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Research Article

Some New Formulas for the Generalized Hypergeometric Series

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Abstract

The aim of this research paper is to provide some new formulas for the generalized hypergeometric series. The results are derived with the help of the extensions of Euler's and Kummer's classical transformations.

As special cases, we have recovered several results due to Sharma. The results established in this paper are simple, interesting, easily obtainable and may be useful.

Keywords: Classical Summation Theorems; Generalized Hypergeometric Series; Euler's Transformation; Kummer's Transformation.

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1 Introduction and Results Required

The generalized hypergeometric function ${}_pF_q$ with p numerator parameters and q denomenator parameters is defined (see, for example [2, 5]) by

$$
{}_pF_q\left[\begin{array}{ccc} a, & \ldots, & a_p; \\ & & & z \\ b_1, & \ldots, & b_q; \end{array}\right] = {}_pF_q\left[\, a, \quad \ldots, \quad a_p; \quad b_1, \quad \ldots, \quad b_q; \quad z \,\right]
$$

$$
= \sum_{n=0}^{\infty} \frac{(a_1)_n \ldots (a_p)_n}{(b_1)_n \ldots (b_q)_n} \frac{z^n}{n!}, \tag{1.1}
$$

where $(a)_n$ denotes the Pochhammer's symbol (or shifted or raised factorial, since $(1)_n = n!$) defined by

$$
(a)_n = \begin{cases} 1, & n = 0, \\ a(a+1)\ldots(a+n-1), & n \in \mathbb{N} := \{1, 2, 3, \ldots\}. \end{cases} \tag{1.2}
$$

By using the fundamental property of the gamma function $\Gamma(a+1) = a\Gamma(a)$, $(a)_n$ can be written in the form

$$
(a)_n = \frac{\Gamma(a+n)}{\Gamma(a)}, \left(a \in \mathbb{C} \setminus \mathbb{Z}_0^-\right), \tag{1.3}
$$

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where $\mathbb{C}$ and $\mathbb{Z}_0^-$ denote the sets of complex numbers and nonpositive integers, respectively.
For more details about the generalized hypergeometric function, see [1, 2, 3].
The following is the well known and very useful result in the theory of generalized hypergeometric series [7]:

$$ {}_pF_q\left[\begin{array}{c} a_p; \\ b_q; \end{array} x\right] = {}_{2p}F_{2q+1}\left[\begin{array}{ccc} \frac{1}{2}a_p, & \frac{1}{2}a_p+\frac{1}{2}; & \\ \frac{1}{2}, & \frac{1}{2}b_q, & \frac{1}{2}b_q+\frac{1}{2}; \end{array} (4)^{p-q-1}x^2\right] + \frac{(a_p)}{(b_q)} x\, {}_{2p}F_{2q+1}\left[\begin{array}{ccc} \frac{1}{2}a_p+\frac{1}{2}, & \frac{1}{2}a_p+1; & \\ \frac{3}{2}, & \frac{1}{2}b_q+\frac{1}{2}, & \frac{1}{2}b_q+1; \end{array} (4)^{p-q-1}x^2\right]. \tag{1.4}$$

We also mention the following two transformations:

- Euler's II transformation [3, 5]:

$$ {}_2F_1\left[\begin{array}{cc} \alpha, & \beta; \\ \gamma; & \end{array} x\right] = (1-x)^{\gamma-\alpha-\beta}\, {}_2F_1\left[\begin{array}{cc} \gamma-\alpha, & \gamma-\beta; \\ \gamma; & \end{array} x\right]. \tag{1.5}$$

- Kummer's first transformation [5]:

$$ e^{-x}\, {}_1F_1\left[\begin{array}{c} \alpha; \\ \beta; \end{array} x\right] = {}_1F_1\left[\begin{array}{c} \beta-\alpha; \\ \beta; \end{array} -x\right]. \tag{1.6}$$

By applying (1.4) in the transformations (1.5) and (1.6), Sharma [7] has obtained the following interesting results on the generalized hypergeometric series

$$ \begin{aligned} & {}_2F_1\left[\begin{array}{cc} \gamma+\delta-\alpha-\beta, & \gamma+\delta-\alpha-\beta+\frac{1}{2}; \\ \frac{1}{2}; & \end{array} x\right] \\ & \times {}_4F_3\left[\begin{array}{cccc} \alpha, & \alpha+\frac{1}{2}, & \beta, & \beta+\frac{1}{2}; \\ \frac{1}{2}, & \gamma, & \gamma+\frac{1}{2}; & \end{array} x\right] \\ & + \left(\frac{\gamma+\delta-\alpha-\beta}{\gamma}\right) 4\alpha\beta x\, {}_2F_1\left[\begin{array}{cc} \gamma+\delta-\alpha-\beta+\frac{1}{2}, & \gamma+\delta-\alpha-\beta+1; \\ \frac{3}{2}; & \end{array} x\right] \\ & \times {}_4F_3\left[\begin{array}{cccc} \alpha+\frac{1}{2}, & \alpha+1, & \beta+\frac{1}{2}, & \beta+1; \\ \frac{3}{2}, & \gamma+\frac{1}{2}, & \gamma+1; & \end{array} x\right] \\ & = {}_2F_1\left[\begin{array}{cc} \delta, & \delta+\frac{1}{2}; \\ \frac{1}{2}; & \end{array} x\right] {}_4F_3\left[\begin{array}{cccc} \gamma-\alpha, & \gamma-\alpha+\frac{1}{2}, & \gamma-\beta, & \gamma-\beta+\frac{1}{2}; \\ \frac{1}{2}, & \gamma, & \gamma+\frac{1}{2}; & \end{array} x\right] \\ & + \frac{4\delta(\gamma-\alpha)(\gamma-\beta)}{\gamma} x\, {}_2F_1\left[\begin{array}{cc} \delta+\frac{1}{2}, & \delta+1; \\ \frac{3}{2}; & \end{array} x\right] \\ & \times {}_4F_3\left[\begin{array}{cccc} \gamma-\alpha+\frac{1}{2}, & \gamma-\alpha+1, & \gamma-\beta+\frac{1}{2}, & \gamma-\beta+1; \\ \frac{3}{2}, & \gamma+\frac{1}{2}, & \gamma+1; & \end{array} x\right]; \end{aligned} \tag{1.7}$$

$$
\begin{aligned}
&(\gamma+\delta-\alpha-\beta)\,{}_2F_1\left[\begin{matrix}\gamma+\delta-\alpha-\beta+\frac{1}{2}, & \gamma+\delta-\alpha-\beta+1; \\ \frac{3}{2}; & \end{matrix}\ x\right] \\
&\times {}_4F_3\left[\begin{matrix}\alpha, & \alpha+\frac{1}{2}, & \beta, & \beta+\frac{1}{2}; \\ \frac{1}{2}, & \gamma, & \gamma+\frac{1}{2}; & \end{matrix}\ x\right] \\
&+\frac{\alpha\beta}{\gamma}\,{}_2F_1\left[\begin{matrix}\gamma+\delta-\alpha-\beta, & \gamma+\delta-\alpha-\beta+\frac{1}{2}; \\ \frac{1}{2}; & \end{matrix}\ x\right] \\
&\times {}_4F_3\left[\begin{matrix}\alpha+\frac{1}{2}, & \alpha+1, & \beta+\frac{1}{2}, & \beta+1; \\ \frac{3}{2}, & \gamma+\frac{1}{2}, & \gamma+1; & \end{matrix}\ x\right] \\
&=\frac{(\gamma-\alpha)(\gamma-\beta)}{\gamma}\,{}_2F_1\left[\begin{matrix}\delta, & \delta+\frac{1}{2}; \\ \frac{1}{2}; & \end{matrix}\ x\right] \\
&\times {}_4F_3\left[\begin{matrix}\gamma-\alpha+\frac{1}{2}, & \gamma-\alpha+1, & \gamma-\beta+\frac{1}{2}, & \gamma-\beta+1; \\ \frac{3}{2}, & \gamma+\frac{1}{2}, & \gamma+1; & \end{matrix}\ x\right] \\
&+\delta\,{}_2F_1\left[\begin{matrix}\delta+\frac{1}{2}, & \delta+1; \\ \frac{3}{2}; & \end{matrix}\ x\right]{}_4F_3\left[\begin{matrix}\gamma-\alpha, & \gamma-\alpha+\frac{1}{2}, & \gamma-\beta, & \gamma-\beta+\frac{1}{2}; \\ \frac{1}{2}, & \gamma, & \gamma+\frac{1}{2}; & \end{matrix}\ x\right];
\end{aligned}
\tag{1.8}
$$

$$
\begin{aligned}
&{}_0F_1\left[\begin{matrix}-, \\ \frac{1}{2};\end{matrix}\ \frac{1}{4}a^2\right]{}_2F_3\left[\begin{matrix}\alpha, & \alpha+\frac{1}{2}; & \\ \frac{1}{2}, & \beta, & \beta+\frac{1}{2};\end{matrix}\ \frac{1}{4}a^2\right] \\
&+\frac{\alpha ab}{\beta}\,{}_0F_1\left[\begin{matrix}-, \\ \frac{3}{2};\end{matrix}\ \frac{1}{4}b^2\right]{}_2F_3\left[\begin{matrix}\alpha+\frac{1}{2}, & \alpha+1; & \\ \frac{3}{2}, & \beta+\frac{1}{2}, & \beta+1;\end{matrix}\ \frac{1}{4}a^2\right] \\
&={}_0F_1\left[\begin{matrix}-, \\ \frac{1}{2};\end{matrix}\ \frac{1}{4}(a+b)^2\right]{}_2F_3\left[\begin{matrix}\beta-\alpha, & \beta-\alpha+\frac{1}{2}; & \\ \frac{1}{2}, & \beta, & \beta+\frac{1}{2};\end{matrix}\ \frac{1}{4}a^2\right] \\
&+\frac{a(\beta-\alpha)(a+b)}{\beta}\,{}_0F_1\left[\begin{matrix}-, \\ \frac{3}{2};\end{matrix}\ \frac{1}{4}(a+b)^2\right] \\
&\times {}_2F_3\left[\begin{matrix}\beta-\alpha+\frac{1}{2}, & \beta-\alpha+1; & \\ \frac{3}{2}, & \beta+\frac{1}{2}, & \beta+1;\end{matrix}\ \frac{1}{4}a^2\right];
\end{aligned}
\tag{1.9}
$$

$$
\begin{aligned}
& b\,{}_0F_1\left[\begin{matrix}-, \\ \frac{3}{2};\end{matrix}\ \frac{1}{4}b^2\right]{}_2F_3\left[\begin{matrix}\alpha, & \alpha+\frac{1}{2}; & \\ \frac{1}{2}, & \beta, & \beta+\frac{1}{2};\end{matrix}\ \frac{1}{4}a^2\right] \\
& +\frac{\alpha a}{\beta}\,{}_0F_1\left[\begin{matrix}-, \\ \frac{1}{2};\end{matrix}\ \frac{1}{4}b^2\right]{}_2F_3\left[\begin{matrix}\alpha+\frac{1}{2}, & \alpha+1; & \\ \frac{3}{2}, & \beta+\frac{1}{2}, & \beta+1;\end{matrix}\ \frac{1}{4}a^2\right] \\
& =(a+b)\,{}_0F_1\left[\begin{matrix}-, \\ \frac{3}{2};\end{matrix}\ \frac{1}{4}(a+b)^2\right]{}_2F_3\left[\begin{matrix}\beta-\alpha, & \beta-\alpha+\frac{1}{2}; & \\ \frac{1}{2}, & \beta, & \beta+\frac{1}{2};\end{matrix}\ \frac{1}{4}a^2\right] \\
& +\frac{a(\beta-\alpha)}{\beta}\,{}_0F_1\left[\begin{matrix}-, \\ \frac{1}{2};\end{matrix}\ \frac{1}{4}(a+b)^2\right]{}_2F_3\left[\begin{matrix}\beta-\alpha+\frac{1}{2}, & \beta-\alpha+1; & \\ \frac{3}{2}, & \beta+\frac{1}{2}, & \beta+1;\end{matrix}\ \frac{1}{4}a^2\right]. \qquad (1.10)
\end{aligned}
$$

Recently a good progress has been made in generalizing the Euler's transformation (1.5) and Kummer's transformation (1.6).
In 2011, Rakha and Rathie [6] obtained the extension of Euler's II transformation (1.5) in the form:

$$
{}_3F_2\left[\begin{matrix}\alpha, & \beta, & d+1; \\ \gamma+1, & d; & \end{matrix}\ x\right]=(1-x)^{\gamma-\alpha-\beta}\,{}_3F_2\left[\begin{matrix}\gamma-\alpha, & \gamma-\beta, & 1+g; \\ \gamma+1, & g; & \end{matrix}\ x\right] \qquad (1.11)
$$

for $d \neq 0,-1,-2,\ldots$, $g = f\left(\frac{\alpha-\gamma}{\alpha-f}\right)$ with $f = \frac{d(\beta-\gamma)}{\beta-d}$ and in 2005, Paris [4] obtained the following extension of the Kummer first transformation (1.6) in the form:

$$
e^{-x}\,{}_2F_2\left[\begin{matrix}\alpha, & c+1; \\ \beta+1, & c;\end{matrix}\ x\right]={}_2F_2\left[\begin{matrix}\beta-\alpha, & f+1; \\ \beta+1, & f;\end{matrix}\ -x\right] \qquad (1.12)
$$

for $c \neq 0,-1,-2,\ldots$ and $f = \frac{c(\alpha-\beta)}{\alpha-c}$.
The main aim of this paper is to obtain four interesting results involving generalized hypergeometric series by utilizing the results (1.4), (1.11) and (1.12).
Several special cases including the results (1.7) to (1.10) obtained earlier by Sharma [7] have also been given.
The results derived in this paper are simple, interesting, easily established and may be useful.

2 Main Results

The following four general results will be established in this paper:

$$
\begin{aligned}
&{}_2F_1\left[\begin{array}{c} \gamma+\delta-\alpha-\beta,\quad \gamma+\delta-\alpha-\beta+\frac{1}{2}; \\ \frac{1}{2}; \end{array} x\right] \\
&\times {}_5F_4\left[\begin{array}{c} \alpha,\quad \alpha+\frac{1}{2},\quad \beta,\quad \beta+\frac{1}{2},\quad d+1; \\ \frac{1}{2},\quad \gamma+1,\quad \gamma+\frac{1}{2},\quad d; \end{array} x\right] \\
&+\frac{4(\gamma+\delta-\alpha-\beta)\alpha\beta(d+1)}{d(\gamma+1)}x \\
&\times {}_2F_1\left[\begin{array}{c} \gamma+\delta-\alpha-\beta+\frac{1}{2},\quad \gamma+\delta-\alpha-\beta+1; \\ \frac{3}{2}; \end{array} x\right] \\
&\times {}_5F_4\left[\begin{array}{c} \alpha+\frac{1}{2},\quad \alpha+1,\quad \beta+\frac{1}{2},\quad \beta+1,\quad d+\frac{3}{2}; \\ \frac{3}{2},\quad \gamma+1,\quad \gamma+\frac{3}{2},\quad d+\frac{1}{2}; \end{array} x\right] \\
&= {}_2F_1\left[\begin{array}{c} \delta,\quad \delta+\frac{1}{2}; \\ \frac{1}{2}; \end{array} x\right] \\
&\times {}_5F_4\left[\begin{array}{c} \gamma-\alpha,\quad \gamma-\alpha+\frac{1}{2},\quad \gamma-\beta,\quad \gamma-\beta+\frac{1}{2},\quad g+1; \\ \frac{1}{2},\quad \gamma+1,\quad \gamma+\frac{1}{2},\quad g; \end{array} x\right] \\
&+\frac{4\delta(\gamma-\alpha)(\gamma-\beta)(1+g)}{g(\gamma+1)}x\,{}_2F_1\left[\begin{array}{c} \delta+\frac{1}{2},\quad \delta+1; \\ \frac{3}{2}; \end{array} x\right] \\
&\times {}_5F_4\left[\begin{array}{c} \gamma-\alpha+\frac{1}{2},\quad \gamma-\alpha+1,\quad \gamma-\beta+\frac{1}{2},\quad \gamma-\beta+1,\quad g+\frac{3}{2}; \\ \frac{3}{2},\quad \gamma+1,\quad \gamma+\frac{3}{2},\quad g+\frac{1}{2}; \end{array} x\right]; \qquad (2.13)
\end{aligned}
$$

$$
\begin{aligned}
&(\gamma+\delta-\alpha-\beta)\,{}_2F_1\left[\begin{matrix}\gamma+\delta-\alpha-\beta+\frac{1}{2}, & \gamma+\delta-\alpha-\beta+1; \\ \frac{3}{2}; & \end{matrix}\; x\right] \\
&\times {}_5F_4\left[\begin{matrix}\alpha, & \alpha+\frac{1}{2}, & \beta, & \beta+\frac{1}{2}, & d+1; \\ \frac{1}{2}, & \gamma+1, & \gamma+\frac{1}{2}, & d; & \end{matrix}\; x\right] \\
&+\frac{\alpha\beta(d+1)}{d(\gamma+1)}\,{}_2F_1\left[\begin{matrix}\gamma+\delta-\alpha-\beta, & \gamma+\delta-\alpha-\beta+\frac{1}{2}; \\ \frac{1}{2}; & \end{matrix}\; x\right] \\
&\times {}_5F_4\left[\begin{matrix}\alpha+\frac{1}{2}, & \alpha+1, & \beta+\frac{1}{2}, & \beta+1, & d+\frac{3}{2}; \\ \frac{3}{2}, & \gamma+1, & \gamma+\frac{3}{2}, & d+\frac{1}{2}; & \end{matrix}\; x\right] \\
&=\frac{(\gamma-\alpha)(\gamma-\beta)(1+g)}{g(\gamma+1)}\,{}_2F_1\left[\begin{matrix}\delta, & \delta+\frac{1}{2}; \\ \frac{1}{2}; & \end{matrix}\; x\right] \\
&\times {}_5F_4\left[\begin{matrix}\gamma-\alpha+\frac{1}{2}, & \gamma-\alpha+1, & \gamma-\beta+\frac{1}{2}, & \gamma-\beta+1, & g+\frac{3}{2}; \\ \frac{3}{2}, & \gamma+1, & \gamma+\frac{3}{2}, & g+\frac{1}{2}; & \end{matrix}\; x\right] \\
&+\delta\,{}_2F_1\left[\begin{matrix}\delta+\frac{1}{2}, & \delta+1; \\ \frac{3}{2}; & \end{matrix}\; x\right] \\
&\times {}_5F_4\left[\begin{matrix}\gamma-\alpha, & \gamma-\alpha+\frac{1}{2}, & \gamma-\beta, & \gamma-\beta+\frac{1}{2}, & g+1; \\ \frac{1}{2}, & \gamma+\frac{1}{2}, & \gamma+1, & g; & \end{matrix}\; x\right];
\end{aligned}
\tag{2.14}
$$

$$
\begin{aligned}
&{}_0F_1\left[\begin{matrix}-, \\ \frac{1}{2};\end{matrix}\; \frac{1}{4}b^2\right]{}_3F_4\left[\begin{matrix}\alpha, & \alpha+\frac{1}{2}, & c+1; & \\ \frac{1}{2}, & \beta+1, & \beta+\frac{1}{2}, & c;\end{matrix}\; \frac{1}{4}a^2\right] \\
&+\frac{\alpha ab(c+1)}{c(\beta+1)}\,{}_0F_1\left[\begin{matrix}-; \\ \frac{3}{2};\end{matrix}\; \frac{1}{4}b^2\right]{}_3F_4\left[\begin{matrix}\alpha+\frac{1}{2}, & \alpha+1, & c+\frac{3}{2}; & \\ \frac{3}{2}, & \beta+1, & \beta+\frac{3}{2}, & c+\frac{1}{2};\end{matrix}\; \frac{1}{4}a^2\right] \\
&={}_0F_1\left[\begin{matrix}-; \\ \frac{1}{2};\end{matrix}\; \frac{1}{4}(a+b)^2\right]{}_3F_4\left[\begin{matrix}\beta-\alpha, & \beta-\alpha+\frac{1}{2}, & f+1; & \\ \frac{1}{2}, & \beta+\frac{1}{2}, & \beta+1, & f;\end{matrix}\; \frac{1}{4}a^2\right] \\
&+\frac{a(a+b)(\beta-\alpha)(f+1)}{f(\beta+1)}\,{}_0F_1\left[\begin{matrix}-; \\ \frac{3}{2};\end{matrix}\; \frac{1}{4}(a+b)^2\right] \\
&\times {}_3F_4\left[\begin{matrix}\beta-\alpha+\frac{1}{2}, & \beta-\alpha+1, & f+\frac{3}{2}; & \\ \frac{3}{2}, & \beta+1, & \beta+\frac{3}{2}, & f+\frac{1}{2};\end{matrix}\; \frac{1}{4}a^2\right];
\end{aligned}
\tag{2.15}
$$

$$
\begin{aligned}
& b\,{}_0F_1\left[\begin{array}{c} -, \\ \frac{3}{2}; \end{array} \frac{1}{4}b^2\right] {}_3F_4\left[\begin{array}{cccc} \alpha, & \alpha+\frac{1}{2}, & c+1; & \\ \frac{1}{2}, & \beta+1, & \beta+\frac{1}{2}, & c; \end{array} \frac{1}{4}a^2\right] \\
& + \frac{\alpha a(c+1)}{c(\beta+1)}\,{}_0F_1\left[\begin{array}{c} -; \\ \frac{1}{2}; \end{array} \frac{1}{4}b^2\right] {}_3F_4\left[\begin{array}{cccc} \alpha+\frac{1}{2}, & \alpha+1, & c+\frac{3}{2}; & \\ \frac{3}{2}, & \beta+1, & \beta+\frac{3}{2}, & c+\frac{1}{2}; \end{array} \frac{1}{4}a^2\right] \\
& = (a+b)\,{}_0F_1\left[\begin{array}{c} -; \\ \frac{3}{2}; \end{array} \frac{1}{4}(a+b)^2\right] {}_3F_4\left[\begin{array}{cccc} \beta-\alpha, & \beta-\alpha+\frac{1}{2}, & f+1; & \\ \frac{1}{2}, & \beta+\frac{1}{2}, & \beta+1, & f; \end{array} \frac{1}{4}a^2\right] \\
& + \frac{a(\beta-\alpha)(f+1)}{f(\beta+1)}\,{}_0F_1\left[\begin{array}{c} -; \\ \frac{1}{2}; \end{array} \frac{1}{4}(a+b)^2\right] \\
& \times {}_3F_4\left[\begin{array}{cccc} \beta-\alpha+\frac{1}{2}, & \beta-\alpha+1, & f+\frac{3}{2}; & \\ \frac{3}{2}, & \beta+1, & \beta+\frac{3}{2}, & f+\frac{1}{2}, \end{array} \frac{1}{4}a^2\right]. \qquad (2.16)
\end{aligned}
$$

3 Derivations

In order to prove our first two results (2.13) and (2.14), we proceed as follows. Writing the result (1.11) in the form

$$
(1-x)^{\alpha+\beta-\gamma}\,{}_3F_2\left[\begin{array}{ccc} \alpha, & \beta, & d+1; \\ \gamma+1, & d; & \end{array} x\right] = {}_3F_2\left[\begin{array}{ccc} \gamma-\alpha, & \gamma-\beta, & 1+g; \\ \gamma+1, & g; & \end{array} x\right]. \qquad (3.17)
$$

Multiplying both sides of (3.17) by $(1-x)^{-\delta}$, we have

$$
\begin{aligned}
& (1-x)^{\alpha+\beta-\gamma-\delta}\,{}_3F_2\left[\begin{array}{ccc} \alpha, & \beta, & d+1; \\ \gamma+1, & d; & \end{array} x\right] \\
& = (1-x)^{-\delta}\,{}_3F_2\left[\begin{array}{ccc} \gamma-\alpha, & \gamma-\beta, & 1+g; \\ \gamma+1, & g; & \end{array} x\right], \qquad (3.18)
\end{aligned}
$$

writing $(1-x)^{-a}$ as ${}_1F_0\left[\begin{array}{c} a; \\ -; \end{array} x\right]$ and using the result (1.4) in the left-hand side as well as in the right-hand side, after some simplification we get

$$\left\{ {}_2F_1\left[\begin{matrix} \frac{1}{2}(\gamma+\delta-\alpha-\beta), & \frac{1}{2}(\gamma+\delta-\alpha-\beta+1); \\ \frac{1}{2}; & \end{matrix}\; x^2\right]\right.$$

$$\left. +(\gamma+\delta-\alpha-\beta)x\,{}_2F_1\left[\begin{matrix} \frac{1}{2}(\gamma+\delta-\alpha-\beta+1), & \frac{1}{2}(\gamma+\delta-\alpha-\beta+2); \\ \frac{3}{2}; & \end{matrix}\; x^2\right]\right\}$$

$$\times\left\{ {}_6F_5\left[\begin{matrix} \frac{1}{2}\alpha, & \frac{1}{2}\alpha+\frac{1}{2}, & \frac{1}{2}\beta, & \frac{1}{2}\beta+\frac{1}{2}, & \frac{1}{2}(d+1), & \frac{1}{2}(d+1)+\frac{1}{2}; \\ \frac{1}{2}, & \frac{1}{2}(\gamma+1), & \frac{1}{2}(\gamma+1)+\frac{1}{2}, & \frac{1}{2}d, & \frac{1}{2}d+\frac{1}{2}; & \end{matrix}\; x^2\right]\right.$$

$$+\frac{\alpha\beta(d+1)}{d(\gamma+1)}x\,{}_6F_5\left[\begin{matrix} \frac{1}{2}\alpha+\frac{1}{2}, & \frac{1}{2}\alpha+1, & \frac{1}{2}\beta+\frac{1}{2}, & \frac{1}{2}\beta+1, \\ \frac{3}{2}, & \frac{1}{2}(\gamma+1)+\frac{1}{2}, & \frac{1}{2}(\gamma+1)+1, & \frac{1}{2}d+\frac{1}{2}, \end{matrix}\right.$$

$$\left.\left.\begin{matrix} \frac{1}{2}(d+1)+\frac{1}{2}, & \frac{1}{2}(d+1)+1; \\ \frac{1}{2}d+1; & \end{matrix}\; x^2\right]\right\}$$

$$=\left\{ {}_2F_1\left[\begin{matrix} \frac{1}{2}\delta, & \frac{1}{2}\delta+\frac{1}{2}; \\ \frac{1}{2}; & \end{matrix}\; x^2\right] + \delta x\,{}_2F_1\left[\begin{matrix} \frac{1}{2}\delta+\frac{1}{2}, & \frac{1}{2}\delta+1; \\ \frac{3}{2}; & \end{matrix}\; x^2\right]\right\}$$

$$\times\left\{ {}_6F_5\left[\begin{matrix} \frac{1}{2}(\gamma-\alpha), & \frac{1}{2}(\gamma-\alpha)+\frac{1}{2}, & \frac{1}{2}(\gamma-\beta), & \frac{1}{2}(\gamma-\beta)+\frac{1}{2}, & \frac{1}{2}(1+g), \\ \frac{1}{2}, & \frac{1}{2}(\gamma+1), & \frac{1}{2}(\gamma+1)+\frac{1}{2}, & \frac{1}{2}g, & \frac{1}{2}g+\frac{1}{2}; \end{matrix}\right.\right.$$

$$\left.\begin{matrix} \frac{1}{2}(1+g)+\frac{1}{2}; \\ \end{matrix}\; x^2\right] + \frac{(\gamma-\alpha)(\gamma-\beta)(1+g)}{g(\gamma+1)}x$$

$$\times\,{}_6F_5\left[\begin{matrix} \frac{1}{2}(\gamma-\alpha)+\frac{1}{2}, & \frac{1}{2}(\gamma-\alpha)+1, & \frac{1}{2}(\gamma-\beta)+\frac{1}{2}, \\ \frac{3}{2}, & \frac{1}{2}(\gamma+1)+\frac{1}{2}, & \frac{1}{2}(\gamma+1)+1, \end{matrix}\right.$$

$$\left.\left.\begin{matrix} \frac{1}{2}(\gamma-\beta)+1, & \frac{1}{2}(1+g)+\frac{1}{2}, & \frac{1}{2}(1+g)+1; \\ \frac{1}{2}g+\frac{1}{2}, & \frac{1}{2}g+1; & \end{matrix}\; x^2\right]\right\}.$$

Further replace x by ix, we have

$$\left\{ {}_2F_1\left[\begin{matrix} \frac{1}{2}(\gamma+\delta-\alpha-\beta), & \frac{1}{2}(\gamma+\delta-\alpha-\beta+1); \\ \frac{1}{2}; \end{matrix} \; (ix)^2\right] \right.$$

$$\left. +(\gamma+\delta-\alpha-\beta)(ix)\,{}_2F_1\left[\begin{matrix} \frac{1}{2}(\gamma+\delta-\alpha-\beta+1), & \frac{1}{2}(\gamma+\delta-\alpha-\beta+2); \\ \frac{3}{2}; \end{matrix} \; (ix)^2\right]\right\}$$

$$\times\left\{ {}_6F_5\left[\begin{matrix} \frac{1}{2}\alpha, & \frac{1}{2}\alpha+\frac{1}{2}, & \frac{1}{2}\beta, & \frac{1}{2}\beta+\frac{1}{2}, & \frac{1}{2}(d+1), & \frac{1}{2}(d+1)+\frac{1}{2}; \\ \frac{1}{2}, & \frac{1}{2}(\gamma+1), & \frac{1}{2}(\gamma+1)+\frac{1}{2}, & \frac{1}{2}d, & \frac{1}{2}d+\frac{1}{2}; \end{matrix} \; (ix)^2\right]\right.$$

$$+\frac{\alpha\beta(d+1)}{d(\gamma+1)}(ix)\,{}_6F_5\left[\begin{matrix} \frac{1}{2}\alpha+\frac{1}{2}, & \frac{1}{2}\alpha+1, & \frac{1}{2}\beta+\frac{1}{2}, & \frac{1}{2}\beta+1, & \frac{1}{2}(d+1)+\frac{1}{2}, & \frac{1}{2}(d+1)+1; \\ \frac{3}{2}, & \frac{1}{2}(\gamma+1)+\frac{1}{2}, & \frac{1}{2}(\gamma+1)+1, & \frac{1}{2}d+\frac{1}{2}, & \frac{1}{2}d+1; \end{matrix} \; (ix)^2\right]\left.\right\}$$

$$=\left\{ {}_2F_1\left[\begin{matrix} \frac{1}{2}\delta, & \frac{1}{2}\delta+\frac{1}{2}; \\ \frac{1}{2}; \end{matrix} \; (ix)^2\right] + \delta(ix)\,{}_2F_1\left[\begin{matrix} \frac{1}{2}\delta+\frac{1}{2}, & \frac{1}{2}\delta+1; \\ \frac{3}{2}; \end{matrix} \; (ix)^2\right]\right\}$$

$$\times\left\{ {}_6F_5\left[\begin{matrix} \frac{1}{2}(\gamma-\alpha), & \frac{1}{2}(\gamma-\alpha)+\frac{1}{2}, & \frac{1}{2}(\gamma-\beta), & \frac{1}{2}(\gamma-\beta)+\frac{1}{2}, & \frac{1}{2}(1+g), & \frac{1}{2}(1+g)+\frac{1}{2}; \\ \frac{1}{2}, & \frac{1}{2}(\gamma+1), & \frac{1}{2}(\gamma+1)+\frac{1}{2}, & \frac{1}{2}g, & \frac{1}{2}g+\frac{1}{2}; \end{matrix} \; (ix)^2\right] + \frac{(\gamma-\alpha)(\gamma-\beta)(1+g)}{g(\gamma+1)}(ix)\right.$$

$$\times\,{}_6F_5\left[\begin{matrix} \frac{1}{2}(\gamma-\alpha)+\frac{1}{2}, & \frac{1}{2}(\gamma-\alpha)+1, & \frac{1}{2}(\gamma-\beta)+\frac{1}{2}, & \frac{1}{2}(\gamma-\beta)+1, & \frac{1}{2}(1+g)+\frac{1}{2}, & \frac{1}{2}(1+g)+1; \\ \frac{3}{2}, & \frac{1}{2}(\gamma+1)+\frac{1}{2}, & \frac{1}{2}(\gamma+1)+1, & \frac{1}{2}g+\frac{1}{2}, & \frac{1}{2}g+1; \end{matrix} \; (ix)^2\right]\left.\right\}$$

On equating real and imaginary parts (and replacing α,β,γ and δ by $2\alpha,2\beta,2\gamma$ and 2δ and $-x^2$ by x), we get the results (2.13) and (2.14) respectively. This completes the proof of our first two results (2.13) and (2.14).□
In exactly the same manner, first by replacing x by ax in (1.12) and multiply both sides by e^{bx} and using the result (1.4), we get (3.17) and (3.18) respectively, so the details of the proof are omitted.

4 Special Cases

In this section, we shall consider several interesting special cases of our main results.

1. In (2.13) and (2.14), if we take $d=\gamma$, then $f=\gamma$ and $g=\gamma$, after some simplification we get, the known results (1.7) and (1.8) (in corrected form) due to Sharma [7].

2. In (2.15) and (2.16), if we take $c=\beta$, then $f=\beta$, after some simplification we get, the known results (1.9) and (1.10) due to Sharma [7].

3. In (2.13) and (2.14), if we put $\delta = 0$, we get the following result:

$$
\begin{aligned}
&{}_5F_4\left[\begin{matrix}\gamma-\alpha, & \gamma-\alpha+\frac{1}{2}, & \gamma-\beta, & \gamma-\beta+\frac{1}{2}, & g+1; \\ \frac{1}{2}, & \gamma+\frac{1}{2}, & \gamma+1, & g; & \end{matrix}\; x\right] \\
&= {}_2F_1\left[\begin{matrix}\gamma-\alpha-\beta, & \gamma-\alpha-\beta+\frac{1}{2}; \\ \frac{1}{2}; & \end{matrix}\; x\right] \\
&\times {}_5F_4\left[\begin{matrix}\alpha, & \alpha+\frac{1}{2}, & \beta, & \beta+\frac{1}{2}, & d+1; \\ \frac{1}{2}, & \gamma+\frac{1}{2}, & \gamma+1, & d; & \end{matrix}\; x\right] \\
&+ \frac{4\alpha\beta(\gamma-\alpha-\beta)(d+1)}{d(\gamma+1)} x\, {}_2F_1\left[\begin{matrix}\gamma-\alpha-\beta+\frac{1}{2}, & \gamma-\alpha-\beta+1; \\ \frac{3}{2}; & \end{matrix}\; x\right] \\
&\times {}_5F_4\left[\begin{matrix}\alpha+\frac{1}{2}, & \alpha+1, & \beta+\frac{1}{2}, & \beta+1, & d+\frac{3}{2}; \\ \frac{3}{2}, & \gamma+1, & \gamma+\frac{3}{2}, & d+\frac{1}{2}; & \end{matrix}\; x\right]
\end{aligned} \tag{4.19}
$$

and

$$
\begin{aligned}
&{}_5F_4\left[\begin{matrix}\gamma-\alpha+\frac{1}{2}, & \gamma-\alpha+1, & \gamma-\beta+\frac{1}{2}, & \gamma-\beta+1, & g+\frac{3}{2}; \\ \frac{3}{2}, & \gamma+\frac{3}{2}, & \gamma+1, & g+\frac{1}{2}; & \end{matrix}\; x\right] \\
&= \frac{g(\gamma-\alpha-\beta)(\gamma+1)}{(\gamma-\alpha)(\gamma-\beta)(1+g)}\, {}_2F_1\left[\begin{matrix}\gamma-\alpha-\beta+\frac{1}{2}, & \gamma-\alpha-\beta+1; \\ \frac{3}{2}; & \end{matrix}\; x\right] \\
&\times {}_5F_4\left[\begin{matrix}\alpha, & \alpha+\frac{1}{2}, & \beta, & \beta+\frac{1}{2}, & d+1; \\ \frac{1}{2}, & \gamma+\frac{1}{2}, & \gamma+1, & d; & \end{matrix}\; x\right] \\
&+ \frac{\alpha\beta(d+1)g(\gamma+1)}{(\gamma-\alpha)(\gamma-\beta)d(\gamma+1)(1+g)}\, {}_2F_1\left[\begin{matrix}\gamma-\alpha-\beta+\frac{1}{2}, & \gamma-\alpha-\beta; \\ \frac{1}{2}; & \end{matrix}\; x\right] \\
&\times {}_5F_4\left[\begin{matrix}\alpha+\frac{1}{2}, & \alpha+1, & \beta+\frac{1}{2}, & \beta+1, & d+\frac{3}{2}; \\ \frac{3}{2}, & \gamma+1, & \gamma+\frac{3}{2}, & d+\frac{1}{2}; & \end{matrix}\; x\right].
\end{aligned} \tag{4.20}
$$

Further, in (4.19) and (4.20), if we set $d=\gamma$ then $g=\gamma$, we get the following result due to Sharma [7]

$$
\begin{aligned}
&{}_4F_3\left[\begin{matrix}\gamma-\alpha, & \gamma-\alpha+\frac{1}{2}, & \gamma-\beta, & \gamma-\beta+\frac{1}{2}; \\ \frac{1}{2}, & \gamma, & \gamma+\frac{1}{2}; & \end{matrix}\; x\right] \\
&= {}_2F_1\left[\begin{matrix}\gamma-\alpha-\beta, & \gamma-\alpha-\beta+\frac{1}{2}; \\ \frac{1}{2}; & \end{matrix}\; x\right] {}_4F_3\left[\begin{matrix}\alpha, & \alpha+\frac{1}{2}, & \beta, & \beta+\frac{1}{2}; \\ \frac{1}{2}, & \gamma, & \gamma+\frac{1}{2}; & \end{matrix}\; x\right] \\
&+ \frac{4\alpha\beta(\gamma-\alpha-\beta)}{\gamma} x\, {}_2F_1\left[\begin{matrix}\gamma-\alpha-\beta+\frac{1}{2}, & \gamma-\alpha-\beta+1; \\ \frac{3}{2}; & \end{matrix}\; x\right] \\
&\times {}_4F_3\left[\begin{matrix}\alpha+\frac{1}{2}, & \alpha+1, & \beta+\frac{1}{2}, & \beta+1; \\ \frac{3}{2}, & \gamma+\frac{1}{2}, & \gamma+1; & \end{matrix}\; x\right]
\end{aligned}
\tag{4.21}
$$

and

$$
\begin{aligned}
&{}_4F_3\left[\begin{matrix}\gamma-\alpha+\frac{1}{2}, & \gamma-\alpha+1, & \gamma-\beta+\frac{1}{2}, & \gamma-\beta+1; \\ \frac{3}{2}, & \gamma+\frac{1}{2}, & \gamma+1; & \end{matrix}\; x\right] \\
&= \frac{\gamma(\gamma-\alpha-\beta)}{(\gamma-\alpha)(\gamma-\beta)}\, {}_2F_1\left[\begin{matrix}\gamma-\alpha-\beta+\frac{1}{2}, & \gamma-\alpha-\beta+1; \\ \frac{3}{2}; & \end{matrix}\; x\right] \\
&\times {}_4F_3\left[\begin{matrix}\alpha, & \alpha+\frac{1}{2}, & \beta, & \beta+\frac{1}{2}; \\ \frac{1}{2}, & \gamma, & \gamma+\frac{1}{2}; & \end{matrix}\; x\right] \\
&+ \frac{\alpha\beta}{(\gamma-\alpha)(\gamma-\beta)}\, {}_2F_1\left[\begin{matrix}\gamma-\alpha-\beta, & \gamma-\alpha-\beta+\frac{1}{2}; \\ \frac{1}{2}; & \end{matrix}\; x\right] \\
&\times {}_4F_3\left[\begin{matrix}\alpha+\frac{1}{2}, & \alpha+1, & \beta+\frac{1}{2}, & \beta+1; \\ \frac{3}{2}, & \gamma+\frac{1}{2}, & \gamma+1; & \end{matrix}\; x\right].
\end{aligned}
\tag{4.22}
$$

4. In (2.15) and (2.16), put $b=0$ and writing $\frac{1}{4}a^2=x$, then we get the following results:

$$
\begin{aligned}
&{}_3F_4\left[\begin{matrix}\alpha, & \alpha+\frac{1}{2}, & c+1; & \\ \frac{1}{2}, & \beta+1, & \beta+\frac{1}{2}, & c; \end{matrix}\; x\right] \\
&= {}_0F_1\left[\begin{matrix}-; \\ \frac{1}{2}; \end{matrix}\; x\right] {}_3F_4\left[\begin{matrix}\beta-\alpha, & \beta-\alpha+\frac{1}{2}, & f+1; & \\ \frac{1}{2}, & \beta+\frac{1}{2}, & \beta+1, & f; \end{matrix}\; x\right] \\
&+ \frac{4x(\beta-\alpha)(f+1)}{f(\beta+1)}\, {}_0F_1\left[\begin{matrix}-; \\ \frac{3}{2}; \end{matrix}\; x\right] \\
&\times {}_3F_4\left[\begin{matrix}\beta-\alpha+\frac{1}{2}, & \beta-\alpha+1, & f+\frac{3}{2}; & \\ \frac{3}{2}, & \beta+1, & \beta+\frac{3}{2}, & f+\frac{1}{2}; \end{matrix}\; x\right]
\end{aligned}
\tag{4.23}
$$

and

$$
\begin{aligned}
&{}_3F_4\left[\begin{array}{cccc} \alpha+\frac{1}{2}, & \alpha+1, & c+\frac{3}{2}; & \\ & & & x \\ \frac{3}{2}, & \beta+1, & \beta+\frac{3}{2}, & c+\frac{1}{2}; \end{array}\right] \\
&= \frac{c(\beta+1)}{\alpha(c+1)}\, {}_0F_1\left[\begin{array}{c} -; \\ \quad x \\ \frac{3}{2}; \end{array}\right] {}_3F_4\left[\begin{array}{cccc} \beta-\alpha, & \beta-\alpha+\frac{1}{2}, & f+1; & \\ & & & x \\ \frac{1}{2}, & \beta+\frac{1}{2}, & \beta+1, & f; \end{array}\right] \\
&+ \frac{(\beta-\alpha)(f+1)x}{f\alpha(c+1)}\, {}_0F_1\left[\begin{array}{c} -; \\ \quad x \\ \frac{1}{2}; \end{array}\right] \\
&\times {}_3F_4\left[\begin{array}{cccc} \beta-\alpha+\frac{1}{2}, & \beta-\alpha+1, & f+\frac{3}{2}; & \\ & & & x \\ \frac{3}{2}, & \beta+1, & \beta+\frac{3}{2}, & f+\frac{1}{2}, \end{array}\right]. \qquad (4.24)
\end{aligned}
$$

Further, (4.23) and (4.24) if we set $c=\beta$ then $f=\beta$, we get the following results due to Sharma [7]:

$$
\begin{aligned}
&{}_2F_3\left[\begin{array}{ccc} \alpha, & \alpha+\frac{1}{2}; & \\ & & x \\ \frac{1}{2}, & \beta, & \beta+\frac{1}{2}; \end{array}\right] \\
&= {}_0F_1\left[\begin{array}{c} -; \\ \quad x \\ \frac{1}{2}; \end{array}\right] {}_2F_3\left[\begin{array}{ccc} \beta-\alpha, & \beta-\alpha+\frac{1}{2}; & \\ & & x \\ \frac{1}{2}, & \beta, & \beta+\frac{1}{2}; \end{array}\right] \\
&+ \frac{4x(\beta-\alpha)}{\beta}\, {}_0F_1\left[\begin{array}{c} -; \\ \quad x \\ \frac{3}{2}; \end{array}\right] {}_2F_3\left[\begin{array}{ccc} \beta-\alpha+\frac{1}{2}, & \beta-\alpha+1; & \\ & & x \\ \frac{3}{2}, & \beta+1, & \beta+\frac{1}{2}; \end{array}\right] \qquad (4.25)
\end{aligned}
$$

and

$$
\begin{aligned}
&{}_2F_3\left[\begin{array}{ccc} \alpha+\frac{1}{2}, & \alpha+1; & \\ & & x \\ \frac{3}{2}, & \beta+\frac{1}{2}, & \beta+1; \end{array}\right] \\
&= \frac{\beta}{\alpha}\, {}_0F_1\left[\begin{array}{c} -; \\ \quad x \\ \frac{3}{2}; \end{array}\right] {}_2F_3\left[\begin{array}{ccc} \beta-\alpha, & \beta-\alpha+\frac{1}{2}; & \\ & & x \\ \frac{1}{2}, & \beta, & \beta+\frac{1}{2}; \end{array}\right] \\
&+ \frac{(\beta-\alpha)}{\alpha}\, {}_0F_1\left[\begin{array}{c} -; \\ \quad x \\ \frac{1}{2}; \end{array}\right] {}_2F_3\left[\begin{array}{ccc} \beta-\alpha+\frac{1}{2}, & \beta-\alpha+1; & \\ & & x \\ \frac{3}{2}, & \beta+1, & \beta+\frac{1}{2}; \end{array}\right]. \qquad (4.26)
\end{aligned}
$$

5. In (2.15) and (2.16), put $\alpha=\beta$ and replacing $\frac{1}{4}b^2=x^2$ and $\frac{1}{4}a^2=y^2$, then we get the following result:

$$
\begin{aligned}
&{}_0F_1\left[\begin{array}{c} -, \\ \quad (x+y)^2 \\ \frac{1}{2}; \end{array}\right] \\
&= {}_0F_1\left[\begin{array}{c} -; \\ \quad x^2 \\ \frac{1}{2}; \end{array}\right] {}_2F_3\left[\begin{array}{ccc} \beta, & c+1; & \\ & & y^2 \\ \frac{1}{2}, & \beta+1, & c; \end{array}\right] \\
&+ 4xy\frac{\beta(c+1)}{c(\beta+1)}\, {}_0F_1\left[\begin{array}{c} -; \\ \quad x^2 \\ \frac{3}{2}; \end{array}\right] {}_2F_3\left[\begin{array}{ccc} \beta+\frac{1}{2}, & c+\frac{3}{2}; & \\ & & y^2 \\ \frac{3}{2}, & \beta+\frac{3}{2}, & c+\frac{1}{2}; \end{array}\right] \qquad (4.27)
\end{aligned}
$$

and

$$(x+y)\,{}_0F_1\left[\begin{matrix} -, \\ \frac{3}{2}; \end{matrix}\; (x+y)^2\right]$$
$$= x\,{}_0F_1\left[\begin{matrix} -; \\ \frac{3}{2}; \end{matrix}\; x^2\right] {}_2F_3\left[\begin{matrix} \beta, \quad c+1; \\ \frac{1}{2}, \quad \beta+1, \quad c; \end{matrix}\; y^2\right]$$
$$+\frac{\beta(c+1)}{c(\beta+1)} y\,{}_0F_1\left[\begin{matrix} -; \\ \frac{1}{2}; \end{matrix}\; x^2\right] {}_2F_3\left[\begin{matrix} c+\frac{3}{2}, \quad \beta+\frac{1}{2}; \\ \frac{3}{2}, \quad c+\frac{1}{2}, \quad \beta+\frac{3}{2}; \end{matrix}\; y^2\right]. \tag{4.28}$$

Further, in (4.27) and (4.28), if we set $c=\beta$, we geth the following results due to Sharma [7]:

$${}_0F_1\left[\begin{matrix} -, \\ \frac{1}{2}; \end{matrix}\; (x+y)^2\right] = {}_0F_1\left[\begin{matrix} -; \\ \frac{1}{2}; \end{matrix}\; x^2\right] {}_0F_1\left[\begin{matrix} -; \\ \frac{1}{2}, \end{matrix}\; y^2\right]$$
$$+4xy\,{}_0F_1\left[\begin{matrix} -; \\ \frac{3}{2}; \end{matrix}\; x^2\right] {}_0F_1\left[\begin{matrix} -; \\ \frac{3}{2}, \end{matrix}\; y^2\right] \tag{4.29}$$

and

$$(x+y)\,{}_0F_1\left[\begin{matrix} -, \\ \frac{3}{2}; \end{matrix}\; (x+y)^2\right] = x\,{}_0F_1\left[\begin{matrix} -; \\ \frac{3}{2}; \end{matrix}\; x^2\right] {}_0F_1\left[\begin{matrix} -; \\ \frac{1}{2}, \end{matrix}\; y^2\right]$$
$$+y\,{}_0F_1\left[\begin{matrix} -; \\ \frac{1}{2}; \end{matrix}\; x^2\right] {}_0F_1\left[\begin{matrix} -; \\ \frac{3}{2}, \end{matrix}\; y^2\right]. \tag{4.30}$$

Similarly, other results can be obtained.

5 Conflict of Interests

The authors declare that they have no any conflict of interests.

Acknowledgments

All authors contributed equally in this paper. They read and approved the final manuscript.

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