

The role of problem solving method on the improvement of mathematical learning

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ABSTRACT:

In history of education, problem solving is one of the important educational goals and teachers or parents have intended that their students have capacity of problem solving. In present research, it is tried that study the problem solving method for mathematical learning. This research is implemented via quasi-experimental method on 49 boy students at high school. The results of Leven test and T-test indicated that problem solving method has more effective on the improvement of mathematical learning than traditional instruction method. Therefore it seems that teachers of mathematics must apply the problem solving method in educational systems till students became self-efficiency in mathematical problem solving.

KEYWORDS: Mathematics, problem solving, learning, teaching, improvement.

1. INTRODUCTION

In current educational systems, often teachers teach students to solve mathematics problems by having them copy standard solution methods provided by the textbook. Little time is devoted to teaching students how to solve problems [3]. The effect is that students have great difficulty in solving nonstandard problems that require the application of domain knowledge and routines [7]. A mathematical problem is a task in which it is not clear to the individual which mathematical actions should be applied, either because the situation does not immediately bring to mind the appropriate mathematical action(s) required to complete the task or because there are several plausible mathematical actions that the individual believes could be useful. In the past three decades, there has been a dramatic shift in the nature of research on mathematical problem solving. In the 1980, problem-solving was primarily conceived of as a goal of mathematical instruction. Researches were undertaken with the aim of understanding the nature of problem-solving and creating instructional programs that developed students' knowledge

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base, heuristics, and dispositions so that they could solve problems more effectively [10]. In recent years, it has become more common to view problem-solving as a means to achieve other pedagogical goals. Problem-solving situations can be used as valuable pedagogical tools for helping students construct sophisticated mathematical knowledge [11]. Under the right circumstances, engaging students in problem-solving can foster the development of deep mathematical insight, useful representations for reasoning about complex mathematical concepts, and powerful problem-solving heuristics [8, 14]. According to Gallagher et. al, (1999) in problem based learning environment, students act as professionals and are confronted with problems that require clearly defining and well-structured problems, developing hypothesis, assessing, analyzing, utilizing data from different sources, revising initial hypothesis as the data collected developing and justifying solutions based on evidence and reasoning. The educators used problem solving method as an educational tool to enhance learning as a relevant and practical experience, to have students' problem solving skills and to promote students' independent learning skill. Eng (2001) opined problem based learning as a philosophy aims to design and deliver a total learning environment that is holistic to student-centered and student empowerment [1]. The National Council of Teachers of Mathematics has advocated that "the very essence of studying mathematics is itself an exercise in exploring, conjecturing, examining and testing, all aspects of problem solving. Students should be given opportunities to formulate problems from given situations and to create new problems by modifying the conditions of given problems" [9, 13]. In view of Schoenfeld, solving a problem is interpreted "as working toward achieving a high-priority personal goal". That is, problem solving requires that a pupil participate in a knowledge-constructing activity arising from a problem to solve. When a pupil faces an obstacle in her/his attempt to construct a solution, there is a conflict between the pupil's initial ineffective knowledge and new knowledge that will permit her/him to progress toward a solution. In this sense, problem-solving activity might be likened to the work of a mathematician constructing new mathematics [5]. Gagne (1985) called problem solving as learning "higher order rule". Regard to this theory, learner combines simple rules till get to higher level rules that led to problem solving. Of course, solver must have dominates on pervious knowledge. Problem solving includes of several categories and strategies. Here we have tried that indicated problem solving process in clear algorithm (see Fig 1). Then in present paper, we have tried to introduce problem solving method for better mathematical learning generally.

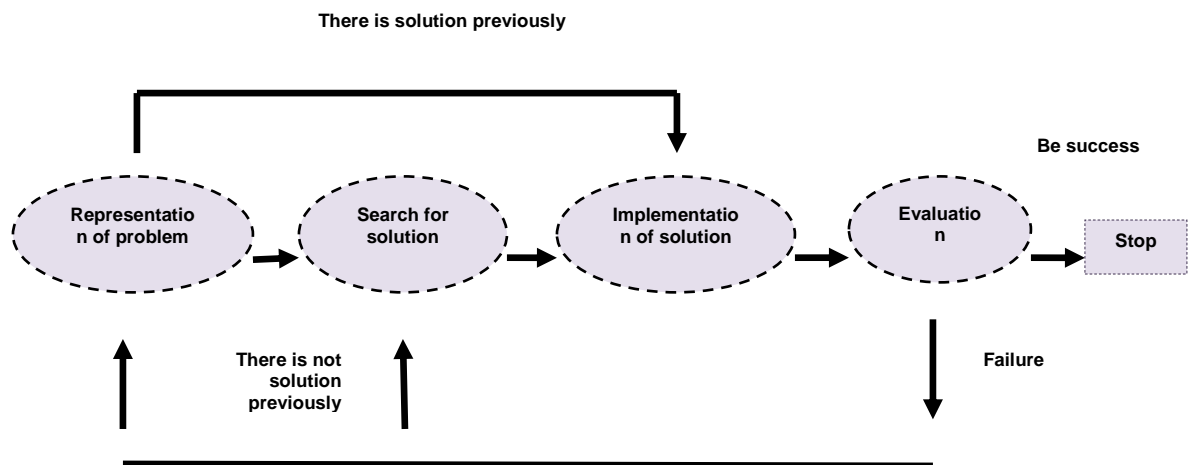


Fig 1- The plan of problem solving process

2. LITERATURE REVIEW

In this field, there are many researches such as; I) Lucia Grugnetti and Francois Jaquet (2005) in their research entitled "a mathematical competition as a problem solving and a mathematical education experience" indicated that the choice of problem situations is fundamental in order to allow pupils to make hypotheses, to mobilize their knowledge, to argue, and finally, to construct new knowledge. Certain factors concerning this choice are discussed, such as; designing features in the problem situation that allow students, by themselves, to check or rectify their method; the importance of collaborative work; what stages typically occur in solving a problem; how a posteriori analysis can reveal students' unexpected approaches; and the use of didactic variables that will compel students to develop, over time, more sophisticated tools and argumentation in problem solving and in constructing proofs [5], II) Margaret Taplin (1995) in study entitled "mathematics through problem solving" believed that there are many reasons why a problem-solving approach can contribute significantly to the outcomes of a mathematics education. Not only is it a vehicle for developing logical thinking, it can provide students with a context for learning mathematical knowledge, it can enhance transfer of skills to unfamiliar situations and it is an aesthetic form in itself. A problem-solving approach can provide a vehicle for students to construct their own ideas about mathematics and to take responsibility for their own learning [12], and also Riasat Ali et al.(2010) in their paper entitled "effect of using problem solving method in teaching mathematics on the achievement of mathematics students" concluded that use of problem solving method enhanced the achievement of the students in mathematics. The results showed that there was significant difference between the effectiveness of traditional teaching method and problem solving method in teaching of mathematics at elementary level. They recommended that the teachers should be encouraged to employ problem solving method in teaching mathematics concepts like set, information handling and geometry etc. Regular training, workshops and seminars

should be arranged for teachers to give them knowledge and understanding of problem based learning [1]. Regard to cited researches, there is the necessity of mathematical research in problem solving field.

3. RESEARCH HYPOTHESIS

Problem solving method has more effective on the improvement of mathematical learning than traditional instruction method.

4. METHODOLOGY

Since this study has occurred in real environment and human's behaviors has under controlled quietly then it may be that all real behaviors of human did not occur. Then, study method is quasi-experimental method. In this method, it is tried that effects of independent variable has studied on dependent variables. Then researchers selected control and experiment groups randomly. It is used of traditional method in control group and new instructional method or problem solving method is used for experiment group. Finally the effects of new instructional method has studied and compared against control group.

5. PARTICIPATION

In present research, statistical society was included of all boy students at three grade of one high school in mathematics strand of Tehran. Vis quasi-cluster sampling, it is selected two classes of respected high school. One class is selected as control group and other class as experiment group randomly. It is selected 49 students as sample generally.

6. RESEARCH INSTRUMENTATION

For studying respected hypothesis, it is used of math exam for pretest and posttest. In pretest, standard math exam was included of six questions of essential and primary concepts in mathematics at high school level and posttest was seven questions. Questions of posttest indicated the knowledge and perception level of students after traditional and problem solving instruction methods in control and experiment groups. Content reliability of math exam was proved via teachers and professors of mathematics and validity rate of these exams are studied via Split-half test with 75 percent.

7. THE IMPLEMENT METHOD OF RESEARCH

Problem solving method includes of five steps; 1) identify, 2) define, 3) explore, 4) act, and 5) look back. In present research, it is used of cited five steps. Researchers have tried that stated and designed mathematical problem regard to mathematics textbook firstly then cited steps are implemented for each problem. All steps are applied accord to Fig 2.

Step 1: Identify

In this step, learner must understand main concept of respected problem.

Example: Find tangent slope on inverse function; $F(x) = x^3 + x - 2$ in $x = 0$.

Learner has to notice that this problem relates to inverse function.



Step 2: Define

In this step, learner has to find essential information of respected problem and withdraw unessential information.

Learner has to know and find this formula; $(f^{-1})(b) = \frac{1}{f'(a)}$.



Step 3: Explore

In this step, there is not pervious solution and learner has to explore new solution.

Learner must to notice that change place of x and y in inverse function that is he/she has to find

$$x^3 + x - 2 = 0 \rightarrow x = 1.$$



Step 4: Act

In this step, learner can apply the self-solution in respected problem.

After derivation, Learner replaces $x = 1$ in derivative function; $f'(x) = 3x^2 + 1 \rightarrow f'(1) = 4$. Then he/she

replaces the values in formula $m = (f^{-1})(0) = \frac{1}{f'(1)} = \frac{1}{4}$.



Step 5: Look back

In this step, learner can evaluate the self-solution in problem solving process for the accuracy of self-performance.

Learner evaluates or look backs whether her/his replacements was corrected in formula.

Fig 2- The snapshot of problem solving method

Firstly researchers study and evaluate pretests of control and experiment groups and they sure that knowledge level of students is same in both groups. After implementing problem solving method in experiment group and the implement of traditional instruction method in control group for three month, researchers collected resulted data through posttests. Resulted data is studied in two phases; descriptive and deductive statistic.

8. FINDINGS

As it is indicated in Table 1, it clears that mean of posttest of experiment group is more than posttest of control group. Also means of pretests of both groups are same. The lowest Std is related to posttest of control group. In Fig 2, the bar of experiment group's posttest is higher than other bars.

Table 1- The results of descriptive statistics

Groups	N	Mean	Std.deviation
Pretest of control group	24	11.13	2.81
Pretest of experiment group	25	11.48	3.3
Posttest of control group	24	12.04	2.75
Posttest of experiment group	25	13.82	3.51

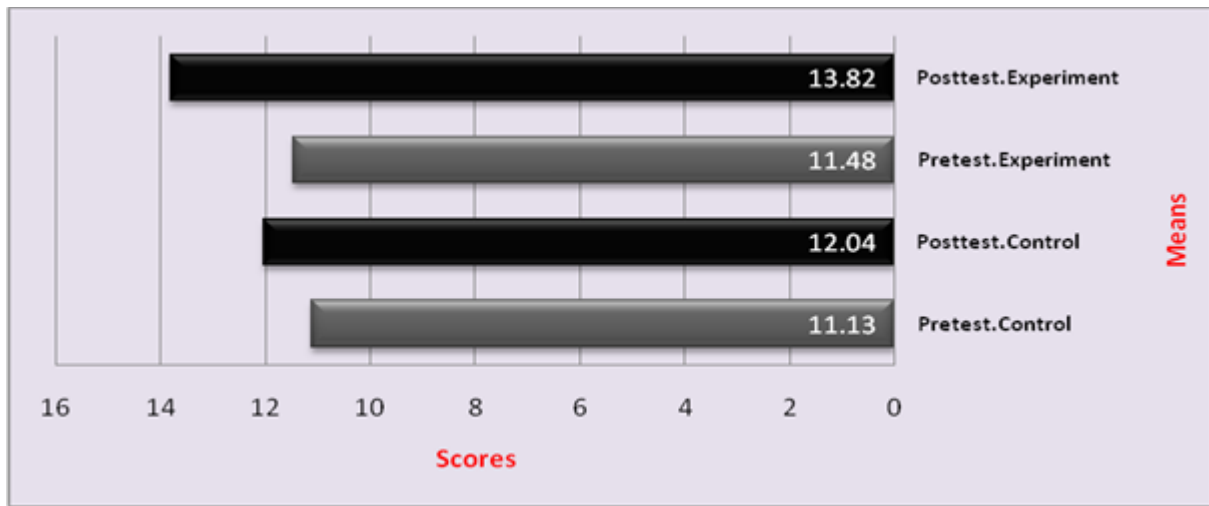


Fig 3- The comparison of means in control and experiment groups

For selecting proper tests, it is used of One-Sample Kolmogorov-Smirnov test firstly. The results of this test indicates that data is normal ($P > 0.05$). Then we can use of parametric test; T-test for the study of means.

Table 2- The results of One-Sample Kolmogorov-Smirnov test

Groups	N	Z	Sig
Pretest of control group	24	0.42	0.99
Pretest of experiment group	25	0.73	0.64
Posttest of control group	24	0.99	0.28
Posttest of experiment group	25	0.39	0.99

The results of Table 3 indicate that variances of pretests equal thereby Levene test ($P>0.05$). Also it clears that means of pretests equal because there are not significant difference between pretests in T-test ($P>0.05$). Therefore there are not differences between knowledge levels of students in pretests.

Table 3- The result of T-test for pretests

Pretests	Levene's Test for Equality of Variances		t-test for Equality of Means						
	F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
								Lower	Upper
Equal variances assumed	0.8	0.37	-0.39	47	0.69	-0.34	0.87	-2.11	1.94
Equal variances not assumed	-	-	-0.39	46	0.69	-0.34	0.87	1.2	1.41

Regard to Table 4, it indicates that variances of posttests equal through Levene test ($P>0.05$). In addition it obvious that means of posttests do not equal that is there are significant difference between posttests in T-test ($P<0.05$). This result shows that problem solving method has more effective on the improvement of mathematical learning than traditional instruction method.

Table 4- The result of T-test for posttests

Posttests	Levene's Test for Equality of Variances		t-test for Equality of Means						
	F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
								Lower	Upper
Equal variances assumed	2.46	0.12	-1.96	47	0.04	-1.77	0.9	-3.59	0.04
Equal variances not assumed	-	-	-1.97	45	0.04	-1.77	0.9	-3.59	0.03

9. CONCLUSION

Problem solving method is a basic skill needed by today's students. Guided by recent research in problem solving method, changing professional standards, new workplace demands, and recent changes in learning theory, educators and teachers are revising curricula to include integrated learning environments which encourage learners to use higher order thinking skills, and in particular, problem solving skills. Learners often learn the facts and rote procedures with few ties to the context and application of knowledge. Problem solving method has become the means to rejoin content and application in a learning environment for basic skills as well as their application in various contexts. Then problem solving methods must apply in educational institutes. There are reasons such as; 1) problem solving method motivates students for learning, 2) students act as creator and will be removed self-fear in moment of force to difficult problem, 3) students can cooperate altogether till solved mathematical problem, 4) misconceptions or misunderstandings will be indicated in problem solving process and teachers or trainers will find these difficulties. Researchers suggest that other researchers must have precaution for using problem solving method because if this method did not implement properly then learners will have not motivation for problem solving. Also next researches can follow the problem solving method at elementary or middle schools. There are limitations such as; this research is limited for mathematics textbook and is limited for boy students.

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