Journal of Soft Computing and Applications 2013 (2013) 1-14

Available online at www.ispacs.com/jsca
Volume 2013, Year 2013 Article ID jsca-00016, 14 Pages
doi:10.5899/2013/jsca-00016
Research Article

Journal of Soft Computing and Applications

Failure mode and effects analysis: A fuzzy group MCDM approach

A. Hadi-Vencheh[1*], M. Aghajani[2]

(1) Department of Mathematics, Mobarakeh Branch, Islamic Azad University, Mobarakeh, Isfahan, Iran
(2) Department of Industrial Management, Mobarakeh Branch, Islamic Azad University, Mobarakeh, Isfahan, Iran

Abstract

In this paper, a new fuzzy group decision making (FGDM) model based on α -level sets, is proposed to generate, more accurate fuzzy using, risk priority numbers (RPNs) and ensure to be robust against the uncertainty. This model allows decision makers (DMs) to evaluate FMEA risk factors using linguistic terms rather than precise numerical values, allows them to express their opinions independently. A case study is investigated using the proposed model to illustrate its applications in RPN assessment.

Keywords: Fuzzy decision making; α -cut; Failure mode and effects analysis

1 Introduction

The main objective of FMEA is to discover and prioritize the potential failure modes by computing risk priority numbers (RPNs), which is a product of the risk factors occurrence (O), severity (S) and detection (D) [6]. Occurrence and severity are the frequency and seriousness (effects) of the failure and detection is the ability to detect the failure before it reaches the customer. The three risk factors are evaluated using the ratings (also called ranks or scores) from 1 to 10. Generally, the higher the RPN of a failure mode, the more important degree it should be assigned. With respect to the scores of RPNs, the failure modes can be ranked and then proper actions will be preferentially taken on the high-risk failure modes. However, the RPN has been criticized for a variety of reasons, some of which are listed as follow: Firstly, different combinations of O, S and D may produce exactly the same value of RPN, but their hidden risk implications may be totally different. For example, two different events with the values of 2, 3, 2 and 4, 1, 3 for O, S and D, respectively, have the same RPN value of 12. However, the hidden risk implications of the two events may not necessarily be the same. This may cause a waste of resources and time, and in some cases a high-risk event may go noticed. Secondly, the relative importance among O, S and D is not taken into consideration. The three risk factors are assumed to be equally important. This may not be the case when considering a practical application of FMEA. To overcome the drawbacks listed above, a number of approaches have been suggested in the literature. For example, Bevilacqua et al. [2] defined

* Corresponding Author. Email address: ahadi@khuisf.ac.ir

RPN as the weighted sum of six parameters (safety, machine importance for the process, maintenance costs, failure frequency, downtime length and operating conditions) multiplied by a seventh factor (machine access difficulty), where the relative importance of the six attributes was estimated using pair-wise comparisons.

So far numerous fuzzy methods have been proposed for raking risk factors in FMEA [3-13]. Generally, evaluators can use fuzzy concepts to express linguistic terms by real-life, everyday language in order to evaluate risk factors for each one of failure modes. Connecting these terms to appropriate membership functions, better and precise analysis are determined for item scores. With these considerations, this study aims to present a simple model on the basis of fuzzy logic.

Furthermore, two other gaps must be deeply considered in the evaluation of risk factors and the final ranking of failure modes using fuzzy RPNs.

1. FMEA is generally performed by a cross-functional team, which requires decision makers (DMs) from different fields [14]. Considering their personal backgrounds, preferences and different understanding levels to the failure modes, DMs may use different linguistic term sets to express their own judgments on evaluating O, S, D, and importance weights of the three factors [15]. These multi-granularity linguistic assessments must be aggregated for reaching a better group consensus.

2. The mathematical expressions adopted for calculating the fuzzy RPNs is strongly sensitive to variations in the evaluating process, which cannot ensure the ranking results to be robust against the uncertain environment [16].

In this study, a new fuzzy TOPSIS model based on α -level sets is proposed for FMEA under uncertainty. Average risk factors and the α -level sets of the above averaged ratings and the weights are fuzzy numbers. The relative importance weights of O, S and D are also considered in the programming model. As a result, several failures having an identical value of RPN that derives from different combinations of O, S and D can be easily discriminated.

The paper is arranged as follows. The preliminaries are described in Section 2. Section 3 proposes a new model for evaluating and ranking the failure modes with use fuzzy TOPSIS method based on α -level sets. In Section 4, an application is used to illustrate the proposed model. Conclusions are then presented in Section 5.

2 Preliminaries

Let X be the universe of discourse. A fuzzy set $\tilde{A}$ of the universe of discourse X is said to be normal if there exists a $x_i \in X$ satisfying $\mu_{\tilde{A}}(x_i) = 1$

Fuzzy sets can also be represented by intervals, which are called $\alpha - level$ sets or $\alpha - cuts$. The $\alpha - level$ sets A_α of a fuzzy set $\tilde{A}$ are defined as

$$A_\alpha = \left\{ x \in X \mid \mu_{\tilde{A}}(x) \ge \alpha \right\} = \left[\min\left\{ x \in X \mid \mu_{\tilde{A}}(x) \ge \alpha \right\}, \max\left\{ x \in X \mid \mu_{\tilde{A}}(x) \ge \alpha \right\} \right] \tag{1}$$

According to Zadeh's extension principle (Zadeh, 1965), the fuzzy set $\tilde{A}$ can be expressed as

$$\tilde{A} = \bigcup_\alpha \alpha A_\alpha, \qquad 0 \le \alpha \le 1 \tag{2}$$

Fuzzy numbers are special cases of fuzzy sets that are both convex and normal. A fuzzy number is a convex fuzzy set, characterized by a given interval of real numbers, each with a grade of membership between 0 and 1. Its membership function is piecewise continuous and satisfies the following conditions:

(a) $\mu_{\tilde{A}}(x) = 0$ for each $x \notin [a,d]$;

(b) $\mu_{\tilde{A}}(x)$ is non-decreasing (monotonic increasing) on [a, b] and non-increasing (monotonic decreasing) on [c, d];

(c) $\mu_{\tilde{A}}(x) = 1$ for each $x \in [b,c]$,

where $a \le b \le c \le d$ are real numbers in the real line $R = (-\infty, +\infty)$.

The most commonly used fuzzy numbers are triangular and trapezoidal fuzzy numbers, whose membership functions are respectively defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} (x-a)/(b-a), & a \le x \le b, \\ (d-x)/(d-b), & b \le x \le d, \\ 0 & otherwise. \end{cases} \tag{3}$$

$$\mu_{\tilde{A}}(x) = \begin{cases} (x-a)/(b-a), & a \le x \le b, \\ 1, & b \le x \le d, \\ (d-x)/(d-c), & c \le x \le d, \\ 0 & otherwise. \end{cases} \tag{4}$$

For brevity, triangular and trapezoidal fuzzy numbers are often denoted as (a,b,c) and (a,b,c,d). It is obvious that triangular fuzzy numbers are special cases of trapezoidal fuzzy numbers with b = c.

Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be two positive triangular fuzzy numbers. Then basic fuzzy arithmetic operations on these fuzzy numbers are defined as (Dubois & Prade, 1980; Kauffman & Gupta, 1991)

Addition:

$$\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3); \tag{5}$$

Subtraction:

$$\tilde{A} - \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1); \tag{6}$$

Multiplication:

$$\tilde{A} \times \tilde{B} \approx (a_1 b_1, a_2 b_2, a_3 b_3); \tag{7}$$

Division:

$$\tilde{A} \div \tilde{B} \approx (\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_3}). \tag{8}$$

In fuzzy MCDM problems, criteria/attribute values and the relative weights are usually characterized by fuzzy numbers. A_α is referred to as abstractly α -level sets or α -cuts of the fuzzy number/set $\tilde{A}$. In order that the TOPSIS method can also be used to deal with fuzzy MCDM problems, several extensions have been suggested. The simplest extension is to change a fuzzy MCDM problem into a crisp one via defuzzification. This way, however, can cause some information lost and only gives a crisp point estimate for the relative closeness of each alternative. An important concept related to the applications of fuzzy numbers is defuzzification, which converts a fuzzy number into a crisp value. Such a transformation is not unique because different methods are possible. The most commonly used defuzzification method is the centroid defuzzification method, which is also known as center of gravity or center of area defuzzification. The centroid defuzzification method can be expressed as follows (Yager, 1981):

$$\bar{x}_0(\tilde{A}) = \frac{\int_a^d x\,\mu_{\tilde{A}}(x)dx}{\int_a^d \mu_{\tilde{A}}(x)dx}, \tag{9}$$

where $\bar{x}_0(\tilde{A})$ is the defuzzified value.

The fuzzy weighted average of fuzzy numbers is referred to as fuzzy weighted average (FWA), which is defined as

$$\tilde{y} = \frac{\tilde{w}_1\tilde{x}_1 + \tilde{w}_2\tilde{x}_2 + \ldots + \tilde{w}_n\tilde{x}_n}{\tilde{w}_1 + \ldots + \tilde{w}_n}, \tag{10}$$

where $\tilde{x}_1, \ldots \tilde{x}_n$ are n fuzzy numbers to be weighted and $\tilde{w}_1, \ldots, \tilde{w}_n$ are fuzzy weights. Fuzzy arithmetic operations are found not suitable for computing $\tilde{y}$ because the weight variables appear in both denominator and numerator simultaneously. Lots of research has been done on how to compute $\tilde{y}$. The most commonly used approach is to calculate $\tilde{y}$ using the extension principle. Let $x_{i\alpha} = [x^L_{i\alpha}, x^U_{i\alpha}]$, $w_{i\alpha} = [w^L_{i\alpha}, w^U_{i\alpha}]$ and $y_\alpha = [y^L_\alpha, x^U_\alpha]$ be the $\alpha - level$ sets of $\tilde{x}_i$, $\tilde{w}_i$ and $\tilde{y}$,respectively. Then $y_\alpha = [y^L_\alpha, x^U_\alpha]$ can be derived by the following pair of fractional programming models:

$$\begin{aligned} y^L_\alpha = Min\, & \frac{w_1x^L_{1\alpha} + w_2x^L_{2\alpha} + \ldots + w_nx^L_{n\alpha}}{w_1 + w_2 + \ldots + w_n} \\ s.t.\quad & w^L_{i\alpha} \le w_i \le w^U_{i\alpha}, i = 1, \ldots, n. \end{aligned} \tag{11}$$

$$\begin{aligned} y^U_\alpha = Max\, & \frac{w_1x^U_{1\alpha} + w_2x^U_{2\alpha} + \ldots + w_nx^U_{n\alpha}}{w_1 + w_2 + \ldots + w_n} \\ s.t.\quad & w^L_{i\alpha} \le w_i \le w^U_{i\alpha}, i = 1, \ldots, n. \end{aligned} \tag{12}$$

Let

$$\begin{aligned} & z = 1/(w_1 + w_2 + \ldots + w_n), \\ & v_i = zw_i, \quad i = 1, \ldots, n. \end{aligned} \tag{13}$$

The above fractional programming models can be simplified as (Kao & Liu, 2001)

$$\begin{aligned} y^L_\alpha = Min\quad & v_1x^L_{1\alpha} + v_2x^L_{2\alpha} + \ldots + v_nx^L_{n\alpha} \\ s.t.\quad & v_1 + v_2 + \ldots + v_n = 1, \\ & z.w^L_{i\alpha} \le v_i \le z.w^U_{i\alpha}, \quad i = 1, \ldots, n. \\ & z \ge 0. \end{aligned} \tag{14}$$

$$\begin{aligned} y^U_\alpha = Max\quad & v_1x^U_{1\alpha} + v_2x^U_{2\alpha} + \ldots + v_nx^U_{n\alpha} \\ s.t.\quad & v_1 + v_2 + \ldots + v_n = 1, \\ & z.w^L_{i\alpha} \le v_i \le z.w^U_{i\alpha}, \quad i = 1, \ldots, n. \\ & z \ge 0. \end{aligned} \tag{15}$$

These are linear programming models and are easy to solve using MS Excel Solver or LINDO software package.

3 The proposed FGDM model for FMEA

Suppose there is m risk events and k decision maker. According to RPN formula, we have:

$$R_i^k = OR_i^k \times SR_i^k \times DR_i^k, \quad i = 1,\ldots,n; k = 1,\ldots,m \tag{16}$$

Each of risk factors is a fuzzy number. For each defined risk event, its O, S and D need to be assessed on the basis of evidence and engineering judgment. This can usually be done by a team of experts or called decision makers (DMs) in the terminology of decision analysis. It turns out to be not easy, if not impossible, to require the DMs to provide precise numerical judgments about the factor risk of each risk event. So, fuzzy linguistic terms are much easier to be accepted and adopted by the DMs.

To determine the overall risk of the risk event, the relative importance weights of each risk factor need to be specified. This can be done by directly assigning a crisp or fuzzy weight or linguistic term to each factor. Thus let $w_j^k, j = 1,\ldots,m$ are assigned weights by DMs that are fuzzy numbers.

Mean rate of each risk factor is according with mean of fuzzy number and are measured as below:

$$\bar{OR}_i = \frac{1}{m}\sum_{k=1}^{m} OR_i^k = (\frac{1}{m}\sum_{k=1}^{m} OR_{iL}^k, \frac{1}{m}\sum_{k=1}^{m} OR_{iM}^k, \frac{1}{m}\sum_{k=1}^{m} OR_{iU}^k), i = 1,\ldots,n \tag{17}$$

$$\bar{SR}_i = \frac{1}{m}\sum_{k=1}^{m} SR_i^k = (\frac{1}{m}\sum_{k=1}^{m} SR_{iL}^k, \frac{1}{m}\sum_{k=1}^{m} SR_{iM}^k, \frac{1}{m}\sum_{k=1}^{m} SR_{iU}^k), i = 1,\ldots,n \tag{18}$$

$$\bar{DR}_i = \frac{1}{m}\sum_{k=1}^{m} DR_i^k = (\frac{1}{m}\sum_{k=1}^{m} DR_{iL}^k, \frac{1}{m}\sum_{k=1}^{m} DR_{iM}^k, \frac{1}{m}\sum_{k=1}^{m} DR_{iU}^k), i = 1,\ldots,n \tag{19}$$

$$\bar{w}_j = \frac{1}{m}\sum_{k=1}^{m} w_j^k = (\frac{1}{m}\sum_{k=1}^{m} w_{jL}^k, \frac{1}{m}\sum_{k=1}^{m} w_{jM}^k, \frac{1}{m}\sum_{k=1}^{m} w_{jU}^k), j = 1,\ldots,m \tag{20}$$

We can consider each risk factor as below:

$$OR_i^k = (OR_{iL}^k, OR_{iM}^k, OR_{iU}^k) \tag{21}$$

$$SR_i^k = (SR_{iL}^k, SR_{iM}^k, SR_{iU}^k) \tag{22}$$

$$DR_i^k = (DR_{iL}^k, DR_{iM}^k, DR_{iU}^k) \tag{23}$$

$$w_j^k = (w_{jL}^k, w_{jM}^k, w_{jU}^k) \tag{24}$$

Let $\tilde{X} = (\tilde{x}_{ij})_{n\times m}$

be a fuzzy decision matrix characterized by membership functions $\mu_{\tilde{x}_{ij}}(x)(i=1,...,n,\ j=1,...,m)$ and $\tilde{W}=(\tilde{w}_1,...,\tilde{w}_m)$ be fuzzy weights characterized by $\mu_{\tilde{w}_j}(x)(j=1,...,m)$ If all the criteria/attributes, $C_1,...,C_m$, are assessed using the same set of fuzzy linguistic variables, then the fuzzy decision matrix $\tilde{X}$ is of the same dimension and therefore needs no normalization. Otherwise, $\tilde{X}$ has to be normalized. Suppose $\tilde{x}_{ij}=(OR_{ij},SR_{ij},DR_{ij})(i=1,...,n,\ j=1,...,m)$ are triangular fuzzy numbers, then normalization process can be conducted by

$$\tilde{r}_{ij}=\left(\frac{OR_{ij}}{Dr_j^*},\frac{SR_{ij}}{DR_j^*},\frac{DR_{ij}}{DR_j^*}\right), i=1,...,n;\ j\in\Omega_b \tag{25}$$

and

$$\tilde{r}_{ij}=\left(\frac{OR_j^-}{DR_{ij}},\frac{OR_j^-}{SR_{ij}},\frac{OR_j^-}{OR_{ij}}\right), i=1,...,n;\ j\in\Omega_c \tag{26}$$

where

$$DR_j^*=\max_i DR_{ij},\ j\in\Omega_b$$

and

$$OR_j^-=\min_i OR_{ij},\ j\in\Omega_c$$

Normalized $\tilde{r}_{ij}$ are still triangular fuzzy numbers. For trapezoidal fuzzy numbers, the normalization process can be conducted in the same way. It is obvious that the normalized criteria/attribute values/ratings $\tilde{r}_{ij}$ are between zero and one. So, the ideal solution can be defined as $A^*=\{1,...,1\}$ .As such, the negative ideal solution can be defined as $A^-=\{0,...,0\}$. Note that if there is no need to normalize the fuzzy decision matrix $\tilde{X}=(\tilde{x}_{ij})_{n\times m}$, then the ideal and the negative ideal solutions can be respectively defined as

$$A^*=\{x_1^*,...,x_m^*\}=\{(\max_j d_{ij}\mid j\in\Omega_b),(\min_j a_{ij}\mid j\in\Omega_c)\} \tag{27}$$

$$A^-=\{x_1^-,...,x_m^-\}=\{(\min_j a_{ij}\mid j\in\Omega_b),(\max_j d_{ij}\mid j\in\Omega_c)\} \tag{28}$$

Let $(r_{ij})_\alpha=[(r_{ij})_\alpha^l,(r_{ij})_\alpha^u]$ and $(w_j)_\alpha=[(w_j)_\alpha^l,(w_j)_\alpha^u]$ be α-level sets of $\tilde{r}_{ij}$ and $\tilde{w}_j$, respectively. Then Eq. (6) can be equivalently rewritten as

$$RC_i=\frac{\sqrt{\sum_{j=1}^m (w_j r_{ij})^2}}{\sqrt{\sum_{j=1}^m (w_j r_{ij})^2}+\sqrt{\sum_{j=1}^m (w_j(r_{ij}-1))^2}}, i=1,...,n, \tag{29}$$

Where

$$(w_j)_\alpha^L\le w_j\le (w_j)_\alpha^U,\qquad j=1,...,m,$$

$$(r_{ij})_\alpha^L\le r_{ij}\le (r_{ij})_\alpha^U,\qquad j=1,...,m,$$

Obviously, RC_i is an interval in this situation, whose lower and upper bounds can be captured by the following pair of fractional programming models (Wang, 2006):

$$(RC_i)_\alpha^L = Min \frac{\sqrt{\sum_{j=1}^{m}(w_j r_{ij})^2}}{\sqrt{\sum_{j=1}^{m}(w_j r_{ij})^2} + \sqrt{\sum_{j=1}^{m}(w_j (r_{ij}-1))^2}}; \tag{30}$$

s.t. $(w_j)_\alpha^L \le w_j \le (w_j)_\alpha^U, \quad j=1,\dots,m,$

$(r_{ij})_\alpha^L \le r_{ij} \le (r_{ij})_\alpha^U, \quad j=1,\dots,m,$

$$(RC_i)_\alpha^U = Max \frac{\sqrt{\sum_{j=1}^{m}(w_j r_{ij})^2}}{\sqrt{\sum_{j=1}^{m}(w_j r_{ij})^2} + \sqrt{\sum_{j=1}^{m}(w_j (r_{ij}-1))^2}} \tag{31}$$

s.t. $(w_j)_\alpha^L \le w_j \le (w_j)_\alpha^U, \quad j=1,\dots,m,$

$(r_{ij})_\alpha^L \le r_{ij} \le (r_{ij})_\alpha^U, \quad j=1,\dots,m,$

RC_i is therefore a monotonically increasing function of $r_{ij}\,(j=1,\dots,m)$, which means RC_i reaches its maximum at $r_{ij} = (r_{ij})_\alpha^U$ and arrives at its minimum when $r_{ij} = (r_{ij})_\alpha^L$. So, the above pair of fractional programming models can be simplified as

$$(RC_i)_\alpha^L = Min \frac{\sqrt{\sum_{j=1}^{m}(w_j (r_{ij})_\alpha^L)^2}}{\sqrt{\sum_{j=1}^{m}(w_j (r_{ij})_\alpha^L)^2} + \sqrt{\sum_{j=1}^{m}(w_j ((r_{ij})_\alpha^L-1))^2}}; \tag{32}$$

s.t. $(w_j)_\alpha^L \le w_j \le (w_j)_\alpha^U, \quad j=1,\dots,m,$

$$(RC_i)_\alpha^U = Max \frac{\sqrt{\sum_{j=1}^{m}(w_j (r_{ij})_\alpha^U)^2}}{\sqrt{\sum_{j=1}^{m}(w_j (r_{ij})_\alpha^U)^2} + \sqrt{\sum_{j=1}^{m}(w_j ((r_{ij})_\alpha^U-1))^2}} \tag{33}$$

s.t. $(w_j)_\alpha^L \le w_j \le (w_j)_\alpha^U, \quad j=1,\dots,m,$

Although this is a pair of non-linear programming (NLP) models, it can be solved using Microsoft Excel Solver or LINGO software package without striking a blowing because their constraints are all linear.

Note that in the case of no normalization that needs to be carried out, the relative closeness RC_i should be determined by the following NLP models:

$$(RC_i)_\alpha^L = Min \frac{\sqrt{\sum_{j=1}^{m}(w_j((x_{ij})_\alpha^L - x_j^-))^2}}{\sqrt{\sum_{j=1}^{m}(w_j((x_{ij})_\alpha^L - x_j^-))^2} + \sqrt{\sum_{j=1}^{m}(w_j((x_{ij})_\alpha^L - x_j^*))^2}}; \quad (34)$$

s.t. $(w_j)_\alpha^L \le w_j \le (w_j)_\alpha^U, \quad j = 1,\dots,m,$

$$(RC_i)_\alpha^U = Max \frac{\sqrt{\sum_{j=1}^{m}(w_j((x_{ij})_\alpha^U - x_j^-))^2}}{\sqrt{\sum_{j=1}^{m}(w_j((x_{ij})_\alpha^U - x_j^-))^2} + \sqrt{\sum_{j=1}^{m}(w_j((r_{ij})_\alpha^U - x_j^*))^2}} \quad (35)$$

s.t. $(w_j)_\alpha^L \le w_j \le (w_j)_\alpha^U, \quad j = 1,\dots,m,$

where $(x_{ij})_\alpha = [(x_{ij})_\alpha^L, (x_{ij})_\alpha^U]$ are the α-level sets of $\tilde{x}_{ij}\,(i = 1,\dots,n;\ j = 1,\dots,m)$, and x_j^* and x_j^- are the ideal and negative ideal solutions determined by Eqs. (27) and (28), respectively.
By setting different α levels, different α -level sets $(RC_i)_\alpha = [(RC_i)_\alpha^L, (RC_i)_\alpha^U]$ can be generated by solving the above pair of NLP models (32) and (33) or (34) and (35). According to the extension principle (Zimmermann, 1991), $R\tilde{C}_i$ can finally be expressed as

$$R\tilde{C}_i = \bigcup_\alpha \alpha (RC_i)_\alpha = \bigcup_\alpha \alpha [(RC_i)_\alpha^L, (RC_i)_\alpha^U], \quad 0 < \alpha \le 1 \quad (36)$$

For n alternatives, we have n fuzzy relative closenesses, which are all expressed by their α-level sets and are usually no longer triangular or trapezoidal fuzzy numbers. In order to select a best alternative or rank the n alternatives, these fuzzy relative closenesses need to be defuzzified. The averaging level cuts (ALC) suggested by Dubios and Parade (see Oussalah, 2002 for discussions) is perhaps the simplest defuzzification method based on α -level sets and is therefore utilized in this paper. Let $\alpha_1 < \dots < \alpha_N$ be different α -level satisfying $0 < \alpha_1 < \dots < \alpha_N = 1$. Then the defuzzified values of $R\tilde{C}_i$ can be determined by

$$(RC_i)_{ALC}^* = \frac{1}{N}\sum_{j=1}^{N}\left(\frac{(RC_i)_{\alpha_j}^L + (RC_i)_{\alpha_j}^U}{2}\right), \quad i = 1,\dots,n \quad (37)$$

As a summary, the fuzzy TOPSIS method based on alpha level sets can be summed up as follows:
* Normalize fuzzy decision matrix $\tilde{X} = (\tilde{x}_{ij})_{n\times m}$ by Eqs. (25) and (26) if necessary.

* Determine the ideal solution and the negative ideal solution by Eqs. (27) and (28) if necessary.
* Calculate the α -level sets of $\tilde{r}_{ij}$ or $\tilde{x}_{ij}$ $(i = 1,...,n;\ j = 1,...,m)$ by setting different α levels.
* Compute the fuzzy relative closeness of each alternative by solving the NLP models (40) and (41) or (34) and (35) for each alpha level.
* Defuzzify the fuzzy relative closeness by Eq. (37)
* Rank alternatives in terms of their defuzzified relative closenesses.

4 Application

In a small ship, four systems have been studied. In one of these systems five failures items have been known that these systems are Contamination, Generator fail, Gearbox seizure, Shaft seizure and loss of pressure. For every failure item, three risk factors have been expressed by a team of three experts. Linguistic terms and their fuzzy numbers used for evaluating the risk factors that can be described as follows.

Tables 1, 2 and 3 show the linguistic terms and their fuzzy numbers used for evaluating the risk factors. These linguistic terms are perfectly consistent with those defined by the traditional FMEA, but they are treated as triangular fuzzy numbers in this paper rather than precise numerical values. It is most desirable that the DMs achieve a consensus on these definitions. If they disagree with each other, then average values should be used for the definitions.

The three DMs express their opinions on the risk factors of every failure items that results have showed in table 4. Tables 5 show the original assessment information provided by the three DMs, where aggregated fuzzy numbers are obtained by averaging the fuzzy opinions of the three DMs. This matrix has been normalized and shows corresponding ideal and negative ideal solutions for every risk factors too. In Table 6 α-cut of risk factors for item1 have been showed. Table 7 shows the relative importance weights and the rating given by the kth DM.

All the NLP models determined by (40) and (41) are implemented in MS-Excel worksheets and are solved using the WinQSB software. The results are presented in Table 9. The defuzzified values computed by Eq. (45) are which give the ranking of A5 $\succ$ A4 $\succ$ A3 $\succ$ A2 $\succ$ A1, where the symbol $\succ$ means “is superior or preferred to”.

Table 1: Fuzzy rating for occurrence of a failure

Rating	Probability of occurrence	Fuzzy number
Very high (VH)	Failure is almost inevitable	(8, 9, 10)
High (H)	Repeated failures	(6, 7, 9)
Moderate (M)	Occasional failures	(3, 4, 7)
Low (L)	Relatively few failures	(1, 2, 4)
Remote (R)	Failure is unlikely	(1, 1, 2)

Table 2: Fuzzy rating for severity of a failure

Rating number	Severity if effect	Fuzzy
Hazardous without warning (HWOW)	Very high severity ranking without warning	(9, 10, 10)
Hazardous with warning (HWW)	Very high severity ranking with warning	(8, 9, 10)
Very high (VH)	System inoperable with destructive failure	(7, 8, 9)
High (H)	System inoperable with equipment damage	(6, 7, 8)
Moderate (M)	System inoperable with minor damage	(5, 6, 7)
Low (L)	System inoperable without damage	(4, 5, 6)
Very low (VL)	System operable with significant degradation of performance	(3, 4, 5)
Minor (MR)	System operable with some degradation of performance	(2, 3, 4)
Very minor (VMR)	System operable with minimal interference	(1, 2, 3)
None (N)	No effect	(1, 1, 2)

Table 3: Fuzzy rating for detection of a failure

Rating	Likelihood of detection	Fuzzy number
Absolute uncertainty (AU)	No chance	(9, 10, 10)
Very remote (VR)	Very remote chance	(8, 9, 10)
Remote (R)	Remote chance	(7, 8, 9)
Very low (VL)	Very low chance	(6, 7, 8)
Low (L)	Low chance	(5, 6, 7)
Moderate (M)	Moderate chance	(4, 5, 6)
Moderately high (MH)	Moderately high chance	(3, 4, 5)
High (H)	High chance	(2, 3, 4)
Very high (VH)	Very high chance	(1, 2, 3)
Almost certain (AC)	Almost certainty	(1, 1, 2)

Table 4: linguistic terms of risk factors by the three DMs

Items	DM1			DM2			DM3		
	OR	SR	DR	OR	SR	DR	OR	SR	DR
1	L	L	VR	L	VL	AU	M	L	R
2	M	VL	R	M	M	VR	M	L	VL
3	H	H	VL	M	M	R	M	H	R
4	H	VH	MH	H	H	VL	H	M	L
5	VH	M	MH	H	VH	VL	H	H	L

Table 5: Normalized fuzzy mean rate of each risk factors and its ideal and negative ideal solutions

Items	OR			SR			DR		
1	0.33	0.63	1.00	0.65	0.79	1.00	0.83	0.93	1.00
2	0.24	0.42	0.56	0.61	0.73	0.92	0.72	0.83	0.93
3	0.22	0.33	0.42	0.48	0.55	0.65	0.69	0.79	0.90
4	0.19	0.24	0.28	0.46	0.52	0.61	0.48	0.59	0.69
5	0.18	0.22	0.25	0.46	0.52	0.61	0.48	0.59	0.69
A+	1.00			1.00			1.00		
A-	0.00			0.00			0.00		

Table 6: $\alpha-cut$ of risk factors for item1

α -cut	Item1					
	OR		SR		DR	
	L	U	L	U	L	U
0	0.33	1	0.65	1.00	0.83	1.00
0.1	0.36	1.04	0.66	1.02	0.84	1.01
0.2	0.39	1.07	0.68	1.04	0.85	1.01
0.3	0.42	1.11	0.69	1.06	0.86	1.02
0.4	0.45	1.15	0.71	1.08	0.87	1.03
0.5	0.48	1.19	0.72	1.11	0.88	1.04
0.6	0.51	1.22	0.73	1.13	0.89	1.04
0.7	0.54	1.26	0.75	1.15	0.9	1.05
0.8	0.57	1.30	0.76	1.17	0.91	1.06
0.9	0.60	1.33	0.78	1.19	0.92	1.06
1	0.63	1.37	0.79	1.21	0.93	1.07

Table 7: Linguistic terms of weights by the three DMs

DMs	DM1			DM2			DM3		
Risk Factors	OR	SR	DR	OR	SR	DR	OR	SR	DR
Weights	VH	M	H	VH	H	M	H	VH	M

Table 8: $\alpha-cut$ of weights

α -cut	weights					
	OR		SR		DR	
	L	U	L	U	L	U
0	0.83	1.00	0.63	0.90	0.43	0.80
0.1	0.85	1.00	0.65	0.91	0.45	0.82
0.2	0.86	1.01	0.67	0.92	0.47	0.83
0.3	0.87	1.01	0.68	0.93	0.49	0.85
0.4	0.89	1.01	0.70	0.94	0.51	0.87
0.5	0.90	1.02	0.72	0.95	0.53	0.88
0.6	0.91	1.02	0.73	0.96	0.55	0.90
0.7	0.93	1.02	0.75	0.97	0.57	0.92
0.8	0.83	1.03	0.77	0.98	0.59	0.93
0.9	0.95	1.03	0.78	0.99	0.61	0.95
1	0.97	1.03	0.80	1.00	0.63	0.97

Table 9: α -level sets of the fuzzy relative closeness of the five failure items and ranking

α	Item1		Item2		Item3		Item4		Item5	
0.1	0.044	0.590	0.430	0.680	0.650	0.740	0.790	0.830	0.810	0.860
0.2	0.079	0.570	0.350	0.670	0.530	0.750	0.800	0.820	0.810	0.840
0.3	0.126	0.550	0.400	0.670	0.640	0.740	0.800	0.830	0.820	0.860
0.4	0.170	0.530	0.380	0.660	0.620	0.750	0.820	0.820	0.840	0.860
0.5	0.222	0.510	0.330	0.660	0.610	0.750	0.830	0.830	0.840	0.850
0.6	0.258	0.490	0.320	0.640	0.600	0.740	0.810	0.830	0.840	0.850
0.7	0.305	0.470	0.340	0.730	0.600	0.870	0.800	0.820	0.830	0.840
0.8	0.355	0.450	0.340	0.720	0.580	0.860	0.780	0.800	0.790	0.840
0.9	0.390	0.420	0.300	0.620	0.510	0.750	0.790	0.850	0.790	0.870
1	0.440	0.390	0.320	0.670	0.560	0.900	0.780	0.870	0.820	0.880
Defuzzified	0.367		0.511		0.687		0.815		0.837	
Rank	5		4		3		2		1	

In evaluation of above result can told, all methods lead to a crisp relative closeness for each alternative. In other words, Crisp relative closeness provides only one possible solution to a fuzzy MCDM problem, but cannot reflect the whole picture of its all possible solutions. Fuzzy TOPSIS method offers a fuzzy relative closeness for each alternative, because the closeness is badly distorted and over exaggerated because of the reason of fuzzy arithmetic operations. So, there is a need to develop an exact fuzzy TOPSIS method for fuzzy MCDM problems. This paper proposes a fuzzy TOPSIS method based on α -level sets and the fuzzy extension principle, which turns out to be a nonlinear programming (NLP) problem and can be solved by MSQSB software. In comparison with other methods, the results are more logical and more close to reality. Since TOPSIS method is applied, the formulas are simple and number of calculations are more less too.

5 Conclusion

FMEA is a very important safety and reliability analysis tool which has been widely used in many areas and industries. In view of its difficulty in acquiring precise assessment information on failure risks such as probability of occurrence, severity and detectability and the difficulty in building a complete fuzzy if-then rule base, this paper proposed a new fuzzy FMEA which allows the risk factors and their relative weights to be evaluated in a linguistic manner rather than in a precise way and a fuzzy RPN rather than a crisp RPN or fuzzy if-then rules to be defined for prioritization of failure modes. The fuzzy RPN was defined as the fuzzy weighted α -level sets and the fuzzy extension principle. The α -level sets of FRPNs are easy to be generated by solving a series of linear programming models. Compared with the traditional RPN and its various fuzzy improvements, the proposed fuzzy FMEA has some advantages

References

[1] A. Segismundo, P. Augusto, C. Miguel, Failure mode and effects analysis (FMEA) in the context of risk management in new product Development, A case study in an automotive company, International Journal of Quality and Reliability Management, 25 (9) (2008) 899-912.
http://dx.doi.org/10.1108/02656710810908061

[2] M. Bevilacqua, M. Braglia, R. Gabbrielli, Monte Carlo simulation approach for a modified FMECA in a power plant, Quality and Reliability Engineering International, 16 (2000) 313-324.
http://dx.doi.org/10.1002/1099-1638(200007/08)16:4<313::AID-QRE434>3.0.CO;2-U

[3] N. R. Sankar, B. S. Prabhu, Modified approach for prioritization of failures in a system failure mode and effects analysis, International Journal of Quality and Reliability Management, 18 (3) (2001) 324-335.
http://dx.doi.org/10.1108/02656710110383737

[4] M. Braglia, MAFMA: multi-attribute failure mode analysis, International Journal of Quality and Reliability Management, 17 (9) (2000) 1017-1033.
http://dx.doi.org/10.1108/02656710010353885

[5] M. Braglia, M. Frosolini, R. Montanari, Fuzzy TOPSIS approach for failure mode, effects and criticality analysis, Quality and Reliability Engineering International, 19 (2003) 425-443.
http://dx.doi.org/10.1002/qre.528

[6] C. L. Chang, C. C. Wei, Y. H. Lee, Failure mode and effects analysis using fuzzy method and grey theory, Kybernetes, 28 (1999) 1072-1080.
http://dx.doi.org/10.1108/03684929910300295

[7] J. B. Bowles, C. E. Pelaez, Fuzzy logic prioritization of failures in a system failure mode, effects and criticality analysis, Reliability Engineering and System Safety, 50 (1995) 203-213.
http://dx.doi.org/10.1016/0951-8320(95)00068-D

[8] J. Yang, H-Z. Huang, L-P. He, S-P. Zhu, D. Wen, Risk evaluation in failure mode and effects analysis of aircraft turbine rotor blades using Dempster-Shafer evidence theory under uncertainty, Engineering Failure Analysis, 18 (8) (2011) 2084-2092.
http://dx.doi.org/10.1016/j.engfailanal.2011.06.014

[9] R. K. Sharma, D. Kumar, P. Kumar, Fuzzy modeling of system behavior for risk and reliability analysis, International Journal of Systems Science, 39 (6) (2008) 563-581.
http://dx.doi.org/10.1080/00207720701717708

[10] P. A. A. Garcia, R. Schirru, E. Frutuoso, P. F. Melo, A fuzzy data envelopment analysis approach for FMEA, Progress in Nuclear Energy, 46 (3-4) (2005) 59-373.
http://dx.doi.org/10.1016/j.pnucene.2005.03.016

[11] Y. M. Wang, K. S. Chin, K. K. G. Poon, J. B. Yang, Risk evaluation in failure mode and effects analysis using fuzzy weighted geometric mean, Expert Systems with Applications, 36 (2) (2009) 1195-1207.
http://dx.doi.org/10.1016/j.eswa.2007.11.028

[12] N. Xiao, H-Z. Huang, Y. Li, L. He, T. Jin, Multiple failure modes analysis and weighted risk priority number evaluation in FMEA, Engineering Failure Analysis, 18 (4) (2011) 1162-1170.
http://dx.doi.org/10.1016/j.engfailanal.2011.02.004

[13] Z. Zhang, X. Chu, Risk prioritization in failure mode and effects analysis under uncertainty, Expert Systems with Applications, 38 (2011) 206-214.
http://dx.doi.org/10.1016/j.eswa.2010.06.046

[14] D. H. Stamatis, Failure mode and effect analysis: FMEA from theory to execution, Milwaukee, WI: ASQC Quality Press, (1995).

[15] F. Herrera, E. Herrera-Viedma, L. Martínez, A fusion approach for managing multi-granularity linguistic term sets in decision-making, Fuzzy Sets and Systems, 114 (2000) 43-58.
http://dx.doi.org/10.1016/S0165-0114(98)00093-1

[16] M. Braglia, M. Frosolini, R. Montanari, Fuzzy TOPSIS approach for failure mode, effects and criticality analysis, Quality and Reliability Engineering International, 19 (5) (2003) 425-443.
http://dx.doi.org/10.1002/qre.528

[17] L. A. Zadeh, Fuzzy sets, Information and Control, 8 (1965) 338-353.
http://dx.doi.org/10.1016/S0019-9958(65)90241-X

[18] Y. M. Wang, T. M. S. Elhag, Fuzzy TOPSIS method based on alpha level sets with an application to bridge risk assessment, Expert Systems with Applications, 31 (2) (2007) 309-319.
http://dx.doi.org/10.1016/j.eswa.2005.09.040