Evaluating customers’ satisfaction in the industrial company with uncertainty

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Abstract
In this paper, the evaluation of customers’satisfaction is investigated by using group decision making method under uncertainty. Here, we have used the Zadeh’s fuzzy concept to model and measure the satisfaction. For this purpose, we have firstly applied the Delphi method with fuzzy Likert scales. Then, the fuzzy ELECTRE technique was used in order to ordering the index and questionnaires. For determining weights of the index we have applied the Zadeh’s extension principle. On order to show the applicability of the proposed fuzzy approach, we have provided a case study which was applied in Isfahan Gas Company.

Keywords: Zadeh’s extension principle, Customers’ satisfaction, Fuzzy Delphi method, Ranking

1 Introduction

Fuzzy set theory [16] has been applied to many areas which need to manage uncertain and vague data. Such areas include approximate reasoning, decision making, optimization, control and so on, where ranking of fuzzy numbers is an important component of the decision process [15, 17]. A key issue in the fuzzy set theory is how to compare fuzzy numbers. Several methods have been developed for ranking fuzzy numbers [6, 7, 13, 14]. Some of these ranking approaches have compared and reviewed by Bortolan and Degani [8]. Recently, Chen and Hwang [10] reviewed the existing methods for ranking fuzzy numbers. Moreover, Wang and Kerre [24] proposed some axioms as reasonable properties to determine the rationality of a fuzzy number ranking approach and systematically compared a wide array of the existing fuzzy number ranking methods. Almost each approach, however, has pitfalls in some aspect, such as inconsistency with human intuition, indiscrimination, and difficulty of interpretation. So far, none of them is commonly and completely accepted.

Chen [9] proposed the maximizing set and minimizing set method. Recently, Wang and Luo [12], suggested an alternative ranking approach for fuzzy numbers so-called area ranking based on positive and negative ideal points. Also, Wang et. al [11] proposed a novel approach to ranking fuzzy numbers based on the left and right deviation degree. To this end, the maximal and minimal reference sets are defined to measure the deviation degree of fuzzy numbers. Please note that, in all of the existing ranking approaches based on the maximizing and minimizing reference sets, these reference sets are chosen as crisp (real) numbers.

Most companies are trying to attract customers and to make them satisfied with the companies’ products or services.
in order to increase customer loyalty. Among various methods to measure a firm’s competitiveness and marketing performance, customer satisfaction is a most universally accepted measurement [19], as well as an influential performance metric [18]. Many firms attempt to measure customer satisfaction in order to evaluate whether they meet their customers’ needs better than their competitors [20]. Theoretically, it can be assumed that increasing customer satisfaction is more likely to bring positive outcomes such as increasing sales volume and market share. Thus, marketplace outcomes such as sales or market share have become a traditional method of evaluating the success of marketing strategies [21].

2 Preliminaries

We now recall some definitions needed through the paper. The basic definition of fuzzy numbers is given in [3]. By $R$, we denote the set of all real numbers. A fuzzy number is a mapping $u : R \rightarrow [0, 1]$ with the following properties:

(a) $u$ is upper semi-continuous,
(b) $u$ is fuzzy convex, i.e., $u(\lambda x + (1-\lambda)y) \geq \min\{u(x), u(y)\}$ for all $x, y \in R$, $\lambda \in [0, 1]$,
(c) $u$ is normal, i.e., $\exists x_0 \in R$ for which $u(x_0) = 1$,
(d) $\text{supp } u = \{x \in R \mid u(x) > 0\}$ is the support of the $u$, and its closure $\text{cl}(\text{supp } u)$ is compact. Let $E$ be the set of all fuzzy number on $R$. The $r$-level set of a fuzzy number $u \in E$, $0 \leq r \leq 1$, denoted by $[u]_r$, is defined as

$$[u]_r = \begin{cases} \{x \in R \mid u(x) \geq r\} & \text{if } 0 < r \leq 1 \\ \text{cl}(\text{supp } u) & \text{if } r = 0 \end{cases}$$

It is clear that the $r$-level set of a fuzzy number is a closed and bounded interval $[u(r), \pi(r)]$, where $u(r)$ denotes the left-hand endpoint of $[u]_r$, and $\pi(r)$ denotes the right-hand endpoint of $[u]_r$. Since each $y \in R$ can be regarded as a fuzzy number $\tilde{y}$ defined by

$$\tilde{y}(t) = \begin{cases} 1 & \text{if } t = y \\ 0 & \text{if } t \neq y \end{cases}$$

**Definition 2.1.** A fuzzy number $u$ in parametric form is a pair $\langle u, \pi \rangle$ of functions $u(r)$, $\pi(r)$, $0 \leq r \leq 1$, which satisfy the following requirements:

1. $u(r)$ is a bounded non-decreasing left continuous function in $(0, 1]$, and right continuous at 0,
2. $\pi(r)$ is a bounded non-increasing left continuous function in $(0, 1]$, and right continuous at 0,
3. $u(r) \leq \pi(r)$, $0 \leq r \leq 1$.

For arbitrary $u = \langle u(r), \pi(r) \rangle$, $v = \langle \psi(r), \varphi(r) \rangle$ and $k > 0$ we define addition $u + v$, subtraction $u \ominus v$ and scalar multiplication by $k$ as (See [1, 3])

$$u + v = \langle u(r) + \psi(r), \pi(r) + \varphi(r) \rangle, u - v = \langle u(r) - \varphi(r), \pi(r) - \psi(r) \rangle, k \oslash u = \left\{ \begin{array}{ll} (ku, ku) & k \geq 0, \\ (k\pi, k\pi) & k < 0. \end{array} \right.$$  

**Lemma 2.1.** [2]. If $u \in E$ then the following properties hold:

(i) $[u]_{r_1} \subseteq [u]_{r_2}$ if $0 < r_1 \leq r_2 \leq 1$;
(ii) $\{r_n\} \subseteq (0, 1]$ is a nondecreasing sequence which converges to $r$ then $[u]_r = \bigcap_{n \geq 1} [u]_{r_n}$ (i.e, $u'_r$, and $u''_r$ are left-continuous with respect to $r$).

Conversely, if $\{A_r \subseteq \{0, 1\} \}$ is a family of closed real intervals verifying (i) and (ii), then $\{A_r \}$ defined a fuzzy number $u \in E$ such that $[u]_r = A_r$.

The Hausdorff distance between fuzzy numbers given by $d : E \times E \rightarrow R^+ \cup \{0\}$,

$$d(u, v) = \sup_{r \in (0, 1]} \max \left\{ |u(r) - \varphi(r)|, |\pi(r) - \psi(r)| \right\},$$
where \( u = (u(r), \overline{u}(r)) \), \( v = (v(r), \overline{v}(r)) \) \( \subset R \) is utilized in [5]. Then, it is easy to see that \( d \) is a metric in \( E \) and has the following properties (See [4])

1. \( d(u \oplus w, v \oplus w) = d(u, v), \ \forall u, v, w \in E \).
2. \( d(k \odot u, k \odot v) = k \| d(u, v), \ \forall k \in R, u, v \in E \).
3. \( d(u \oplus v, w \oplus e) \leq d(u, w) + d(v, e), \ \forall u, v, w, e \in E \).
4. \((d, E)\) is a complete metric space.

**Definition 2.2.** Let \( x, y \in E \). If there exists \( z \in E \) such that \( x = y + z \), then \( z \) is called the \( H \)-difference of \( x, y \) and it is denoted \( x \ominus y \). In what follows, we fixed \( I = (a, b), \) for \( a, b \in R, a < b \).

### 2.1 The proposed ranking method for triangular fuzzy numbers

Here, we use the following ranking formula for ordering the fuzzy numbers as follows:

Let \( A_i = (a_i, b_i, c_i) \) is a triangular fuzzy number, then we allocate to this fuzzy number the following real number as its value

\[
D(A_i) = \frac{a_i + 2b_i + c_i}{4},
\]

Finally, the ranking of \( n \) fuzzy numbers \( A_1, A_2, \ldots, A_n \) is determined by the following rules for all \( i, j = 1, 2, \ldots, n \):

1. \( A_i \prec A_j \) if and only if \( D(A_i) < D(A_j) \).
2. \( A_i \succ A_j \) if and only if \( D(A_i) > D(A_j) \).
3. \( A_i \approx A_j \) if and only if \( D(A_i) = D(A_j) \).

In this regard, the Wang’s and Kerre’s reasonable properties are investigated for the ordering approach (see[24]).

(A-1) For an arbitrary finite subset \( \mathcal{A} \) of \( S \) and \( u \in \mathcal{A} \); \( u \succeq u \).

(A-2) For an arbitrary finite subset \( \mathcal{A} \) of \( S \) and \( (u, v) \in \mathcal{A}^2 \); \( u \succeq v \) and \( v \succeq u \) by \( D \) on \( \mathcal{A} \), this method should have \( u \sim v \).

(A-3) For an arbitrary finite subset \( \mathcal{A} \) of \( S \) and \( (u, v, w) \in \mathcal{A}^3 \); \( u \succeq v \) and \( v \succeq w \) by \( D \) on \( \mathcal{A} \), this method should have \( u \succeq w \).

(A-4) For an arbitrary finite subset \( \mathcal{A} \) of \( S \) and \( (u, v) \in \mathcal{A}^2 \); \( \inf \supp(u) \geq \sup \supp(v) \), this method should have \( u \succeq v \).

(A-4) For an arbitrary finite subset \( \mathcal{A} \) of \( S \) and \( (u, v) \in \mathcal{A}^2 \); \( \inf \supp(u) > \sup \supp(v) \), this method should have \( u \succ v \).

(A-5) Let \( S, S' \) be two arbitrary finite sets of fuzzy quantities in which \( D \) can be applied and \( u, v \) are in \( S \cap S' \). This method obtain the ranking order \( u \succeq v \) on \( S \) iff \( u \succeq v \) on \( S \).

**Proposition 2.1.** The ranking \( D \) has the above properties.

### 3 Main section

#### 3.1 Delphi method

Here, we just mentioned the steps of Delphi method for the deterministic cases as following:

- Creating a group of analyzers
- Choosing an expert group
- Distributing the questionnaires as the first phase
- Collecting and Analyzing the first questionnaires
- Distributing the questionnaires for the second time
- Collecting and Analyzing the second questionnaires
- Preparing some reports from Delphi process
- Reporting the results to the experts.
3.2 ELECTRE method

As a well-known way for ranking outputs, we recall basic steps as follows:

- Converting decision-making matrix $D$ into non scaled matrix
- Formation of "weighted non scaled" matrix using weights vector related to indexes.
- Identifying concordance and discordance sets for each pair of alternatives. The concordance set for two desired alternatives $j$ and $k$ include all indexes that alternative $k$ is preferred to alternative $j$. Supplement of the above set forms a discordance set.
- Calculating the concordance criterion and concordance matrix.
- Discordance criterion reflects how an alternative could be worse. This standard and its corresponding matrix are obtained similar to the previous step.
- Formation of effective concordance matrix and effective discordance matrix (Boolean matrices).
- Determining the general and effective matrix. Two matrixes of the previous step are obtained through Boolean multiplication of two matrixes and it shows order of the relative priorities of alternatives being obtained by comparing number of zeros or ones in each column (related to one alternative).

3.3 A new fuzzy model

Here, we introduce a novel method to appropriately determine the weights with uncertainty. In fact, for this purpose, we have used the Zadeh’s extension principle to solve the related fuzzy linear programming system.

Let the prioritization of $m$ alternatives are stated as follows:

$$A^{(i)}_1 \succ A^{(i)}_2 \succ A^{(i)}_3 \succ \ldots \succ A^{(i)}_{m-1} \succ A^{(i)}_m.$$  

Then, we should have

$$W_1 \succ W_2 \succ W_3 \succ \ldots \succ W_{m-1} \succ W_m.$$  

Parameter $j$ equal to the highest rank of each inequality in the last relation is used in order to observe the possible intensity of different priorities, i.e.

$$1(D(W_1) - D(W_2)) > 0, 2(D(W_2) - D(W_3)) > 0, \ldots, j(D(W_j) - D(W_{j+1})) > 0, \ldots, (m-1)(D(W_{m-1}) - D(W_m)) > 0.$$  

Finally, in order to appropriately determine the values of $W_j$, we solve the following problem:

$$\max \{ 1(D(W_1) - D(W_2)), 2(D(W_2) - D(W_3)), \ldots, j(D(W_j) - D(W_{j+1})), \ldots, (m-1)(D(W_{m-1}) - D(W_m)) \}$$

s.t. $\sum_{j=1}^{m} W_j = 1.$

To maximize the multiple-objective decision-making (MCDM) model it is just sufficient to maximize the minimum of those purposes that is as below:

$$\max : \quad Z$$

s.t. $\quad Z \leq j(D(W_j^{(i)}) - D(W_{j+1}^{(i)}))$

$$\sum_{j=1}^{m} W_j^{(i)} = 1,$$

$$D(W_j^{(i)}) \geq 0, \quad j = 1, 2, \ldots, m.$$
3.4 The project process and implementation

One of the most important tasks of the public relations and control and planning sector in Isfahan Gas Company is to evaluate customer satisfaction to analyze its performance in the past and represent proper strategies in the future, determine strengths and weaknesses of the organization, encourage and penalize various sections and other managerial policies related to evaluation of customer satisfaction [22]. To do so, designing a questionnaire and implementing suitable weighted coefficients are necessary for the above organization that is accomplished by the following steps.

3.5 Compiling general indexes

In this step, experienced managers were invited and work procedures were determined through Delphi method. Several sessions were held in which nine specialized experts participated and then five general indexes were proposed after selecting the indexes in the above company and conducting studies with the help of research centers to evaluate customer satisfaction and sending such information to experts’ sessions. These indexes are: A1 - services of the officer who reads counters, A2 - informing about security issues, A3 - 24-hour gas relief (telephone number 194), A4 - behavior of employees of the Gas Company, A5 - changing the counter or replacement of the regulator.

3.6 Compiling secondary indexes

The same steps were conducted to determine secondary indexes after preparing general indexes using Delphi method [23].

What are secondary indexes? Due to the difficulty of measuring general indexes, they are divided into several indexes which are called secondary indexes. The most important feature of these indexes is that it can easily be perceive and measure by customers.

After selecting the following secondary indexes were obtained which were agreed upon by experts and managers: Four secondary indexes (B1 to B4) for A1; four secondary indexes (C1 to C4) for A2; five secondary indexes (D1 to D5) for A3; four secondary indexes (E1 to E4) for A4 and three secondary indexes (F1 to F4) for A5. Because secondary indexes should conduct the measurement, they were compiled in a questionnaire containing 20 questions with Likert scale, as presented in Table 1. In fact, we have used the following fuzzy values for this scale:

\[
\tilde{1} = (0, 1, 2), \tilde{2} = (1, 2, 3), \tilde{3} = (2, 3, 4), \tilde{4} = (3, 4, 5), \tilde{5} = (4, 5, 6).
\]

3.7 Ranking of indexes

In this part, we ordering the obtained indexes using the proposed ranking approach. Then, we obtained the following results:

General indexes: \( A_1 \succ A_2 \succ A_3 \succ A_4 \succ A_5 \).

Secondary indexes: \( B_1 \succ B_4 \succ B_3 \succ B_2, C_4 \succ C_1 \succ C_3 \succ C_2, D_2 \succ D_5 \succ D_3 \succ D_4 \succ D_1, E_3 \succ E_2 \succ E_4 \succ E_1, F_1 \approx F_2 \approx F_3 \).

Table 1: Fuzzy Likert scale

<table>
<thead>
<tr>
<th>Very weak</th>
<th>Weak</th>
<th>Moderate</th>
<th>Good</th>
<th>Very Good</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tilde{1} )</td>
<td>( \tilde{2} )</td>
<td>( \tilde{3} )</td>
<td>( \tilde{4} )</td>
<td>( \tilde{5} )</td>
</tr>
</tbody>
</table>

3.8 Value of the weights

Results of determining cardinal weights of indexes using mathematical model of linear programming and LINDO software presented in Table 2.
Then, for evaluating each general index (for example for $A_1$), we consider the weighted sum for each parameter as follows:

$$W_{A_1} = 0.4560 \quad W_{A_2} = 0.2560 \quad W_{A_3} = 0.09 \quad W_{A_4} = 0.1560 \quad W_{A_5} = 0.0400 \quad Z = 0.2000$$

$$W_{B_1} = 0.5209 \quad W_{B_2} = 0.0625 \quad W_{B_3} = 0.1458 \quad W_{B_4} = 0.2708 \quad Z = 0.2500$$

$$W_{C_1} = 0.2708 \quad W_{C_2} = 0.0625 \quad W_{C_3} = 0.1458 \quad W_{C_4} = 0.0520 \quad Z = 0.2500$$

$$W_{D_1} = 0.0400 \quad W_{D_2} = 0.4560 \quad W_{D_3} = 0.1560 \quad W_{D_4} = 0.0900 \quad W_{D_5} = 0.2560 \quad Z = 0.2$$

$$W_{E_1} = 0.0625 \quad W_{E_2} = 0.2708 \quad W_{E_3} = 0.5209 \quad W_{E_4} = 0.1458 \quad Z = 0.2500$$

$$W_{F_1} = 0.3330 \quad W_{F_2} = 0.3330 \quad W_{F_3} = 0.3330 \quad Z = 0.3330$$

### 3.9 Sample size

The following formula was applied to determine sample size:

$$n = \frac{Z^2 \theta^2}{\epsilon^2}.$$  

(3.2)

Indeed, questionnaires are distributed in $n$ numbers and are collected when sampling method is identified.

### 3.10 Classification of the obtained results form questionnaires

For the Isfahan Gas Company different $N_j$ show given responses to each case. For instance, counting for the general index $A_1$ are duplicated in Table 3 which were obtained using Zadeh’s extension principle in the triangular fuzzy numbers form.

### Table 3: Fuzzy general index for $A_1$

<table>
<thead>
<tr>
<th>Secondary index</th>
<th>Very weak</th>
<th>Weak</th>
<th>Moderate</th>
<th>Good</th>
<th>Very good</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_1$</td>
<td>$\tilde{N}_1$</td>
<td>$\tilde{N}_2$</td>
<td>$\tilde{N}_3$</td>
<td>$\tilde{N}_4$</td>
<td>$\tilde{N}_5$</td>
</tr>
<tr>
<td>$B_2$</td>
<td>$\tilde{N}_6$</td>
<td>$\tilde{N}_7$</td>
<td>$\tilde{N}_8$</td>
<td>$\tilde{N}_9$</td>
<td>$\tilde{N}_{10}$</td>
</tr>
<tr>
<td>$B_3$</td>
<td>$\tilde{N}_{11}$</td>
<td>$\tilde{N}_{12}$</td>
<td>$\tilde{N}_{13}$</td>
<td>$\tilde{N}_{14}$</td>
<td>$\tilde{N}_{15}$</td>
</tr>
<tr>
<td>$B_4$</td>
<td>$\tilde{N}_{16}$</td>
<td>$\tilde{N}_{17}$</td>
<td>$\tilde{N}_{18}$</td>
<td>$\tilde{N}_{19}$</td>
<td>$\tilde{N}_{20}$</td>
</tr>
</tbody>
</table>

### 3.11 A new aggregate formula for Customers’ satisfaction

In this part, we propose a novel approach for aggregating the results, as new formula for measuring the customers’ satisfaction. For this purpose, we consider the weighted sum for each parameter as follows:

$$\tilde{H}_{B_1} = \tilde{1} \odot \tilde{N}_{1} \oplus \tilde{2} \odot \tilde{N}_{2} \oplus \tilde{3} \odot \tilde{N}_{3} \oplus \tilde{4} \odot \tilde{N}_{4} \oplus \tilde{5} \odot \tilde{N}_{5}$$  

(3.3)

$$\tilde{H}_{B_2} = \tilde{1} \odot \tilde{N}_{6} \oplus \tilde{2} \odot \tilde{N}_{7} \oplus \tilde{3} \odot \tilde{N}_{8} \oplus \tilde{4} \odot \tilde{N}_{9} \oplus \tilde{5} \odot \tilde{N}_{10}$$  

(3.4)

$$\tilde{H}_{B_3} = \tilde{1} \odot \tilde{N}_{11} \oplus \tilde{2} \odot \tilde{N}_{12} \oplus \tilde{3} \odot \tilde{N}_{13} \oplus \tilde{4} \odot \tilde{N}_{14} \oplus \tilde{5} \odot \tilde{N}_{15}$$  

(3.5)

$$\tilde{H}_{B_4} = \tilde{1} \odot \tilde{N}_{16} \oplus \tilde{2} \odot \tilde{N}_{17} \oplus \tilde{3} \odot \tilde{N}_{18} \oplus \tilde{4} \odot \tilde{N}_{19} \oplus \tilde{5} \odot \tilde{N}_{20}.$$  

(3.6)

Then, for evaluating each general index (for example for $A_1$), we have:
\[ H_1 = W_B_1 \oplus W_B_2 \oplus W_B_3 \oplus W_B_4 . \]  

Finally, we use the following aggregate function for collecting the results as follows:
\[ H_{\text{final}} = W_A_1 \oplus W_A_2 \oplus W_A_3 \oplus W_A_4 . \]  

4 Conclusion

Customer satisfaction has been investigated as one of the most valuable parameters in the evaluating of marketing strategies. This paper was conducted in order to propose a suitable model to evaluate customer satisfaction. To do so, we employed group decision-making techniques in Isfaham Gas Company for our purpose. The proposed model is simple and easy to understand. Generally, group decision-making techniques are usable with linear programming as well as ordinal and cardinal prioritization of indexes in similar cases.

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