1}{2}(T^2 - t_1^2) \right).
\]
So the total average cost per cycle is,
\[
\text{Ta}c_1(t_1) = \frac{1}{T} [\text{Fc} + \text{Sc} + \text{Dc} + \text{Sc}]
\]
\[
= \frac{1}{T} \left[ c_f + c_n \left( (Q(t_0 - \frac{D t_0^2}{2}) + (Q - D t_0)(t_e - t_0) + Q(t_d - t_e) + D(t_e - t_0)(t_d - t_e) - \frac{D}{2}(t_d^2 - t_e^2) + D \left[ t_1(t_1 - t_d) - \frac{1}{4}(t_1^2 - t_d^2) - \frac{\theta t_1}{3}(t_1^3 - t_d^3) + \frac{\theta}{4}(t_1^4 - t_d^4) \right] + \frac{D\theta}{3} \left[ t_1^3(t_1 - t_d) - \frac{1}{4}(t_1^4 - t_d^4) \right] - c_s D \left( t_1(T - t_1) - \frac{1}{2}(T^2 - t_1^2) \right) \right) \right].
\]

**Case - 2** (Here demand falls to zero, after the deterioration set in).

Let \( I(t) \) be the inventory level at any time \( t \), \( 0 \leq t \leq T \). During \( 0 \leq t \leq t_d \) the inventory level \( I(t) \) decrease due to customer demand only, in \( t_d \leq t \leq t_0 \) the inventory level decrease due to customer demand and deterioration, in \( t_0 \leq t \leq t_e \) the inventory level decrease due to customer demand only, and finally in \( t_e \leq t \leq t_1 \), the inventory level decrease due to customer demand and deterioration, and reaches to zero at \( t = t_1 \). During the time interval \( t_1 \leq t \leq T \), the shortages with fully backlogged continued.

![Figure 2: Inventory model (Case -2)](http://www.ispacs.com/journals/ojids/2016/ojids-00006/)

So the differential equation describing as follows;

\[
\frac{dI(t)}{dt} = \begin{cases} 
-D, & 0 \leq t \leq t_d, \\
-D - 2\theta I(t), & t_d \leq t \leq t_0, \\
-2\theta I(t), & t_0 \leq t \leq t_e, \\
-D - 2\theta I(t), & t_e \leq t \leq t_1, \\
-D, & t_1 \leq t \leq T.
\end{cases}
\]

(3.3)

With boundary conditions \( I(t_1) = Q \) and \( I(t_1) = 0 \).

Corresponding solutions of the above differential equations after applying the boundary condition are,
I(t) =

\begin{align*}
(Q - D_t) (1 + \theta t^2 - \theta t^2) + D \left( t_d + \frac{\theta}{3} t^3 - \frac{\theta t}{6} t^2 \right) - D \left( t - \frac{\theta}{3} t^3 \right), \\
(Q - D_t) (1 + \theta t^2 - \theta t^2) + D \left( t_d + \frac{\theta}{3} t^3 - \frac{\theta t}{6} t^2 \right) - D \left( t - \frac{\theta}{3} t^3 \right), \\
+ (Q - D_t) \theta (t_0^2 - t^2) + D \theta t_d (t_0^2 - t^2) - D \theta t \theta (t_0^2 - t^2), \\
D(t_1 - t)(1 - \theta t^2) + \frac{D \theta}{3} (t^3 - t^3),
\end{align*}

\[ D(t_1 - t), \]

\[ t_1 \leq t \leq T. \]

Now the holding cost per cycle is,

\[ H_c = c_h \int_{t_0}^{t_1} I(t) dt + \int_{t_0}^{t_1} I(t) dt + \int_{t_0}^{t_1} I(t) dt \]

\[ = c_h \left[ Q t_d - \frac{D t_d^2}{2} + (Q - D_t) \left\{ (t_0 - t_d) + \theta t_d^2 (t_0 - t_d) - \frac{\theta}{3} (t_0^3 - t_d^3) \right\} + D \left\{ t_d (t_0 - t_d) + \frac{\theta}{3} t_d^3 (t_0 - t_d) - \frac{\theta}{3} (t_0^3 - t_d^3) \right\} - D \left\{ t_0^2 - t^2 \right\} - \frac{\theta}{6} (t_0^4 - t^4) \right\} \right] + \left\{ (Q - D_t) (1 + \theta t^2 - \theta t^2) + D \left( t_d + \frac{\theta}{3} t^3 - \frac{\theta t}{6} t^2 \right) - D \left( t - \frac{\theta}{3} t^3 \right) \right\} + \left\{ (Q - D_t) \theta (t_0^2 - t^2) + D \theta t_d (t_0^2 - t^2) - D \theta t \theta (t_0^2 - t^2) \right\} + D \left\{ t_1 (t_1 - t) - \frac{1}{2} (t^2 - t^2) \right\} \right] \]

\[ \int_{t_0}^{t_1} I(t) dt + \int_{t_0}^{t_1} I(t) dt \]

\[ = c_d D \left\{ (Q - D_t) \left\{ (t_0 - t_d) + \theta t_d^2 (t_0 - t_d) - \frac{\theta}{3} (t_0^3 - t_d^3) \right\} + D \left\{ t_1 (t_1 - t_d) - \frac{1}{2} (t^2 - t^2) \right\} \right\} \]

\[ = -c_s D \left\{ (t_1 (T - t_1) - \frac{1}{2} (T^2 - t_1^2) \right\}. \]

So the total average cost per cycle is,

\[ T_{ac_2}(t_1) = \frac{1}{T} \left[ F_c + S_c + D_c + S_c \right] \]

\[ = \frac{1}{T} \left[ c_f + c_h Q t_d - \frac{D t_d^2}{2} + (Q - D_t) \left\{ (t_0 - t_d) + \theta t_d^2 (t_0 - t_d) - \frac{\theta}{3} (t_0^3 - t_d^3) \right\} + D \left\{ t_1 (t_1 - t_d) - \frac{1}{2} (t^2 - t^2) \right\} \right] \]
\[\frac{\theta}{3}t_d^3(t_0 - t_d) - \frac{\theta}{3}t_d^3 - t_d^3) \right\} - D \left\{ \frac{1}{2} (t_0^2 - t_d^2) - \frac{1}{6} (t_0^4 - t_d^4) \right\} + \left\{ (D - Dt_d)(1 + \theta t_d^2 - \theta t_0^2) + D \left( \left( t_d + \frac{\theta}{3} t_d^3 - \theta t_0^2 \right) - D \left( t_0 - \frac{2\theta}{3} t_0^3 \right) \right) \right\} (t_e - t_0) + (D - Dt_d) \theta \left\{ t_0^2(t_e - t_0) - \frac{1}{3} (t_e^3 - t_0^3) \right\} + D \left\{ t_1(t_1 - t_e) - \frac{1}{2} \left( t_1^2 - t_e^2 \right) - \frac{\theta}{3} t_1^3 - \frac{1}{4} (t_1^4 - t_e^4) \right\} + D \left\{ t_1(t_1 - t_e) - \frac{1}{2} \left( t_1^2 - t_e^2 \right) \right\} \right] \right\}. \] (3.4)

**Case 3** (Without account any special condition).

Let \( I(t) \) be the inventory level at any time \( t \), \( (0 \leq t \leq T) \). During \( 0 \leq t \leq t_d \) the inventory level decrease due to customer demand only, in \( t_d \leq t \leq t_1 \) the inventory level decrease due to customer demand and deterioration, and reaches to zero at \( t = t_1 \). During the time interval \( t_1 \leq t \leq T \), the shortages with fully backlogged continued.

![Figure 3: Inventory model (Case-3)](image)

So the differential equation describing as follows,

\[ \frac{dI(t)}{dt} = \begin{cases} 
-D, & 0 \leq t \leq t_d, \\
-D - 2\theta tI(t), & t_d \leq t \leq t_1, \\
-D, & t_1 \leq t \leq T. 
\end{cases} \] (3.5)

With boundary conditions \( I(0) = Q \) and \( I(t_1) = 0 \).

Corresponding solutions of the above differential equations after applying the boundary condition are,

\[ I(t) = \begin{cases} 
Q - Dt, & 0 \leq t \leq t_d, \\
D(t_1 - t)(1 - \theta t^2) + \frac{\theta}{3} t_1^3 - t_3, & t_d \leq t \leq t_1, \\
D(t_1 - t), & t_1 \leq t \leq T. 
\end{cases} \] (3.6)

Now the holding cost per cycle is,

\[ He = c_h \left\{ \int_0^{t_d} I(t) dt + \int_{t_d}^{t_1} I(t) dt \right\} \]

\[ = c_h \left\{ Qt_d - \frac{\theta t_d^3}{2} + D \left\{ t_1(t_1 - t_d) - \frac{1}{2} (t_1^2 - t_d^2) - \frac{\theta}{3} t_1^3 - t_d^3 \right\} + \frac{\theta}{4} (t_1^4 - t_d^4) \right\} + \frac{\theta}{3} \left\{ t_1^3(t_1 - t_d) - \frac{1}{4} (t_1^4 - t_d^4) \right\}. \]

Deteriorating cost per cycle is,

\[ Dc = c_d \int_{t_d}^{t_1} DI(t) dt \]
\[ T \alpha c_3(t_1) = \frac{1}{T} \left( c_f + c_h \left[ Q t_d - \frac{D t_d^2}{2} + D \left( t_1(t_1 - t_d) - \frac{1}{2}(t_1^2 - t_d^2) - \frac{3}{4}(t_1^3 - t_d^3) \right) + \frac{D^2\theta}{3} \left( t_1^3(t_1 - t_d) - \frac{1}{4}(t_1^4 - t_d^4) \right) \right] + c_f \left[ \left( t_1^3(t_1 - t_d) - \frac{1}{4}(t_1^4 - t_d^4) \right) \right] \right) - c_s D \left( t_1(T - t_1) - \frac{1}{2}(T^2 - t_1^2) \right) \].

For minimum cost it should be, 
\[ \frac{dT \alpha c_1(t_1)}{dt_1} = 0 \text{ and } \frac{d^2T \alpha c_1(t_1)}{dt_1^2} > 0, \ i = 1, 2, 3. \]

### 3.4. Fuzzy model

Due to uncertainty let us assume that, 
\[ \tilde{c}_f = (c^1_f, c^2_f, c^3_f), \tilde{c}_h = (c^1_h, c^2_h, c^3_h), \tilde{c}_s = (c^1_s, c^2_s, c^3_s), \tilde{c}_d = (c^1_d, c^2_d, c^3_d), \]
be triangular fuzzy number then using nearest interval approximation method, we transform all triangular fuzzy number into interval number i.e., \([c^L_f, c^U_f],[c^L_h, c^U_h],[c^L_s, c^U_s] \text{ and } [c^L_d, c^U_d] \) and according to section 2.4 the total average cost is given by,

#### For case -1:

\[ T \alpha c_1(t_1) = \frac{1}{T} \left[ (c^L_f)^{1-s}(c^U_f)^s + (c^L_h)^{1-s}(c^U_h)^s \left[ (Q t_d - \frac{D t_d^2}{2} + (Q - D t_d)(t_e - t_0) + Q(t_d - t_e) + D(t_e - t_0)(t_d - t_e) - \frac{D}{2}(t_d^2 - t_e^2) + D \left( t_1(t_1 - t_d) - \frac{1}{2}(t_1^2 - t_d^2) - \frac{3}{4}(t_1^3 - t_d^3) + \frac{D}{3}(t_1^3 - t_d^3) \right) \right) \right] \right) - c_s D \left( t_1(T - t_1) - \frac{1}{2}(T^2 - t_1^2) \right) \].

#### For case -2:

\[ T \alpha c_2(t_1) = \frac{1}{T} \left[ (c^L_f)^{1-s}(c^U_f)^s + (c^L_h)^{1-s}(c^U_h)^s \left[ (Q t_d - \frac{D t_d^2}{2} + (Q - D t_d)(t_e - t_0) + \theta t_d(t_0 - t_d) - \frac{3}{2}(t_0^3 - t_d^3) \right) + \left( \left( Q - D t_d \right)(1 + \theta t_d^2 - \theta t_0^2) + D \left( t_1(t_1 - t_d) - \frac{1}{2}(t_1^2 - t_d^2) - \frac{3}{4}(t_1^3 - t_d^3) \right) \right) \right] \right) - c_s D \left( t_1(T - t_1) - \frac{1}{2}(T^2 - t_1^2) \right) \].
The optimal solution of the fuzzy model is presented in Table 3.

For case -3:

\[
\text{For crisp solutions,} \\
\text{For fuzzy solutions:}
\]

\[
\begin{align*}
    &\text{Table 1: Input values (Case 1)} \\
    &\begin{array}{cccccccc}
    c_f & c_h & c_d & c_s & D & Q & t_0 & t_e & t_d & \theta & T \\
    \hline
    100 & 60 & 20 & 10 & 6 & 200 & 0.1 & 0.2 & 0.4 & 0.1 & 10 \\
    \end{array}
\end{align*}
\]

Then out-put values are:

\[
\begin{align*}
    &\text{Table 2: output values} \\
    &\begin{array}{cc}
    t_1 & Tac_3(t_1^+) \\
    \hline
    0.843 & 751.237 \\
    \end{array}
\end{align*}
\]

For fuzzy solutions:

When the input data of inventory model is taken as triangular fuzzy number i.e., $\tilde{c_f} = (80,100,120)$, $\tilde{c_h} = (50,60,70)$, $\tilde{c_d} = (16,20,24)$ and $\tilde{c_s} = (8,10,12)$, and others input values are same as table-1. Using nearest interval approximation method, we get the corresponding interval number and interval-valued function i.e., $c_f = [90,110]$, $c_h = (90)^{1-s}(110)^s \in [90,110]$, $c_d = [55,65]$, $c_h = (55)^{1-s}(65)^s \in [55,65]$, $c_d = [18,22]$, $c_d = (18)^{1-s}(22)^s \in [18,22]$, $c_s = [9,11]$, $c_s = (9)^{1-s}(11)^s \in [9,11]$, where $s \in [0,1]$. The optimal solution of the fuzzy inventory model is presented in Table 3.
Here we have given a rough graph, which shows how change the values of $\bar{T\bar{a}c}_1(t_1^*)$ for different values of $s$.

Case-2:
For crisp solutions,
Let us take the input values;

Then output values are;

For fuzzy solutions:
When the input data of inventory model is taken as triangular fuzzy number i.e., $\bar{c}_f = (80,100,120)$, $\bar{c}_h = (50,60,70)$, $\bar{c}_d = (16,20,24)$ and $\bar{c}_s = (8,10,12)$, and others input values are same as table-1. Using nearest interval approximation method, we get the corresponding interval number and interval-valued function i.e.,
\(c_f = [90,110], \Rightarrow \bar{c}_f = (90)^{1-s}(110)^s \in [90,110],\)
\(c_h = [55,65], \Rightarrow \bar{c}_h = (55)^{1-s}(65)^s \in [55,65],\)
\(c_d = [18,22], \Rightarrow \bar{c}_d = (18)^{1-s}(22)^s \in [18,22],\)
\(c_s = [9,11], \Rightarrow \bar{c}_s = (9)^{1-s}(11)^s \in [9,11], \) where \(s \in [0,1].\)

The optimal solution of the fuzzy inventory model is presented in Table 6.

<table>
<thead>
<tr>
<th>(s)</th>
<th>(t_1^*)</th>
<th>(\bar{T}ac_2(t_1^*))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1.155</td>
<td>4206.323</td>
</tr>
<tr>
<td>0.2</td>
<td>1.156</td>
<td>4366.517</td>
</tr>
<tr>
<td>0.4</td>
<td>1.158</td>
<td>4532.861</td>
</tr>
<tr>
<td>0.6</td>
<td>1.159</td>
<td>4705.593</td>
</tr>
<tr>
<td>0.8</td>
<td>1.160</td>
<td>4884.960</td>
</tr>
<tr>
<td>1.0</td>
<td>1.161</td>
<td>5071.220</td>
</tr>
</tbody>
</table>

Here we have given a rough graph, which shown how change the values of \(\bar{T}ac_2(t_1^*)\) for Different values of \(s,\)

![Rough graph](image)

**Figure 5: Rough graph**

**Case-3:**
For crisp solutions,
Let us take the in-put value;

<table>
<thead>
<tr>
<th>(c_f)</th>
<th>(c_h)</th>
<th>(c_d)</th>
<th>(c_s)</th>
<th>(D)</th>
<th>(Q)</th>
<th>(t_d)</th>
<th>(\theta)</th>
<th>(T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>60</td>
<td>20</td>
<td>10</td>
<td>6</td>
<td>200</td>
<td>0.4</td>
<td>0.1</td>
<td>10</td>
</tr>
</tbody>
</table>

Then the out-put values:

<table>
<thead>
<tr>
<th>(t_1)</th>
<th>(\bar{T}ac_3(t_1^*))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.905</td>
<td>749.048</td>
</tr>
</tbody>
</table>
For fuzzy solutions:
When the input data of inventory model is taken as triangular fuzzy number i.e., \( \tilde{c}_f = (80,100,120), \tilde{c}_h = (50,60,70), \tilde{c}_d = (16,20,24) \) and \( \tilde{c}_s = (8,10,12) \), and others input values are same as table-1. Using nearest interval approximation method, we get the corresponding interval number and interval-valued function i.e.,
\[
\begin{align*}
\tilde{c}_f &= [90,110], \Rightarrow \hat{c}_f = (90)^{1-s}(110)^s \in [90,110], \\
\tilde{c}_h &= [55,65], \Rightarrow \hat{c}_h = (55)^{1-s}(65)^s \in [55,65], \\
\tilde{c}_d &= [18,22], \Rightarrow \hat{c}_d = (18)^{1-s}(22)^s \in [18,22], \\
\tilde{c}_s &= [9,11], \Rightarrow \hat{c}_s = (9)^{1-s}(11)^s \in [9,11], \text{ where } s \in [0,1].
\end{align*}
\]

The optimal solution of the fuzzy inventory presented in Table 9.

<table>
<thead>
<tr>
<th>s</th>
<th>( \epsilon_1^* )</th>
<th>( T^{\tilde{ac}_3}(\epsilon_1^*) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.902</td>
<td>682.172</td>
</tr>
<tr>
<td>0.2</td>
<td>0.903</td>
<td>707.027</td>
</tr>
<tr>
<td>0.4</td>
<td>0.904</td>
<td>732.796</td>
</tr>
<tr>
<td>0.6</td>
<td>0.906</td>
<td>759.511</td>
</tr>
<tr>
<td>0.8</td>
<td>0.907</td>
<td>787.208</td>
</tr>
<tr>
<td>1.0</td>
<td>0.908</td>
<td>815.924</td>
</tr>
</tbody>
</table>

Here we have given a rough graph, which shown how change the values of \( T^{\tilde{ac}_3}(\epsilon_1^*) \) for Different values of s,

For \( s=0 \), the lower bound of the interval value of the parameter is used to find the optimal solution. For \( s=1 \), the upper bound of interval value of the parameter is used for the optimal solution. These results yield the lower and upper bounds of the optimal solution. The main advantage of the proposed technique is that one can get the intermediate optimal result using proper value \( s \).

![Rough graph](image-url)
6 Conclusion

In this paper, we have proposed a real life inventory problem in a crisp and fuzzy environment. The inventory model developed with constant demand and shortages. Shortages have been allowed with fully backlogged in this model. In this paper we have considered an unexpected condition (Flood, Strike, Earthquake, etc.) when demand falls to zero and considered three cases. From portion-4 (numerical solution) it seen that Tac3(t1∗) ≤ Tac1(t1∗) ≤ Tac2(t1∗), i.e., the total average cost function maximum when demand falls to zero after deterioration start, it minimum when no such unexpected condition arise.

In this paper, we have considered triangular fuzzy number and use nearest interval approximation method to convert a triangular fuzzy number to an interval number. We transformed this interval number to a parametric interval-valued functional form and solved. In future, the other type of membership functions such as piecewise linear hyperbolic, L-R fuzzy number etc can be considered to construct the membership function and then model can be easily solved.

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Reference

http://dx.doi.org/10.3923/tasr.2011.294.303

http://dx.doi.org/10.1016/S0360-8352(03)00030-5

http://dx.doi.org/10.1287/mnsc.17.4.B141


http://dx.doi.org/10.1016/S0165-0114(86)80028-8

http://dx.doi.org/10.2298/YJOR150202002Y

http://dx.doi.org/10.1007/s12543-011-0066-9


http://dx.doi.org/10.1007/BF02936045

http://dx.doi.org/10.1016/j.amc.2005.10.001

http://dx.doi.org/10.2298/YJOR1002213I


http://dx.doi.org/10.1016/j.ejor.2004.04.046


http://dx.doi.org/10.1016/j.apm.2010.11.014
   http://dx.doi.org/10.1016/S0019-9958(65)90241-X

   http://dx.doi.org/10.1016/0020-0255(85)90025-8

   http://dx.doi.org/10.1007/978-1-4615-3640-6_5