Impulse control and its application in portfolio and hedging with both fixed and proportional transaction costs

A. Delavarkhalafi¹, M. Karbaschi¹

(¹) Faculty of Mathematics, Yazd University, Yazd, P.O. Box 89197/741, Iran.

Abstract

In this paper, we will state impulse control and its application in portfolio selection. For this purpose, we first introduce Quasi Variational Inequalities. Introducing impulse control, in stochastic stopping times we have jumps with stochastic size. Then by using approximative Markov chain, the optimal impulse control is obtained in portfolio, including European option. Finally, considering default parameters, the numerical results in optimal impulse control is obtained.

Keywords: Impulse control, Option pricing; Transaction costs, Utility maximization, Markov chain approximation.

1 Introduction

For many problems in the area of economics and operation research, it is more realistic to allow jumps in the state variable. [1] In the modern finance, it is custom to describe risk preference by a utility function. Expected utility theory maintains that individuals behave as if they maximize the expectation of some utility function of the possible outcomes. Hodges and Neuberger (1989) are pioneers of option pricing and hedging approach which are based on this theory. [9] A non-transaction region was formed where the boundaries defined and the investor is indifferent between the utility of rehedging and the utility of not making any changes to the portfolio. It has been seen as one of the most effective developments in optimal hedging with transaction costs. It has become a common approach developed further in the subsequent studies. Whalley and Wilmott (1994) provided an asymptotic analysis to model Hodges and Neuberger (1989) in the case of any linear transaction costs structure with assumption small transaction costs. [8] Zakamouline (2006) investigations that are based on this approach have reached good empirical results. [10]

2 Quasi Variational Inequalities

Let φ denotes the maximum expected cost for the investor at time t, provided that optimal control would be enforced after t. The quasi-variational inequalities (QVI) characterization can be derived as follows. At any time t, the investor can either effect a control on the asset dynamics or let evolve process. A control would only be effected at time t, if the cost of the control plus the future maximum cost of the future controlled asset, are exactly the maximum
cost of the investor. In accordance with the definition
\[
\phi = M \phi.
\]
where \(M\) is a monotone nonlinear operator that is defined as:

1. impulse control
\[
M\phi(y) = \sup \{ \phi(\Gamma(y, \zeta)) + K(y, \zeta); \zeta \in Z, \Gamma(y, \zeta) \in S \}.
\]

2. optimal stopping
\[
M\phi = g.
\]
That \(g\) is the Terminal cost function.

If on the contrary, one has \(\phi > M\phi\), for the investor it would be better not to effect a control at \(t\) and the process \(Y(t)\) should be left to evolve freely at least in a short time interval. [3]

3 Impulse control of jump diffusions
Suppose that at any time \(\tau\) and any state \(y\) we are free to intervene and to give an impulse \(\zeta\) to the system, where \(\zeta \in Z \subset R^p\) and \(Z\) is a given set. Suppose the result of giving the impulse \(\zeta\) when the state is \(y\), \(y = Y(t^-)\) immediately jumping to \(Y(t) = \Gamma(y, \zeta)\), where \(\Gamma: R^k \rightarrow Z\) is a given function.

An impulse control for this system is a double sequence
\[
\nu = (\tau_1, \tau_2, ..., \tau_j; \xi_1, \xi_2, ..., \xi_j; ...) \quad j \leq M \quad M \leq \infty
\]
where \(\tau_1 < \tau_2 < ...\), are \(F_t\) - stopping times and \(\xi_1, \xi_2, ...\) are the corresponding impulses at these times. We assume that \(\xi_j\) is \(F_{\tau_j}\) -measurable for all \(j\).

the performance criterion is defined as:
\[
J^{(\nu)}(y) = E^y \left[ \int_0^\tau f(Y^{(\nu)}(t)) dt + g(Y^{(\nu)}(\tau_j)) \chi_{\{\tau < \infty\}} + \sum_{\tau_j \leq \tau} K(\tilde{Y}^{(\nu)}(\tau_j), \xi_j) \right].
\]
That \(f, g\) and \(K\) represent the profit rate, terminal payoff and payoff due to intervention, respectively. The impulse control problem is as follows: Finding \(\Phi(y)\) and \(\nu^* \in V\) such that [5]:
\[
\Phi(y) = \sup \{ J^{(\nu)}(y), \nu \in V \}
\]

4 Hedging portfolio
consider Risk-free investment portfolio to be included assets such as bank accounts, risky asset such as stock and option of type stock in portfolio. This portfolio has risk because of stock and option. Therefore, in hedging strategy, risk in portfolio is controlled. Strategy is that hedger beginning to Purchase or selling stock on the base of its position for prevent of stock fluctuations price risk. This means if the intent purchases stock, by using option, the required stock is purchased in specified price at maturity and hedging increase of stock prices. Also if the Intent sells Stock, by using option, the required stock is sold in specified price at maturity and hedging decrease of stock prices. Since the option and stock have uncertainty, portfolio has risk. In this step, for elimination uncertainty and hedging we apply delta hedge Black-Scholes. In portfolio, for hedging with an option, considered difference between optimal number of stock with option liability and without option liability equal with delta hedge Black-Scholes [4]. It means, any time \(\Delta\) stock hold in portfolio. because the high transaction costs delta hedge Black-Scholes is very costly. For this reason
in this method if transaction cost which is considered with both fixed and proportional transaction cost, by increasing number of transactions and thus profits from them transaction costs would be hedged. In next step consider utility function

$$U(z) = -\exp(-\gamma z), \quad \gamma > 0$$

that $\gamma$ is risk aversion to purchase and sell of stock. Hedging strategy is that hedger which use utility function and risk aversion in any time, determine sell or buy stock or don’t transaction. Hedging is buying and selling stocks which consider risk aversion.

5 Portfolio hedging with both fixed and proportional transaction costs

We consider a continuous-time economy, with a risky and a risk-free asset. Let $(\Omega, \mathcal{F}, P)$ be a probability space with a given filtration as $\{\mathcal{F}_t\}_{0 \leq t \leq T}$. The risk-free asset, which we would refer to as the bank account, pays a constant interest rate of $r \geq 0$ and consequently the evolution of the invested amount in the bank, $(x_t)$ that is given by the ordinary differential equation

$$dx_t = r x_t \, dt.$$

We would refer to the risky asset as the stock, and would assume that the price of the stock, $S_t$ evolves according to a geometric Brownian motion that is defined by

$$S_t = S_0 \exp\{\left(\mu - \frac{1}{2} \sigma^2\right)t + \sigma B_t\}. \quad (5.1)$$

Where $\mu$ and $\sigma$ are constants and $B(t)$ is a one-dimensional $\mathcal{F}_t$ – brownian motion. We assume that are purchasing or selling of $\zeta$ has transaction costs consist sum of fixed cost $k \geq 0$ an a cost $\lambda S_t |\zeta|$ proportional to the transaction ($\lambda \geq 0$) These costs are drawn from the bank account.

We suppose that at any time the investor can decide to transfer money from the bank account to the stock and conversely. The control of the investor is a impulse control $\nu = (\tau_1, \tau_2, \ldots; \zeta_1, \zeta_2, \ldots)$ Here $0 \leq \tau_1 < \tau_2 < \ldots$ are $\mathcal{F}_t$ – stopping times giving the times when the investor decides to change his portfolio, and $\zeta_i$ are $\mathcal{F}_\tau$ – measurable random variables give the sizes of the transactions at these times. If such control is applied to the system $(x(t), y(t))$, it assumes the form

$$x(\tau_{i+1}) = x(\tau_{i+1}) - k - (\zeta_{i+1} + \lambda |\zeta_{i+1}| S(\tau_{i+1})),$$

$$y(\tau_{i+1}) = y(\tau_{i+1}) + \zeta_{i+1}.$$

Thus a positive value of $\zeta_{i+1}$ corresponds to buying shares of the stock, and conversely if $\zeta_{i+1}$ is negative.

The starting point for the utility based option pricing and hedging approach is to consider the optimal portfolio selection problem of the investor, who faces with transaction costs and maximizes expected utility of his terminal wealth. The investor has a finite horizon $[0, T]$ and it is assumed that there are no transaction costs at terminal time $T$.

We define the value function of the investor with no option liability at time $t$ as

$$J_0(t, x(t), y(t), S(t)) = \max_{v \in \Lambda(x, y, S)} E_t[U(x(T) + y(T)S(T))]. \quad (5.2)$$

where $U(.)$ is the investor utility function and $\Lambda(x, y, S)$ denotes the set of available admissible controls to the investor who starts at time $t$.

The option contract is a cash settled European call with expiration time $T$, the strike $K$, and payoff $(S(T) - K)^+$ at expiration.

The value function of the investor with option liability is defined by

$$J_o(t, x(t), y(t), S(t)) = \max_{v \in \Lambda(x, y, S)} E_t[U(x(T) + y(T)S(T) - (S(T) - K)^+)]. \quad (5.3)$$
Definition 5.1. The reservation write price of a European call option is defined as the compensation $P$ such that

$$J_w(0,x+p,y,S) = J_0(0,x,y,S).$$

$P$ is the lowest price at which the investor is willing to sell an option.

We denote the investors optimal trading policy without option liability by $y_0(t)$ is Similar to, the optimal trading policy of the investor with option liability that is denoted by $y_w(t)$

Definition 5.2. The option hedging strategy of the investor is defined as this difference,

$$y_w(t) - y_0(t).$$

that is between the investors trading strategies with and without option liability.

In the framework of stochastic impulse control theory one assumes that the investors portfolio space is divided into two disjoint regions:

1. a continuation region
2. an intervention region

The intervention region is where that it is optimal to make a transaction. We define the intervention operator $M$ by

$$MJ_j(t,x,y,S) = \max_{(x',y') \in A(t,x,y,S)} J_j(t,x',y',S).$$

where $x'$ and $y'$ are the new values of $x$ and $y$, and

$$x' = x - k - (\zeta + \lambda |\zeta|)S, \quad y' = y + \zeta,$$

where $\zeta$ is the size of transaction. In other words, $MJ_j(t,x,y,S)$ represents the value of the strategy that consists to choosing the best transaction. The continuation region is the region where it is not optimal to rebalance the investors portfolio. We define the continuation region $D$ by

$$D = \{(t,x,y,S): J_j(t,x,y,S) > MJ_j(t,x,y,S)\}.$$

The investors net wealth is given by

$$X_j(t,x,y,S) = \begin{cases} \max\{x + y(1 - \lambda)S - k, x\} & y_j S_j \geq 0, \\ x + y(1 + \lambda)S - k & y_j S_j < 0. \end{cases}$$

Now, by giving heuristic arguments, we intend to characterize the value function and the associated optimal strategy. If for some initial point $(t,x,y,S)$ the optimal strategy is to not transact, the utility associated with this strategy is $J_j(t,x,y,S)$ Choosing the best transaction and then following the optimal strategy gives the utility $MJ_j(t,x,y,S)$. The necessary condition for the optimality of the first strategy is $J_j(t,x,y,S) \geq MJ_j(t,x,y,S)$. This inequality holds with equality when it is optimal to rebalance the portfolio. Moreover, in the continuation region, the application of the dynamic programming principle gives $LJ_j(t,x,y,S) = 0$, where the operator $L$ is defined as:

$$LJ_j(t,x,y,S) = \frac{\partial J_j}{\partial t} + rx \frac{\partial J_j}{\partial x} + \mu S \frac{\partial J_j}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 J_j}{\partial S^2}.$$
which is defined as:

\[ J_0(T, x, y) = U(x + yS), \]
\[ J_w(T, x, y) = U(x + yS - (S - K)^+). \]

Now by using Verification theorem, optimal impulse control would obtain.

**Verification theorem**

**Theorem 5.1.** Suppose there exists a function \( w_j(t, x, y) \in C^{1,2} \) that satisfies the growth conditions and an admissible control \( \eta = (\tau_1, \tau_2, \ldots; \zeta_1, \zeta_2, \ldots) \) such that

1. on \([0, T] \times S_0\)
   \[ \max\{Lw_j, Mw_j - w_j\} = 0, \]

2. outside \( D \)
   \[ Mw_j - w_j = 0, \] \hspace{1cm} (5.4)

3. in \( D \)
   \[ Lw_j(t, x, y) = 0, \] \hspace{1cm} (5.5)

4. \[ w_j(T, x, y) = U(X_T), \] \hspace{1cm} (5.6)

Then

\[ w_j(t, x, y) = J_j(t, x, y), \]

and the control \( \eta \) that is given by

\[ \tau_i = \inf\{t > \tau_{i-1} : (t, x, y) \notin D\}, \]
\[ \zeta_i = \arg \max \{w_j(\tau_i, x - k - \zeta_i, y + \zeta_i)\}, \]

is optimal.

**Proof.** Using the classical Itos rule between the stopping times \( \tau_i \) when the control \( \zeta_i \) is applied, we have

\[ E_{x,y}[w_j(T, x_T, y_T)] = w_j(t, x_t, y_t) + \int_t^T Lw_j(s, x_s, y_s)ds \]
\[ + \sum_{s \leq t \leq T} [w_j(s, x_s, y_s) - w_j(s^-, x_s^-, y_s^-)]. \]

The last term in (5.7) represents the change in the value function when some control is applied (assuming \( s \equiv \tau_i \)). First, note that the transactions are made in order to maximize the expected utility, that

\[ w_j(s, x_s, y_s) = Mw_j(s^-, x_s^-, y_s^-), \]
Then, using (5.4) we obtain
\[
\sum_{t \leq s \leq T} [w_j(s, x_s, y_s) - w_j(s^-, x_{s^-}, y_{s^-})]
= \sum_{t \leq s \leq T} [M w_j(s^-, x_{s^-}, y_{s^-}) - w_j(s, x_s, y_s)] = 0.
\]

Between the transactions, the portfolio lies inside the continuation region. This means, according to (5.5), the second term in (5.7) is also equal to zero. That is,
\[
\int_T^t L w_j(s, x_s, y_s) ds = 0,
\]
Finally, using (5.6) we get
\[
w_j(t, x, y) = E_t^{X,Y} [w_j(T, x, y)] = J_j(t, x, y).
\]

on can find another proofs for the above theorem in [5, 9].

We further assume that the investor has the negative exponential utility function
\[U(X_t) = -\exp(-\gamma X_t), \quad \gamma > 0\]
where \(\gamma\) is the measure of the investors absolute risk aversion.

Asset value of the investor’s bank account at maturity would be as:
\[
x(T) = \frac{x(t)}{\delta(t, T)} - \sum_{i=0}^n \frac{k + (\zeta_i + \lambda |\zeta_i|) S(\tau_i)}{\delta(\tau_i, T)}.
\]

where \(\delta(t, T)\) is the discount factor that is defined by
\[
\delta(t, T) = \exp(-r(T-t)).
\]

\(n\) is a random number of transactions in \([t, T]\), and \(t \leq \tau_1 < \tau_2 < ... < \tau_n < T\)

In the absence of any transaction costs, the solutions for the optimal number of shares that investor would hold without and with option liability are given by
\[
y_0^* = \frac{\delta(t, T)}{\gamma S} \frac{(\mu - r)}{\sigma^2},
\]
\[
y_w^* = \frac{\delta(t, T)}{\gamma S} \frac{(\mu - r)}{\sigma^2} + \frac{\partial V}{\partial S},
\]
that \(V\) is the option price in the financial markets, with no transaction costs, and is the Black-Scholes price. In particular, the definition of the option hedging strategy with transaction costs would reduce to the Black-Scholes hedging strategy in the absence of transaction costs as:
\[
y_w^* - y_0^* = \frac{\partial V}{\partial S}.
\]

[2, 9, 10]
6 Utility

The utility maximization strategy would maximize the utility of the investor to form the optimal strategy. It was first presented by Neuberger and Hodges in 1989. A no transaction region formed at the region where the boundaries were defined where the investor is indifferent between the utility of rehedging and the utility of not making any changes to the portfolio. It has been seen as one of the most effectual developments for optimal hedging with transaction costs and has become a common approach which is developed further in subsequent studies. To deal with the complexity of the utility-maximization based strategies, Whalley and Wilmot 1997 have presented an asymptotic approximation of the hedging strategies with assuming small transaction costs. In these strategies, the boundaries are simply computed depend on the risk aversion with respect to the movements of the underlying asset.

If the utility function be as

\[ U_j(\gamma, X_t) = -e^{-\gamma X_t}. \]  

Which \( \gamma \) is constant absolute risk aversion, that is independent of the investor's wealth. This utility function is defined even for negative wealths. Thus, we do not require that the state process \((x, y)\) should remain in the some solvency region.

Recall there are no explicit solutions for the utility based option pricing and hedging model with transaction costs. As a result, exact solutions have to be obtained by numerical methods. However, the numerical methods are computationally rather hard. One of the alternatives for numerical methods is to obtain an asymptotic solution to a problem. In asymptotic analysis we study the solution for problem such that some parameters in the problem assume large or small values. [8]

7 Markov chain

The investor value function in a market with both fixed and proportional transaction costs is characterized by quasi-variational HJB inequalities where one makes use of the maximum utility operator. To find the solution of the continuous-time continuous-space stochastic control problem that is described by

\[ \max \{L_j(x, y, S), \max_{(x', y') \in A(x, y, S)} J_j(x', y', S) - J_j(x, y, S)\} = 0. \]  

we apply the method of the Markov chain approximation that is suggested by Kushner that is the basic idea which involves a consistent approximation of the problem under considered by Markov chain. Then the solution of an appropriate optimization problem for the Markov chain modeled. First, according to the Markov chain approximation method, we construct discrete time approximations of the continuous time price process. Then the discrete time program is solved by using the discrete time dynamic programming algorithm (that is, the forward recursion algorithm). Consider the partition \( 0 = t_0 < t_1 < ... < t_n = T \) of the time interval \([0, T]\) and assume \( t_i = i\Delta t \) for \( i = 0, 1, ..., n \) where \( \Delta t = \frac{T}{n} \).

Let \( \epsilon \) be a stochastic variable as:

\[ \epsilon = \begin{cases} 
  u & \text{with probability } p, \\
  d & \text{with probability } 1 - p.
\end{cases} \]

We define the discrete time stochastic process of the stock as [7]

\[ S(t_{i+1}) = S(t_i)\epsilon. \]

and the discrete time process of the risk-free asset as

\[ x(t_{i+1}) = x(t_i)\rho. \]

If we choose

\[ d = e^{(\mu - \frac{1}{2} \sigma^2)\Delta t - \sigma \sqrt{\Delta t}}, \quad u = e^{(\mu - \frac{1}{2} \sigma^2)\Delta t + \sigma \sqrt{\Delta t}}, \quad \rho = e^{ri\Delta t}. \]
And \( P = \frac{1}{2} \) we would obtain the binomial model that is proposed by He (1990). As \( n \) goes to infinity, the discrete time process (7.14) converges in distribution to its continuous counterpart (7.15) This is what is called the local consistency conditions for a Markov chain.

The following discretization scheme is proposed to find the value function \( J_j(t, y, S) \) that is defined by (7.13):

\[
J_j^\Delta(t, y, S) = \max \left\{ \max_m J_j^\Delta(t, x - (k + (1 + \lambda)m)\Delta y, y + m\Delta y, S), \right. \\
\left. \max_m J_j^\Delta(t, x - (k + (\lambda - 1)m)\Delta y, y - m\Delta y, S), \right. \\
\left. E\{J_j^\Delta(t, x + P, y, S)\} \right\}, \tag{7.16}
\]

where \( m \) runs through the positive integer numbers, and \((m = 1, 2, \ldots)\) are positive integers. The first and second terms (7.16) are impulse control operator, and are

\[
J_j^\Delta(t, x - (k + (1 + \lambda)m)\Delta y, y + m\Delta y, S) \\
= E\{J_j^\Delta(t, x - (k + (1 + \lambda)m)\Delta y, y + m\Delta y, S)P, y + m\Delta y, S)\}
\]

and

\[
J_j^\Delta(t, x - (k + (\lambda - 1)m)\Delta y, y - m\Delta y, S) \\
= E\{J_j^\Delta(t, x - (k + (\lambda - 1)m)\Delta y, y + m\Delta y, S)P, y - m\Delta y, S)\}
\]

also, when \( \Delta t \to 0 \) we have the following equation

\[
\sup_{d \zeta} J_j(t, x - k - (1 + \lambda)d \zeta, S) = \lim_{\Delta t \to 0} \max_m \left\{ J_j^\Delta(t, x - (k + (1 + \lambda)m)\Delta y, y + m\Delta y, S), \right. \\
\left. J_j^\Delta(t, x - (k + (\lambda - 1)m)\Delta y, y - m\Delta y, S) \right\}
\]

at time \( t_{i+1} \) we do not know yet the value function. In this case we use the known values at the Previous time instant \( t_i \). This scheme is a dynamic programming formulation of the discrete time problem. The solution procedure is as follow. Start at the first date and give the value function values by using the boundary conditions as the continuous value function over the discrete state space. Then work forwards in the time. That is, at every time instant \( t_{i+1} \) and every particular state \((x, y, S)\) by knowing the value function for all the states in the previous time instant \( t_i \), and finding the investor optimal policy. This is carried out by comparing maximum attainable utilities from buying, selling or doing nothing.

So far, the outputs of the discretization scheme are presented above are the value function and the optimal transaction policies that are described as the mapping \((x, y, S) \rightarrow (x', y', S)\), we implicitly have assumed that for every point \((x, y, S)\) the algorithm finds a new point \((x', y', S)\) that represents the optimal transaction. A direct implementing of such an algorithm is extremely time consuming. Below we show how that computational time can be substantially reduced by exploiting the knowledge of the form of the optimal portfolio strategy. The value function \( J_j^\Delta(t, y, S) \) inside the \( NT \) region is found by assuming the \( NT \) policy. As the continuous time case, if the value function \( J_j^\Delta(t, y, S) \) be known in the \( NT \) region, it can be calculated in the buy and sell region by using the discrete space version of

\[
M J_j(t, x, y, S) = \begin{cases} 
J_j(t, x - (k + \lambda - 1)m(y - y_S)S, y, S) \geq y_u(t, S), \\
J_j(t, x - (k + 1 + \lambda)m(y_S - y)S, y, S) \leq y_L(t, S).
\end{cases} \tag{7.17}
\]

as follows:

\[
M J_j^\Delta(t, x, y, S) = \begin{cases} 
J_j(t, x - (k + \lambda - 1)m(y - y_S)S, y, S) \geq y_u(t, S), \\
J_j(t, x - (k + 1 + \lambda)m(y_S - y)S, y, S) \leq y_L(t, S).
\end{cases}
\]
It is crucial to note that inequalities (7.17) would hold equalities on the points that are belonging to the $NT$ boundaries. This means in particular that

$$J_j(t,x,y_u,S) = J_j(t,x - (k - 1)m(y_u - y^*_j)S,y^*_j,S),$$

$$J_j(t,x,y_L,S) = J_j(t,x - (k + 1)m(y^*_j - y_L)S,y^*_j,S).$$

Consequently, assume we know the value function at $t$. Following sequence of steps is performed at the time $t_{i+1}$ to find the value function and impulse control. [6, 9, 10]

8 Conclusion

8.1 Numerical method for impulse control

In this section, optimal impulse control, that is the best time to buy or sell stock, and the best size of buying or selling when a stock is bought or sold is to be achieved. Using with (7.16) the utility at the time $t_i$ is searched, without any transaction cost or buying and selling stocks. This is done by comparing the first, second, and third terms. If the first expression be maximum, we have maximum utility for the purchase transaction, if the second term be the maximum, maximum utility for the sale transaction and the third term is the maximum desirable that transaction is not done. [11]

use the Markov chain approximation method, it starts from the initial point $(x_0,y_0)$ and begins to hedge transaction cost, by increasing investors transaction size. Using of asymptotic analysis, the transaction cost of trading is considered low and the optimum number of shares is considered without transaction cost. At any time, if the first and second terms have more utility, it is the best time for buying or selling shares, and attempting to buy or sell stocks until the number of shares reach’s the number of shares in equations (5.9) And (5.10), so we have optimal impulse control $\nu = (\tau_1, \tau_2, ..., \zeta_1, \zeta_2, ...)$ that $\tau_i$ are the best stopping times in impulse control, and $\zeta_i$ show optimum size of purchase or sell stocks.

8.1.1 Optimal impulse control for portfolio without option liability

Using this numerical method for impulse control, we get optimal impulse control for portfolio that its value function, as equation (5.2) without option liability. It includes a risk-free asset with initial value $x(1) = 1000$, risky asset with initial number $y(1) = 1$ and initial price $S(1) = 100$. Investors by increasing transaction volume, would hedge transaction costs. As a result, low transaction costs and default parameters are considered using with asymptotic analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$</td>
<td>$5 \times 10^{-20}$</td>
<td>$\lambda$</td>
<td>$10^{-20}$</td>
</tr>
<tr>
<td>$r$</td>
<td>$5 \times 10^{-2}$</td>
<td>$\mu$</td>
<td>0.1</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$10^{-3}$</td>
<td>$\sigma$</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Table 1: The default values

Optimal impulse control in the period $[0, 20]$ is as follows:

<table>
<thead>
<tr>
<th>$t$</th>
<th>$\zeta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>2.0214</td>
</tr>
<tr>
<td>12</td>
<td>1.1140</td>
</tr>
<tr>
<td>14</td>
<td>0.7033</td>
</tr>
<tr>
<td>17</td>
<td>0.3031</td>
</tr>
</tbody>
</table>

Table 2: Jump time and its corresponding size in impulse control
In this figure $S$ shows Stock prices in the discrete case, $x$ Risk-free asset, $y$ risky asset, and $W$ investor’s wealth. Since the stock price is a stochastic process, for each sample path stock price, the optimal impulse control is different. If a different sample path for stock price be considered, optimal impulse control in the period of $[0, 20]$ would be as follows:

<table>
<thead>
<tr>
<th>$t$</th>
<th>$\zeta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>4.7301</td>
</tr>
<tr>
<td>7</td>
<td>1.6922</td>
</tr>
<tr>
<td>15</td>
<td>1.9088</td>
</tr>
<tr>
<td>16</td>
<td>0.4375</td>
</tr>
<tr>
<td>20</td>
<td>0.6859</td>
</tr>
</tbody>
</table>

Table 3: jump time and its corresponding size in impulse control

Figure 1: Maximum of asset by using optimal impulse control
8.1.2 Optimal impulse control for portfolio with option liability

Using this numerical method for impulse control, we get optimal impulse control for portfolio that its value function, as equation (5.3) without option liability. It includes a risk-free asset with initial value \( x(1) = 1000 \) risky asset with initial number \( y(1) = 1 \), and initial price \( S(1) = 100 \).

Financial theory generally assumes that investors are indifferent to risk. So it seems the peoples risk aversion as a basis in the analysis of the financial decision is undeniable importance. By using asymptotic analysis, can be considered low risk aversion and reservation option price that is independent of risk aversion is approximated with the following formula:

\[
P = V + \lambda S \frac{\partial V}{\partial S},
\]

Also investors by increase transaction volume, hedging Transaction costs. As a result by using asymptotic analysis considered low transaction costs and by default parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>value</th>
<th>Parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k )</td>
<td>( 5 \times 10^{-20} )</td>
<td>( \lambda )</td>
<td>( 10^{-20} )</td>
</tr>
<tr>
<td>( r )</td>
<td>( 5 \times 10^{-2} )</td>
<td>( \mu )</td>
<td>0.1</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>( 10^{-18} )</td>
<td>( \sigma )</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Table 4: The default values

Optimal impulse control in the Period \([0, 20]\) would be as follows:

![Image of graphs](image-url)
<table>
<thead>
<tr>
<th>$t$</th>
<th>$\zeta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>$5.73 \times 10^{15}$</td>
</tr>
<tr>
<td>3</td>
<td>$1.0618 \times 10^{15}$</td>
</tr>
<tr>
<td>5</td>
<td>$2.7504 \times 10^{15}$</td>
</tr>
<tr>
<td>14</td>
<td>$3.7307 \times 10^{15}$</td>
</tr>
<tr>
<td>17</td>
<td>$6.6156 \times 10^{15}$</td>
</tr>
<tr>
<td>18</td>
<td>$1.2336 \times 10^{15}$</td>
</tr>
</tbody>
</table>

Table 5: jump time and its corresponding size in impulse control

Figure 3: Maximum of asset by using optimal impulse control

Acknowledgement

The authors wish to thank referee for their useful comments.

References


http://dx.doi.org/10.1017/CBO9780511616747


http://dx.doi.org/10.1111/1467-9965.00034


http://dx.doi.org/10.1016/j.jedc.2004.11.002

http://dx.doi.org/10.5899/2012/jnaa-00135