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Research Article

# A common fixed point theorem for weakly compatible mappings in Menger probabilistic quasi metric space

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**Abstract**

In this paper, we prove a common fixed point theorem for finite number of self mappings in Menger probabilistic quasi metric space. Our result improves and extends the results of Rezaiyan et al. [A common fixed point theorem in Menger probabilistic quasi-metric spaces, Chaos, Solitons and Fractals 37 (2008) 1153-1157.], Miheţ [A note on a fixed point theorem in Menger probabilistic quasi-metric spaces, Chaos, Solitons and Fractals 40 (2009) 2349-2352], Pant and Chauhan [Fixed points theorems in Menger probabilistic quasi metric spaces using weak compatibility, Internat. Math. Forum 5 (6) (2010) 283-290] and Sastry et al. [A fixed point theorem in Menger PQM-spaces using weak compatibility, Internat. Math. Forum 5 (52) (2010) 2563-2568].

**Keywords:** t-norm, Menger probabilistic quasi metric space, Weakly compatible mappings, Fixed point.

## 1 Introduction

Classical metrical fixed point theory plays an important role in general topology. In 1942, Menger [10] introduced the notion of probabilistic metric spaces as a generalization of metric spaces. Since then, the theory of probabilistic metric space has developed in many directions [1, 20]. In 1989, Kent and Richardson [7] introduced the class of probabilistic quasi-metric spaces (briefly, PQM-spaces) and proved common fixed point theorems. The study of fixed points of mappings in probabilistic quasi metric spaces is in nascent stage.

With a view to improve commutativity conditions in fixed point theorems, Sessa [22] introduced the notion of weakly commuting mappings. Inspired by this concept, Jungck [5] weakened the notion of weak commutativity by introducing compatible mappings. Further, Jungck and Rhoades [6] introduced the notion of weak compatibility which is the most general among all commutativity concepts. Many authors formulated the definitions of weakly commuting [25], compatibility [14] and weakly compatible mappings [24] in framework of probabilistic settings and proved several fixed point results. Fixed point theorems in PQM-spaces have appeared in [2, 12, 13, 15, 17, 18, 19, 21, 23]. The theory of quasi metric spaces can be used as an efficient tool to solve so many several problems like theoretical computer science, approximation theory and topological algebra (see, for instance [3, 9, 15]).

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The aim of this paper is to obtain a common fixed point theorem for a pair of finite number of self mappings in Menger PQM-space using weak compatibility.

## 2 Preliminaries

In this section, we recall some definitions and known results in Menger PQM-spaces.

**Definition 2.1.** *([20]) A mapping* $\mathrm{T}:[0,1]\times[0,1]\to[0,1]$ *is t-norm if* T *satisfies the following conditions:*

1. T *is commutative and associative,*
2. $\mathrm{T}(a,1)=a$ *for all* $a\in[0,1]$,
3. $\mathrm{T}(a,b)\le\mathrm{T}(c,d)$ *whenever* $a\le c$ *and* $b\le d$ *and* $a,b,c,d\in[0,1]$.

The following are some examples of basic t-norms:

1. $\mathrm{T}_M(a,b)=\min\{a,b\}$,
2. $\mathrm{T}_P(a,b)=ab$,
3. $\mathrm{T}_L(a,b)=\max\{a+b-1,0\}$.

Each t-norm T can be extended [8] (by associativity) in a unique way by taking $(x_1,\dots,x_n)\in[0,1]^n(n\in N)$ the values $\mathrm{T}^1(x_1,x_2)=\mathrm{T}(x_1,x_2)$ and $\mathrm{T}^n(x_1,\dots,x_{n+1})=\mathrm{T}(\mathrm{T}^{n-1}(x_1,\dots,x_n),x_{n+1})$ for $n\ge 2$ and $x_i\in[0,1]$, for all $i\in\{1,2,\dots,n+1\}$.

**Definition 2.2.** *([20]) A mapping* $F:\mathbb{R}\to\mathbb{R}^+$ *is called a distribution function if it is non-decreasing and left continuous with* $\inf\{F(t):t\in\mathbb{R}\}=0$ *and* $\sup\{F(t):t\in\mathbb{R}\}=1$.
*We shall denote* $\Im$ *by the set of all distribution functions defined on* $(-\infty,\infty)$ *while* $\varepsilon_0$ *will always denote the specific distribution function, i.e.,*

$$\varepsilon_0(t)=\begin{cases}0, & \text{if } t\le 0;\\ 1, & \text{if } t>0.\end{cases}$$

*If* $X$ *is a non-empty set,* $\mathscr{F}:X\times X\to\Im$ *is called a probabilistic distance on* $X$ *and the value of* $\mathscr{F}$ *at* $(x,y)\in X\times X$ *is represented by* $F_{x,y}$.

**Definition 2.3.** *([15, 17]) A Menger PQM-space is a triplet* $(X,\mathscr{F},\mathrm{T})$*, where* $X$ *is a non-empty set,* $\mathscr{F}$ *is a probabilistic distance and* T *is a continuous t-norm satisfying: for all* $x,y,z\in X$ *and* $t,s>0$

1. $F_{x,y}(t)=\varepsilon_0(t)$ *and* $F_{y,x}(t)=\varepsilon_0(t)$ *then* $x=y$,
2. $F_{x,z}(t+s)\ge\mathrm{T}(F_{x,y}(t),F_{y,z}(s))$.

*A Menger PQM-space is called a Menger PM-space if it satisfies the symmetry condition, i.e.,* $F_{x,y}(t)=F_{y,x}(t)$.

**Definition 2.4.** *([13]) Let* $(X,\mathscr{F},\mathrm{T})$ *be a Menger PQM-space. A sequence* $\{x_n\}$ *is said to be*

1. $F$*-convergent to* $x\in X$ *if for every* $\varepsilon>0$ *and* $\lambda>0$ *there exists* $k\in N$ *such that* $F_{x_n,x}(\varepsilon)>1-\lambda$ *whenever* $n\ge k$.
2. *left Cauchy if for every* $\varepsilon>0$ *and* $\lambda>0$*, there is* $k\in N$ *such that* $F_{x_r,x_s}(\varepsilon)>1-\lambda$ *for all* $s\ge r\ge k$.

A Menger PQM-space $(X,\mathscr{F},\mathrm{T})$ is called left complete if every left Cauchy sequence is $F$-convergent to a point in $X$.

**Definition 2.5.** *([4]) A t-norm* T *is of Hadžić-type (briefly,* H*-type) and* $\mathrm{T}\in\mathscr{H}$ *if the family* $\{\mathrm{T}^n\}_{n\in N}$ *of its iterates defined, for each* $x$ *in* $[0,1]$*, by* $\mathrm{T}^0(x)=1,\mathrm{T}^{n+1}(x)=\mathrm{T}(\mathrm{T}^n(x),x)$*, for all* $n\ge 0$ *is equicontinuous at* $x=1$*, that is*
$\varepsilon\in(0,1)\ \exists\,\delta\in(0,1):x>1-\delta\Rightarrow\mathrm{T}^n(x)>1-\varepsilon$ *for all* $n\ge 1$.
*There is a nice characterization of continuous t-norm* T *of the class* $\mathscr{H}$ *[16].*
*The t-norm* $\mathrm{T}_M$ *is an trivial example of a t-norm of* H*-type, but there are t-norms* T *of Hadžić-type with* $\mathrm{T}\neq\mathrm{T}_M$ *(see examples in [4]).*

**Definition 2.6.** *([4]) If* T *is a t-norm and* $\{x_1, x_2, \ldots, x_n\} \in [0,1]^n (n \in N)$*, then* $\mathrm{T}_{i=1}^n x_i$ *is defined recurrently by 1, if* $n = 0$ *and* $\mathrm{T}_{i=1}^n x_i = \mathrm{T}(\mathrm{T}_{i=1}^{n-1} x_i, x_n)$ *for all* $n \geq 1$*. If* $\{x_i\}_{i \in N}$ *is a sequence of numbers from* $[0,1]$ *then* $\mathrm{T}_{i=1}^{\infty} x_i$ *is defined as* $\lim_{n\to\infty} \mathrm{T}_{i=1}^n x_i$ *(this limit always exists) and* $\mathrm{T}_{i=n}^{\infty} x_i$ *as* $\mathrm{T}_{i=1}^{\infty} x_{n+i}$*.*

In probabilistic metric spaces, there are of particular interest the t-norms T and sequences $\{x_n\} \subset [0,1]$ such that $\lim_{n\to\infty} x_n = 1$ and $\lim_{n\to\infty} \mathrm{T}_{i=1}^{\infty} x_{n+i} = 1$.

**Proposition 2.1.** *([4])*

1. *If* $\mathrm{T} \geq \mathrm{T}_L$*, then the following implication holds:*

$$\lim_{n\to\infty} \mathrm{T}_{i=1}^{\infty} x_{n+i} = 1 \Leftrightarrow \sum_{n=1}^{\infty} (1 - x_n) < \infty.$$

2. *If* $\mathrm{T} \in \mathscr{H}$*, then for every sequence* $\{x_n\}_{n \in N}$ *in* $[0,1]$ *such that* $\lim_{n\to\infty} x_n = 1$*, one has* $\lim_{n\to\infty} \mathrm{T}_{i=1}^{\infty} x_{n+i} = 1$.

Note that if T is a t-norm for which there exists $\{x_n\} \subset [0,1]$ such that $\lim_{n\to\infty} x_n = 1$ and $\lim_{n\to\infty} \mathrm{T}_{i=1}^{\infty} x_{n+i} = 1$, then $\sup_{t<1} \mathrm{T}(t,t) = 1$.

**Proposition 2.2.** *([4]) Let* $\{x_n\}_{n \in N}$ *be a sequence of numbers from* $[0,1]$ *such that* $\lim_{n\to\infty} x_n = 1$ *and t-norm* T *is of* H*-type. Then*

$$\lim_{n\to\infty} \mathrm{T}_{i=n}^{\infty} x_i = \lim_{n\to\infty} \mathrm{T}_{i=1}^{\infty} x_{n+i} = 1.$$

**Lemma 2.1.** *If a Menger PQM-space* $(X, \mathscr{F}, \mathrm{T})$ *satisfies* $F_{x,y}(t) = C$*, for all* $t > 0$ *with fixed* $x, y \in X$*. Then we have* $C = 1$ *and* $x = y$*.*

**Lemma 2.2.** *([4]) Let the function* $\phi(t)$ *satisfy the condition* $(\Phi) : \phi(t) : [0,\infty) \to [0,\infty)$ *is non-decreasing and* $\sum_{n=1}^{\infty} \phi^n(t) < \infty$ *for all* $t > 0$*, when* $\phi^n(t)$ *denotes the* $n^{th}$ *iterative function of* $\phi(t)$*. Then* $\phi(t) < t$ *for all* $t > 0$*.*

**Definition 2.7.** *([14]) A pair* $(A, S)$ *of self mappings defined on a Menger PQM-space* $(X, \mathscr{F}, \mathrm{T})$ *is said to be compatible if* $F_{ASx_n, SAx_n}(t) \to \varepsilon_0(t)$ *for all* $t > 0$*, whenever* $\{x_n\}$ *is a sequence in* $X$ *such that* $Ax_n, Sx_n \to x$ *for some* $x$ *in* $X$ *as* $n \to \infty$*.*

**Definition 2.8.** *([18]) A pair* $(A, S)$ *of self mappings defined on a Menger PQM-space* $(X, \mathscr{F}, \mathrm{T})$ *is said to be weakly compatible (or coincidentally commuting) if they commute at their coincidence points, i.e.,* $Ax = Sx$ *for some* $x \in X$ *then* $ASx = SAx$*.*

**Remark 2.1.** ([24]) If two self mappings of a Menger PQM-space $(X, \mathscr{F}, \mathrm{T})$ are compatible then they are weakly compatible but the converse is not true.

## 3 Main result

In 2008, Rezaiyan et al. [17] proved the following common fixed point theorem as a probabilistic generalization of Banach's contraction principle.

**Theorem 3.1.** *([17, Theorem 2.2]) Let* $(X, \mathscr{F}, \mathrm{T})$ *be a left complete Menger PQM-space and let* $f, g : X \to X$ *be two self mappings satisfying:*

1. $g(X) \subseteq f(X)$*,*
2. $f$ *is continuous,*
3. $F_{g(x),g(y)}(kt) \geq F_{f(x),f(y)}(t)$*, for all* $x, y \in X$ *and for some* $0 < k < 1$*.*

*Then* $f$ *and* $g$ *have a unique common fixed point provided* $f$ *and* $g$ *commute.*

Since then, Miheţ [13] obtained that the result of Rezaiyan et al. [17] is invalid and proved a common fixed point theorem in Menger PQM-space under two necessary additional conditions. Some examples are also mentioned in [13, Examples 2.1-2.2] which support the necessity of these additional conditions.

**Theorem 3.2.** *([13, Theorem 2.1]) Let $(X,\mathscr{F},\mathrm{T})$ be a left complete Menger PQM-space and let $f,g:X\to X$ be two self mappings satisfying:*

1. $\mathrm{T}$ *is of Hadžić-type,*
2. *every convergent sequence in $X$ has a unique limit,*
3. $g(X)\subseteq f(X)$*,*
4. $f$ *is continuous,*
5. $F_{g(x),g(y)}(kt)\geq F_{f(x),f(y)}(t)$*, for all $x,y\in X$ and for some $0<k<1$.*

*Then $f$ and $g$ have a unique common fixed point provided $f$ and $g$ commute.*

Pant and Chauhan [18] improved the result of Miheţ [13] by taking closedness of one of the underlying subspaces and weakly compatibles mappings which is more general than commutativity.

**Theorem 3.3.** *([18, Theorem 2.1]) Let $(X,\mathscr{F},\mathrm{T})$ be a left complete Menger PQM-space and let $A,B,L:X\to X$ be two self mappings satisfying:*

1. $\mathrm{T}$ *is of Hadžić-type,*
2. *every convergent sequence in $X$ has a unique limit,*
3. $AB(X)\subseteq L(X)$*,*
4. $L(X)$ *is a closed subset of $X$,*
5. *there exists a constant $k\in(0,1)$ such that*
$$F_{AB(x),AB(y)}(kt)\geq F_{L(x),L(y)}(t), \tag{3.1}$$
*for all $x,y\in X$ and $t>0$.*

*Then $A,B$ and $L$ have a unique common fixed point provided the pair $(L,AB)$ is weakly compatible.*

Most recently, Sastry et al. [19] improved the result of Pant and Chauhan [18] by using the commutative condition amongst involved mappings (see [19, Corollary 2.5]). Now, we prove a common fixed point theorem for finite number of self mappings in Menger PQM-spaces using weak compatibility.

**Theorem 3.4.** *Let $(X,\mathscr{F},\mathrm{T})$ be a left complete Menger PQM-space. Further, let $f_1,f_2,\dots,f_n$ and $g$ be the finite number of self mappings satisfying:*

1. $\mathrm{T}$ *is of Hadžić-type,*
2. *every convergent sequence in $X$ has a unique limit,*
3. $g(X)\subset f_1f_2\dots f_n(X)$*,*
4. *for all $x,y\in X$ and $t>0$*
$$F_{g(x),g(y)}(\phi(t))\geq F_{f_1f_2\dots f_n(x),f_1f_2\dots f_n(y)}(t), \tag{3.2}$$
*where the function $\phi(t):[0,\infty)\to[0,\infty)$ is onto, strictly increasing and satisfies condition $(\Phi)$,*
5. *one of $g(X)$ and $f_1f_2\dots f_n(X)$ is a complete subspace of $X$, then $f_1f_2\dots f_n$ and $g$ have a coincidence point.*

6. *Suppose that*

$$g(f_2\dots f_n)=(f_2\dots f_n)g,$$
$$g(f_3\dots f_n)=(f_3\dots f_n)g,$$
$$\vdots$$
$$gf_n=f_ng,$$
$$f_1(f_2\dots f_n)=(f_2\dots f_n)f_1,$$
$$f_1f_2(f_3\dots f_n)=(f_3\dots f_n)f_1f_2,$$
$$\vdots$$
$$f_1\dots f_{n-1}(f_n)=(f_n)f_1\dots f_{n-1}.$$

*Moreover, the mappings $f_1,f_2,\dots,f_n$ and $g$ have a unique common fixed point in $X$ provided that the pair $(f_1f_2\dots f_n,g)$ is weakly compatible.*

*Proof.* Let $x_0$ be an arbitrary element in $X$. By (3), we can find $x_1$ such that $f_1f_2\dots f_n(x_1)=g(x_0)$. By induction, we can find a sequence $\{x_n\}\in X$ such that $f_1f_2\dots f_n(x_n)=g(x_{n-1})$.

$$\begin{aligned}F_{f_1f_2\cdots f_n(x_n),f_1f_2\cdots f_n(x_{n-1})}(t) &= F_{g(x_{n-1}),g(x_n)}(t)\\ &\geq F_{f_1f_2\cdots f_n(x_{n-1}),f_1f_2\cdots f_n(x_n)}(\phi^{-1}(t))\\ &\geq \dots\geq F_{f_1f_2\cdots f_n(x_0),f_1f_2\cdots f_n(x_1)}(\phi^{-n}(t)),\end{aligned}$$

for $n=1,2,\dots$. First we show that $\{y_n\},y_n=f_1f_2\dots f_n(x_n)$ is a left Cauchy sequence. Let $\varepsilon>0$ be given and $\lambda\in(0,1)$ be such that $\mathrm{T}^{m-1}(1-\lambda,\dots,1-\lambda)>1-\varepsilon$. Also assume that for $t>0$ such that $F_{y_0,y_1}(t)>1-\lambda$, $\delta$ be a positive number and $n_1\in N$ be such that $\sum_{n_1}^{\infty}\phi^i(t)\leq\delta$. Then, for every $n\geq n_1$ and $m\in N$ we have

$$\begin{aligned}F_{y_n,y_{n+m}}(\delta) &\geq F_{y_n,y_{n+m}}\left(\sum_{i=n}^{n+m-1}\phi^i(t)\right)\\ &\geq \mathrm{T}^{m-1}\left(F_{y_n,y_{n+1}}(\phi^n(t)),\dots,F_{y_{n+m-1},y_{n+m}}(\phi^{n+m-1}(t))\right)\\ &\geq \mathrm{T}^{m-1}(1-\lambda,\dots,1-\lambda)\\ &> 1-\varepsilon.\end{aligned}$$

Hence, $\{y_n\}$ is a left Cauchy sequence in $X$. Since the space $(X,\mathscr{F},\mathrm{T})$ is left complete, there exists $z\in X$ such that $\lim_{n\to\infty}g(x_{n-1})=\lim_{n\to\infty}f_1f_2\dots f_n(x_n)=z$. Suppose that $f_1f_2\dots f_n(X)$ is a complete subspace of $X$, then the subsequence $\{y_{2n+1}\}$ which is contained in $f_1f_2\dots f_n(X)$ must get a limit $z$ in $f_1f_2\dots f_n(X)$, i.e., $f_1f_2\dots f_n(u)=z$ for some $u\in X$. As $\{y_n\}$ is a Cauchy sequence containing a convergent subsequence $\{y_{2n+1}\}$. Therefore, the sequence $\{y_n\}$ also converges implying thereby the convergence of another subsequence $\{y_{2n}\}$.
Putting $x=u$ and $y=x_{2n+1}$ in (3.2), we get

$$\begin{aligned}F_{g(u),g(x_{2n+1})}(\phi(t)) &\geq F_{f_1f_2\cdots f_n(u),f_1f_2\cdots f_n(x_{2n+1})}(t),\\ F_{g(u),z}(\phi(t)) &\geq F_{z,z}(t)=1.\end{aligned}$$

Hence, $F_{g(u),z}(\phi(t))=1$. Thus $g(u)=z$. Since the pair $(g,f_1f_2\dots f_n)$ is weakly compatible, we have $f_1f_2\dots f_nz=f_1f_2\dots f_n(gu)=g(f_1f_2\dots f_nu)=gz$.
On using (3.2) with $x=z$ and $y=x_{2n+1}$, we have

$$\begin{aligned}F_{g(z),g(x_{2n+1})}(\phi(t)) &\geq F_{f_1f_2\cdots f_n(z),f_1f_2\cdots f_n(x_{2n+1})}(t),\\ F_{g(z),z}(\phi(t)) &\geq F_{g(z),z}(t).\end{aligned}$$

Since $F$ is non-decreasing, we get $F_{g(z),z}(\phi(t)) \leq F_{g(z),z}(t)$. Hence $F_{g(z),z}(t) = C$ for all $t > 0$. From Lemma 2.1, we conclude that $C = 1$, that is $g(z) = z$. Therefore, $f_1 f_2 \dots f_n(z) = gz = z$. We show that $z$ is the fixed point of all the component mappings. Putting $x = f_2 \dots f_n z$, $y = z$ and $f_1^{'} = f_1 f_2 \dots f_n$ in (3.2), we obtain

$$\begin{aligned} F_{g(f_2 \dots f_n z), g(z)}(\phi(t)) &\geq F_{f_1^{'}(f_2 \dots f_n z), f_1^{'} z}(t), \\ F_{f_2 \dots f_n z, z}(\phi(t)) &\geq F_{f_2 \dots f_n(z), z}(t). \end{aligned}$$

On the other hand, since $F$ is non-decreasing, we get $F_{f_2 \dots f_n(z),z}(\phi(t)) \leq F_{f_2 \dots f_n(z),z}(t)$. Hence $F_{f_2 \dots f_n(z),z}(t) = C$ for all $t > 0$. In view of Lemma 2.1, we get $C = 1$, i.e., $f_2 \dots f_n(z) = z$. Thus, $f_1 z = f_1(f_2 \dots f_n)z = z$. Similarly, we have $f_2 z = f_3 z = \dots = f_n z = z$. So $z$ is the common fixed point of $f_1, f_2, \dots, f_n$ and $g$.
Uniqueness: Let $w (w \neq z)$ be another common fixed point of $f_1, f_2, \dots, f_n$ and $g$. Putting $x = z$ and $y = w$ in (3.2), we get

$$\begin{aligned} F_{g(z), g(w)}(\phi(t)) &\geq F_{f_1 f_2 \dots f_n(z), f_1 f_2 \dots f_n(w)}(t) \\ F_{z,w}(\phi(t)) &\geq F_{z,w}(t). \end{aligned}$$

Since $F$ is non-decreasing, we have $F_{z,w}(\phi(t)) \leq F_{z,w}(t)$. Hence $F_{z,w}(t) = C$ for all $t > 0$. Appealing to Lemma 2.1, we obtain $z = w$ and so the uniqueness of the common fixed point.
The proof is similar when $g(X)$ is assumed to be a complete subspace of $X$. □

**Corollary 3.1.** *Let $(X, \mathscr{F}, \mathrm{T})$ be a left complete Menger PQM-space. Further, let $f$ and $g$ be two self mappings of $X$ satisfying:*

1. T *is of Hadžić-type,*
2. *every convergent sequence in $X$ has a unique limit,*
3. *$g(X) \subset f(X)$,*
4. *for all $x, y \in X$ and $t > 0$*
$$F_{g(x),g(y)}(\phi(t)) \geq F_{f(x),f(y)}(t), \tag{3.3}$$
*where the function $\phi(t) : [0, \infty) \to [0, \infty)$ is onto, strictly increasing and satisfies condition $(\Phi)$,*
5. *one of $g(X)$ and $f(X)$ is a complete subspace of $X$, then $f$ and $g$ have a coincidence point.*

*Moreover, the mappings $f$ and $g$ have a unique common fixed point in $X$ provided the pair $(f, g)$ is weakly compatible.*

*Proof.* If we set $f_1 f_2 \dots f_n = f$ in Theorem 3.4, then the result easily follows. □

**Remark 3.1.** Theorem 3.4 improves and extends the results of Rezaiyan et al. [17, Theorem 2.2], Miheţ [13, Theorem 2.1], Pant and Chauhan [18, Theorem 2.1, Corollary 2.2] and Sastry et al. [19, Theorem 2.3 and Corollary 2.5].

It should be noticed (see [11, Theorem 3.3] for the case $gx = x$) that the condition T is of Hadžić-type in Theorem 3.4 and Corollary 3.1 may be replaced by $\lim_{n\to\infty} \mathrm{T}_{i=n}^{\infty} F_{f_1 f_2 \dots f_n(x), gx}\left(\frac{1}{\mu^i}\right) = 1$ and $\lim_{n\to\infty} \mathrm{T}_{i=n}^{\infty} F_{f(x),g(x)}\left(\frac{1}{\mu^i}\right) = 1$, for some $x \in X$ and some $\mu \in (0,1)$. Taking into account Proposition 2.1, we get the following natural results:

**Theorem 3.5.** *Let $(X, \mathscr{F}, \mathrm{T})$ be a left complete Menger PQM-space. Further, let $f_1, f_2, \dots, f_n$ and $g$ be the finite number of self mappings satisfying:*

1. *Every convergent sequence in $X$ has a unique limit,*
2. *$g(X) \subset f_1 f_2 \dots f_n(X)$,*

3. *for all $x, y \in X$ and $t > 0$*
$$F_{g(x),g(y)}(\phi(t)) \geq F_{f_1 f_2 \ldots f_n(x), f_1 f_2 \ldots f_n(y)}(t), \tag{3.4}$$
*where the function $\phi(t) : [0, \infty) \to [0, \infty)$ is onto, strictly increasing and satisfies condition $(\Phi)$,*

4. *one of $g(X)$ and $f_1 f_2 \ldots f_n(X)$ is a complete subspace of $X$, then $f_1 f_2 \ldots f_n$ and $g$ have a coincidence point.*

5. *Suppose that*
$g(f_2 \ldots f_n) = (f_2 \ldots f_n)g$,
$g(f_3 \ldots f_n) = (f_3 \ldots f_n)g$,
$\vdots$
$g f_n = f_n g$,
$f_1(f_2 \ldots f_n) = (f_2 \ldots f_n) f_1$,
$f_1 f_2 (f_3 \ldots f_n) = (f_3 \ldots f_n) f_1 f_2$,
$\vdots$
$f_1 \ldots f_{n-1}(f_n) = (f_n) f_1 \ldots f_{n-1}$.

*Moreover, the mappings $f_1, f_2, \ldots, f_n$ and $g$ have a unique common fixed point in $X$ provided that the pair $(f_1 f_2 \ldots f_n, g)$ is weakly compatible and*
$$\sum_{i=1}^{\infty} \left(1 - F_{f_1 f_2 \ldots f_n(x), g(x)}\left(\frac{1}{\mu^i}\right)\right) < \infty,$$
*for some $x \in X$ and $\mu \in (0,1)$.*

**Theorem 3.6.** *Let $(X, \mathscr{F}, \mathrm{T})$ be a left complete Menger PQM-space. Further, let $f$ and $g$ be two self mappings satisfying:*

1. *Every convergent sequence in $X$ has a unique limit,*
2. *$g(X) \subset f(X)$,*
3. *for all $x, y \in X$ and $t > 0$*
$$F_{g(x),g(y)}(\phi(t)) \geq F_{f(x),f(y)}(t), \tag{3.5}$$
*where the function $\phi(t) : [0, \infty) \to [0, \infty)$ is onto, strictly increasing and satisfies condition $(\Phi)$,*
4. *one of $g(X)$ and $f(X)$ is a complete subspace of $X$, then $f$ and $g$ have a coincidence point.*

*Moreover, the mappings $f$ and $g$ have a unique common fixed point in $X$ provided that the pair $(f, g)$ is weakly compatible and*
$$\sum_{i=1}^{\infty} \left(1 - F_{f(x), g(x)}\left(\frac{1}{\mu^i}\right)\right) < \infty,$$
*for some $x \in X$ and $\mu \in (0,1)$.*

**Remark 3.2.** Theorem 3.5 and Theorem 3.6 extend the results of Miheţ [13, Theorem 2.2] and Pant and Chauhan [18, Corollary 2.3] to finite number of self mappings.

**Remark 3.3.** The conclusion of Theorems 3.4-3.6 and Corollary 3.1 remain true for $\phi(t) = kt$, where $k \in (0,1)$ and $t > 0$.

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