Solving fully fuzzy multiple objective linear programming problems: A new perspective

A. Hadi-Vencheh 1*, Z. Rezaei 2, S. Razipour 3

(1) Department of Mathematics, Isfahan (Khorasgan) Branch, Islamic Azad University, Isfahan, Iran
(2) Department of Mathematics, Mobarakeh Branch, Islamic Azad University, Mobarakeh, Isfahan, Iran
(3) Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran

Abstract
In this paper a systematic process has been proposed to solve a fully fuzzy multi objective linear programming problem (FFMOLPP). Using the utility vector the MOLPP is transferred to a single objective programming and this single fuzzy object problem is simply solved by one of the fuzzy approaches. A numerical example is then given to show applicability of the proposed approach.

Keywords: Fuzzy set theory; Multiple-objective linear programming; fully fuzzy linear system

1 Introduction

Bellman and Zadeh [5] proposed the concept of decision making in fuzzy environment. Dehghan et al. [4] proposed a fuzzy linear programming approach for finding the exact solution of fully fuzzy linear system (FFLS) of equations. Lotfi et al. [2] proposed a method to obtain the approximate solution of FFLP problems. To the best of our knowledge, till now there is no method in the literature to obtain the exact solution of FFLP problems with equality constraints. Kumar et al [1] proposed a method for solving fully fuzzy linear programming problems. In this study, a method for solving a FFMLP is presented. An FFMLP problem with m fuzzy equality constraints and n fuzzy variables may be formulated as follows:

Maximize (Minimize) \( (\bar{C}^r) \in X, \)
subject to \( \bar{A} \otimes \bar{X} = \bar{b}, \quad \bar{X} \geq 0, \)

where \( (\bar{C}^r)^T = [\bar{c}^r_j]_{1 \times n}, \quad \bar{X} = [\bar{x}_j]_{m \times 1}, \quad \bar{A} = [\bar{a}_{ij}]_{m \times n}, \quad \bar{b} = [\bar{b}_i]_{m \times 1}, \quad \bar{a}_{ij}, \bar{b}_i, \bar{c}^r_j, \bar{x}_j \in F(R). \)
2 The Proposed Method

In this section we describe the proposed method. The steps are as follow.

Step 1: The weighting problem of (FFMLP) takes the form:

Maximize \( \min_{k=1}^{m} x \times (C^{r})^{T} \otimes X \)

subject to \( \tilde{X} \geq 0, \sum_{r=1}^{k} w^{r} = 1, w^{r} \geq 0 \).

Step 2: By preference vector approach (FFMLP) is transferred to a single fuzzy object linear problem:

Maximize \( \min_{k=1}^{m} x \times (C^{r})^{T} \otimes X \)

subject to \( \tilde{X} \geq 0 \).

Step 3: Substituting \( (C^{r})^{T} = \tilde{c}^{r} \times n \), \( X = \tilde{x}^{r} \times n \), \( \tilde{A} = [\tilde{a}^{i,j}]_{n \times n} \), \( \tilde{b} = [\tilde{b}^{i}]_{n \times n} \), the above FFLP problem may be written as:

Maximize \( \min_{k=1}^{m} \sum_{j=1}^{n} x \times (\tilde{c}^{r} \times j) \)

subject to \( \sum_{j=1}^{n} \tilde{a}^{i,j} \otimes X = \tilde{b}^{i}, \forall i = 1,2,...,m, \tilde{x} \geq 0 \).

Step 4: If all the parameters \( \tilde{c}_{j}, \tilde{x}_{j}, \tilde{a}_{ij}, \tilde{b}_{i} \) are represented by triangular fuzzy numbers \( p_{j}, q_{j}, r_{j}, x_{j}, y_{j}, z_{j}, a_{ij}, b_{ij} \), \( c_{ij}, g_{i}, h_{i} \) respectively then the FFLP problem, obtained in Step 3, may be written as:

Maximize \( \min_{k=1}^{m} \sum_{j=1}^{n} (p_{j}, q_{j}, r_{j}) \otimes (x_{j}, y_{j}, z_{j}) \),

subject to \( \sum_{j=1}^{n} (a_{ij}, b_{ij}, c_{ij}) \otimes (x_{j}, y_{j}, z_{j}) = (b_{ij}, g_{i}, h_{i}), \forall i = 1,2,...,m, \)

\( (x_{j}, y_{j}, z_{j}) \geq 0 \).

Step 5: Assuming \( (a_{ij}, b_{ij}, c_{ij}) \otimes (x_{j}, y_{j}, z_{j}) = (m_{ij}, n_{ij}, o_{ij}) \) the FFLP problem, obtained in Step 4, may be written as:

Maximize \( \min_{k=1}^{m} \left\{ \sum_{j=1}^{n} (p_{j}, q_{j}, r_{j}) \otimes (x_{j}, y_{j}, z_{j}) \right\} \),

subject to \( \sum_{j=1}^{n} (m_{ij}, n_{ij}, o_{ij}) = (b_{ij}, g_{i}, h_{i}), \forall i = 1,2,...,m, \)

\( (x_{j}, y_{j}, z_{j}) \geq 0 \).

where \( \min \tilde{A} = \frac{1}{4} (a + 2b + c) \) for \( \tilde{A} = (a, b, c) \).
Step 6: The fuzzy linear programming problem, obtained in Step 5, is converted into the following CLP problem:

Maximize \( \sum_{j=1}^{n} (p_j, q_j, r_j) \otimes (x_j, y_j, z_j) \),

subject to \( \sum_{j=1}^{n} m_{ij} = b_i, \quad \forall \ i = 1, 2, ..., m, \)

\( \sum_{j=1}^{n} n_{ij} = g_i, \quad \forall \ i = 1, 2, ..., m, \)

\( \sum_{j=1}^{n} o_{ij} = h_i, \quad \forall \ i = 1, 2, ..., m, \)

\( y_j - x_j \geq 0, \quad z_j - y_j \geq 0, \quad \forall j = 1, 2, ..., n. \)

Step 7: Find the optimal solution \( x_j, y_j, z_j \) by solving the CLP problem obtained in step 6.

Step 8: Find the fuzzy optimal solution by putting the values of \( x_j, y_j, z_j \) in \( \tilde{x}_j = (x_j, y_j, z_j) \).

Step 9: Find the fuzzy optimal value by putting \( \tilde{x}_j \) in \( \sum_{j=1}^{n} \tilde{r}_j^T \otimes \tilde{x}_j \)

3 Numerical Example

In this section a numerical example is given to show applicability of the proposed method. Let us consider the following FFMLP:

Maximize \( \left( (1,4,4) \otimes \tilde{x}_1 + (2,4,8) \otimes \tilde{x}_2 \right) \otimes \left( (0,2,4) \otimes \tilde{x}_1 + (2,6,4) \otimes \tilde{x}_2 \right) \),

subject to \( (2,3,4) \otimes \tilde{x}_1 + (1,2,3) \otimes \tilde{x}_2 = (6,16,30), \)

\( (-1,1,2) \otimes \tilde{x}_1 + (1,3,4) \otimes \tilde{x}_2 = (1,17,30), \quad \tilde{x}_1, \tilde{x}_2 \geq 0. \)

Solution: Let \( w = (.5, .5) \), then given FFMLP problem may be written as:

Maximize \( \left( (5,2,2) \otimes \tilde{x}_1 + (1,2,4) \otimes \tilde{x}_2 \right) \otimes \left( (0,1,2) \otimes \tilde{x}_1 + (1,3,2) \otimes \tilde{x}_2 \right) \)

Maximize \( \left( (5,3,4) \otimes \tilde{x}_1 + (2,5,6) \otimes \tilde{x}_2 \right) \)

subject to \( (2,3,4) \otimes \tilde{x}_1 + (1,2,3) \otimes \tilde{x}_2 = (6,16,30), \)

\( (-1,1,2) \otimes \tilde{x}_1 + (1,3,4) \otimes \tilde{x}_2 = (1,17,30), \quad \tilde{x}_1, \tilde{x}_2 \geq 0. \)

Let \( \tilde{x}_1 = (x_1, y_1, z_1) \) and \( \tilde{x}_2 = (x_2, y_2, z_2) \) then given FFLP problem may be written as:

Maximize \( \left( (5,3,4) \otimes (x_1, y_1, z_1) + (2,5,6) \otimes (x_2, y_2, z_2) \right) \),

subject to \( (2,3,4) \otimes (x_1, y_1, z_1) + (1,2,3) \otimes (x_2, y_2, z_2) = (6,16,30), \)

\( (-1,1,2) \otimes (x_1, y_1, z_1) + (1,3,4) \otimes (x_2, y_2, z_2) = (1,17,30), \)

\( (x_1, y_1, z_1), (x_2, y_2, z_2) \geq 0. \)

Using Step 5, the above FFLP problem may be written as:
Maximize \( 0.5x_1 + 2x_2 + 3y_1 + 5y_2 + 4z_1 + 6z_2 \),
subject to \( 2x_1 + x_2 + 3y_1 + 2y_2 + 4z_1 + 3z_2 = (6, 16, 30), \)
\( -x_1 + x_2 + y_1 + 3y_2 + 2z_1 + 4z_2 = (1, 17, 30), \)
\( (x_1, y_1, z_1), (x_2, y_2, z_2) \geq 0. \)

Using Step 6 of the proposed method the above fuzzy linear programming problem is converted into the following CLP problem
Maximize \( 0.25(0.5x_1 + 2x_2 + 6y_1 + 10y_2 + 4z_1 + 6z_2) \),
subject to \( 2x_1 + x_2 = 6, 3y_1 + 2y_2 = 16, 4z_1 + 3z_2 = 30, \)
\( -x_1 + x_2 = 1, y_1 + 3y_2 = 17, 2z_1 + 4z_2 = 30, \)
\( y_1 - x_1 \geq 0, z_1 - y_1 \geq 0, y_2 - x_2 \geq 0, z_2 - y_2 \geq 0. \)

The optimal solution of the above CLP problems \( x_1 = 1.6667, x_2 = 2.6667, y_1 = 2, y_2 = 5, z_1 = 3, z_2 = 6. \)

Using Step 8, the fuzzy optimal solution is given by \( \tilde{x}_1 = (1.6667, 2.0000, 3.0000), \)
\( \tilde{x}_2 = (2.6667, 5.0000, 6.0000) \). Hence, using Step 9, we have \( \tilde{f}_1 = (7.0001, 28, 60), \tilde{f}_2 = (5.3334, 34, 36) \).

4 Conclusion

In this article a systematic approach for solving fully fuzzy Multi objective program is used. By utility vector approach multi objective optimization problem transferred to a single objective programming and this single fuzzy object problem is simply solved by one of the fuzzy approaches. Finally, an example is illustrated for the proposed algorithm.

References
http://dx.doi.org/10.1016/j.apm.2010.07.037

http://dx.doi.org/10.1016/j.apm.2008.10.020


http://dx.doi.org/10.1016/j.amc.2005.11.124

http://dx.doi.org/10.1287/mnsc.17.4.B141