Available online at www.ispacs.com/jiasc
Volume 2012, Year 2012 Article ID jiasc-00014, 10 pages
doi:10.5899/2012/jiasc-00014
Research Article

Journal of Interpolation and Approximation in Scientific Computing

Minimal solution of linear formed fuzzy matrix equations

Maryam Mosleh *
Department of Mathematics, Firoozkooh Branch, Islamic Azad University, Firoozkooh, Iran

Abstract
In this paper according to the structured element method, the $m \times n$ inconsistent fuzzy matrix equation $A\tilde{X} = \tilde{B}$, which are linear formed by fuzzy structured element, is investigated. The necessary and sufficient condition for the existence of a fuzzy solution is also discussed. some examples are presented to illustrate the proposed method.
Keywords: Fuzzy number; Inconsistent Fuzzy Matrix Equations; Minimal Solution

1 Introduction

The concept of fuzzy numbers and fuzzy arithmetic operations were first introduced by Zadeh [35], Dubois and Prade [14]. We refer the reader to [24] for more information on fuzzy numbers and fuzzy arithmetic. Fuzzy systems are used to study a variety of problems ranging from fuzzy topological spaces [13] to control chaotic systems [18, 23], fuzzy metric spaces [30], fuzzy differential equations [5], fuzzy linear systems [3, 4, 11, 9, 10, 28, 29] and particle physics [15, 16, 17, 27, 32].

One of the major applications of fuzzy number arithmetic is treating fuzzy linear systems [7, 8], several problems in various areas such as economics, engineering and physics boil down to the solution of a linear system of equations. Friedman *et al.* [19] introduced a general model for solving a fuzzy $n \times n$ linear system whose coefficient matrix is crisp and the right-hand side column is an arbitrary fuzzy number vector. They used the parametric form of fuzzy numbers and replaced the original fuzzy $n \times n$ linear system by a crisp $2n \times 2n$ linear system and studied duality in fuzzy linear systems $Ax = Bx + y$ where A , B are real $n \times n$ matrix, the unknown vector x is vector consisting of n fuzzy numbers and the constant y is vector consisting of n fuzzy numbers, in [20]. In [1, 2, 3, 4, 11] the

*Corresponding author. Email address: mosleh@iaufb.ac.ir Tel:+98 912 6076308

authors presented conjugate gradient, LU decomposition method for solving general fuzzy linear systems or symmetric fuzzy linear systems. Also, Wang *et al.* [33] presented an iterative algorithm for solving dual linear system of the form $x = Ax + u$, where A is real $n \times n$ matrix, the unknown vector x and the constant u are all vectors consisting of fuzzy numbers. Recently, Muzziloi *et al.* [26] considered fuzzy linear systems of the form $A_1x + b_1 = A_2x + b_2$ with $A_1,\ A_2$ square matrices of fuzzy coefficients and $b_1,\ b_2$ fuzzy number vectors. Abbasbany *et al.* [6] gave the solving method of dual general fuzzy linear systems by using the theory of singular value decomposition.

However, the fuzzy number of fuzzy linear systems often appears in the similar shape of membership functions in the actual problems. In order to simplify the solution of this kind of fuzzy linear system, Sun *et al.* [31] investigated the solving problem of linear formed general fuzzy linear systems and we develop this method to inconsistent fuzzy matrix equations of the form $A\tilde{X} = \tilde{B}$ where A is a real matrix, $\tilde{B}$ and $\tilde{X}$ are known and unknown fuzzy matrices.

2 Preliminaries

The minimal solution of an arbitrary linear system is formally defined such that:

1. If the system is consistent and has a unique solution, then this solution is also the minimal solution.

2. If the system is consistent and has a set solution, then the minimal solution is a member of this set that has the least Euclidean norm.

3. If the system is inconsistent and has a unique least squares solution, then this solution is also the minimal solution.

4. If the system is inconsistent and has a least squares set solution, then the minimal solution is a member of this set that has the least Euclidean norm.

Let E be a fuzzy set on $\mathbb{R}$, $E(x)$ is the membership function of E. Then E is called a symmetrical fuzzy structured element on $\mathbb{R}$, If (i) $\forall x \in (-1, 1), E(x) > 0$; (ii) $E(0) = 1$; (iii) $E(x)$ is a function of monotone increasing and right continuous on $[-1, 0]$, monotone decreasing and left continuous on $(0, 1]$ and $E(x) = E(-x)$.

For example, let E be fuzzy set, the membership function

$$E(x) = \begin{cases} 1 + x, & x \in [-1, 0], \\ 1 - x, & x \in [0, 1], \\ 0, & other \end{cases}$$

is called triangle structured element and the membership function

$$E(x) = \begin{cases} 1, & x \in [-1, 1], \\ 0, & other \end{cases}$$

is called rectangle structured element. Triangle structured element and rectangle structured element are both symmetrical fuzzy structured element.

Theorem 2.1. *[21] (Partial mapping theorem) Let E be a fuzzy structured element and $E(x)$ is its membership function and the function $f(x)$ is continuous and monotone on $[-1, 1]$, then $f(E)$ is a fuzzy number and the membership function of $f(E)$is $E(f^{-1}(x)$. (where $f^{-1}(x)$ is rotational symmetry function for variable x and y. If f is a strictly monotone function, then $f^{-1}(x)$ is the inverse function of $f(x)$.)*

Theorem 2.2. *[21] For a given canonical fuzzy structured element E and any finite fuzzy number V, there always exists a monotone bounded function f on $[-1, 1]$, having the form $V = f(E)$.*

Suppose that E is a given symmetrical fuzzy structured element. If fuzzy number $V = a+bE$, where $a, b \in \mathbb{R}, b \geq 0$, then we call that V is linear formed by fuzzy structured element E.

For a given symmetrical fuzzy structured element E, we can obtain a kind of fuzzy number, which is linear formed by E and has the similar shape of membership function. For example, we let E be triangle structured element, then fuzzy numbers that are linear formed by triangle structuring element E are all triangle number, that is, the shape of their membership function is triangle.

Theorem 2.3. *[22] Suppose that fuzzy numbers U, V are linear formed by the same symmetrical fuzzy structured element E and*

$$U = c_1 + b_1 E, V = c_2 + b_2 E,$$

where c_1, c_2, b_1, b_2 are real numbers and $b_1, b_2 > 0$, then

$$\forall k \in \mathbb{R}, kU = kc_1 + |k|b_1 E,$$

$$U + V = (c_1 + c_2) + (b_1 + b_2)E$$

and the membership function of fuzzy number $U + V$ is

$$(U + V)(x) = E(\frac{x - (c_1 + c_2)}{b_1 + b_2}).$$

Obviously, fuzzy numbers $U + V$ and kU are also linear formed by E.

3 Linear formed general fuzzy matrix equation

Definition 3.1. *The matrix system*

$$\begin{pmatrix} a_{11} & \dots & a_{1n} \\ a_{21} & \dots & a_{2n} \\ \dots & \dots & \dots \\ a_{m1} & \dots & a_{mn} \end{pmatrix} \begin{pmatrix} \tilde{x}_{11} & \dots & \tilde{x}_{1l} \\ \tilde{x}_{21} & \dots & \tilde{x}_{2l} \\ \dots & \dots & \dots \\ \tilde{x}_{n1} & \dots & \tilde{x}_{nl} \end{pmatrix} = \begin{pmatrix} \tilde{b}_{11} & \dots & \tilde{b}_{1l} \\ \tilde{b}_{21} & \dots & \tilde{b}_{2l} \\ \dots & \dots & \dots \\ \tilde{b}_{m1} & \dots & \tilde{b}_{ml} \end{pmatrix}, \tag{3.1}$$

where a_{ij}, $1 \leq i \leq m$, $1 \leq j \leq n$ are crisp numbers and the fuzzy number elements $\tilde{b}_{ij}$ in the right-hand matrix are linear formed by the same symmetrical fuzzy structured elements E, is called a linear formed general fuzzy matrix equation (LFGFME) by fuzzy structured element E.

Using matrix notation, we have

$$A\tilde{X} = \tilde{B}. \tag{3.2}$$

Suppose that fuzzy number $\tilde{b}_{ij}$ is linear formed by the same symmetrical fuzzy structured element E and unknown fuzzy solution $\tilde{x}_{ij}$ be also linear formed by structured element E. A fuzzy number matrix

$$\tilde{X} = (x_1, x_2, \ldots, x_l),$$

given by $x_j = (x_{1j} + y_{1j}E, x_{2j} + y_{2j}E, \ldots, x_{nj} + y_{nj}E)^T$, $1 \leq j \leq l$, where $y_{1j}, ..., y_{nj}$ are nonnegative real numbers, is called a solution of the fuzzy matrix system (3.1) if

$$Ax_j = b_j, \quad j = 1, 2, ..., l,$$

where $b_j = (b_{1j} + c_{1j}E, b_{2j} + c_{2j}E, \ldots, b_{mj} + c_{mj}E)^T$ is the jth column of fuzzy number matrix $\tilde{B}$ and $c_{1j}, ..., c_{nj}$ are nonnegative real numbers.

3.1 Model to the LFGFME

Using the technique in [34], we extend the matrix systems (3.1) into two crisp matrix equation.

Theorem 3.1. *The fuzzy matrix equation (3.1) can be extended into two crisp matrix equation as follows:*

$$\begin{pmatrix} a_{11} & \ldots & a_{1n} \\ a_{21} & \ldots & a_{2n} \\ \ldots & \ldots & \ldots \\ a_{m1} & \ldots & a_{mn} \end{pmatrix} \begin{pmatrix} x_{11} & \ldots & x_{1l} \\ x_{21} & \ldots & x_{2l} \\ \ldots & \ldots & \ldots \\ x_{n1} & \ldots & x_{nl} \end{pmatrix} = \begin{pmatrix} b_{11} & \ldots & b_{1l} \\ b_{21} & \ldots & b_{2l} \\ \ldots & \ldots & \ldots \\ b_{m1} & \ldots & b_{ml} \end{pmatrix} \tag{3.3}$$

and

$$\begin{pmatrix} a'_{11} & \ldots & a'_{1n} \\ a'_{21} & \ldots & a'_{2n} \\ \ldots & \ldots & \ldots \\ a'_{m1} & \ldots & a'_{mn} \end{pmatrix} \begin{pmatrix} y_{11} & \ldots & y_{1l} \\ y_{21} & \ldots & y_{2l} \\ \ldots & \ldots & \ldots \\ y_{n1} & \ldots & y_{nl} \end{pmatrix} = \begin{pmatrix} c_{11} & \ldots & c_{1l} \\ c_{21} & \ldots & c_{2l} \\ \ldots & \ldots & \ldots \\ c_{m1} & \ldots & c_{ml} \end{pmatrix}, \tag{3.4}$$

where

$$\begin{aligned} \tilde{x}_{kj} &= x_{kj} + y_{kj}E, \quad k = 1, 2, ..., n, \quad j = 1, 2, ..., l, \\ \tilde{b}_{kj} &= b_{kj} + c_{kj}E, \quad k = 1, 2, ..., m, \quad j = 1, 2, ..., l. \end{aligned}$$

Proof. *Firstly, we rewrite the original system (3.1) in the forms of matrix*

$$A(x_1, x_2, ..., x_l) = (b_1, b_2, ..., b_l), \tag{3.5}$$

where x_j, b_j denote the jth column of unknown matrix $\tilde{X}$ and fuzzy number matrix $\tilde{B}$, respectively. Thus the original system (3.1) is equivalent to the following fuzzy linear equation

$$Ax_j = b_j, \quad j = 1, 2, .., l. \tag{3.6}$$

Let $\tilde{x}_{kj} = x_{kj} + y_{kj}E,\ k = 1, 2, ..., n,\ \tilde{b}_{kj} = b_{kj} + c_{kj}E,\ k = 1, 2, ..., m$. Then the above equation (3.6) is as follows:

$$\begin{pmatrix} a_{11} & \dots & a_{1n} \\ a_{21} & \dots & a_{2n} \\ \dots & \dots & \dots \\ a_{m1} & \dots & a_{mn} \end{pmatrix} \begin{pmatrix} x_{1j} + y_{1j}E \\ x_{2j} + y_{2j}E \\ \vdots \\ x_{nj} + y_{nj}E \end{pmatrix} = \begin{pmatrix} b_{1j} + c_{1j}E \\ b_{2j} + c_{2j}E \\ \vdots \\ b_{mj} + c_{mj}E \end{pmatrix}, j = 1, ..., l. \tag{3.7}$$

Therefore we have

$$\begin{aligned} &\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \begin{pmatrix} x_{1j} \\ x_{2j} \\ \vdots \\ x_{nj} \end{pmatrix} + \begin{pmatrix} a'_{11} & a'_{12} & \dots & a'_{1n} \\ a_{21} & a'_{22} & \dots & a'_{2n} \\ \dots & \dots & \dots & \dots \\ a'_{m1} & a'_{m2} & \dots & a'_{mn} \end{pmatrix} \\ &\begin{pmatrix} y_{1j} \\ y_{2j} \\ \vdots \\ y_{nj} \end{pmatrix} E = \begin{pmatrix} b_{1j} \\ b_{2j} \\ \vdots \\ b_{mj} \end{pmatrix} + \begin{pmatrix} c_{1j} \\ c_{2j} \\ \vdots \\ c_{mj} \end{pmatrix} E, \end{aligned} \tag{3.8}$$

where a'_{ij} are determined as follows:

$$\begin{aligned} &a_{ij} \geq 0 \Rightarrow a'_{ij} = a_{ij}, \\ &a_{ij} < 0 \Rightarrow a'_{ij} = -a_{ij},\ i = 1, 2, ..., m,\ j = 1, 2, ..., n. \end{aligned}$$

Moreover, A' is a nonnegative matrix.

Finally, we restore the Eq.(3.6) and obtain the following two matrix equation:

$$\begin{cases} AX = B, \\ A'Y = C, \end{cases} \tag{3.9}$$

where

$$X = \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1l} \\ x_{21} & x_{22} & \dots & x_{2l} \\ \dots & \dots & \dots & \dots \\ x_{n1} & x_{n2} & \dots & x_{nl} \end{pmatrix},\ Y = \begin{pmatrix} y_{11} & y_{12} & \dots & y_{1l} \\ y_{21} & y_{22} & \dots & y_{2l} \\ \dots & \dots & \dots & \dots \\ y_{n1} & y_{n2} & \dots & y_{nl} \end{pmatrix},$$

$$B = \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1l} \\ b_{21} & b_{22} & \dots & b_{2l} \\ \dots & \dots & \dots & \dots \\ b_{m1} & b_{m2} & \dots & b_{ml} \end{pmatrix},\ C = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1l} \\ c_{21} & c_{22} & \dots & c_{2l} \\ \dots & \dots & \dots & \dots \\ c_{m1} & c_{m2} & \dots & c_{ml} \end{pmatrix}.$$

The proof is completed. □

Corollary 3.1. *Let T be $p \times q$ real column full rank or row full rank. There exists a $p \times p$ orthogonal matrix U, a $q \times q$ orthogonal matrix V, and a $p \times q$ diagonal matrix Σ with $\langle \Sigma \rangle_{ij} = 0$ for $i \neq j$ and $\langle \Sigma \rangle_{ii} = \sigma_i > 0$ with $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_s > 0$, where $s = \min\{p, q\}$, such that the singular value decomposition*

$$T = U \Sigma V^t,$$

is valid. And if Σ^+ is that $q \times p$ matrix whose only nonzero entries are $\langle \Sigma^+ \rangle_{ii} = 1/\sigma_i$ for $1 \leq i \leq s$, then $T^+ = V \Sigma^+ U^t$ is the unique pseudo-inverse of T. Also if $p = q$ then $T^+ = T^{-1}$.

We refer the reader to [12] for more information on finding pseudo-inverse of an arbitrary matrix, and when we work with full rank matrices, there are not any problem and all calculations are stable and well-posed.

According to singular value decomposition, we can obtain the minimal solution of Eq.(3.3) and Eq.(3.4).

Corollary 3.2. *[25] Let A and A' are row full rank (for $m \leq n$) or column full rank (for $n < m$). The minimal solution of Eq.(3.3) and Eq.(3.4) are obtained by*

$$\begin{array}{l} X = A^{+}B, \\ Y = A'^{+}C. \end{array} \tag{3.10}$$

However, the unique minimal solution X and Y can't always satisfy the solution conditions for LFGFME Eq.(3.1). Next, we will study the necessary and sufficient conditions for the existence of one fuzzy solution.

Theorem 3.2. *If the solution x_{kj}, y_{kj} exists in (3.9), then fuzzy matrix equation (3.1) has fuzzy matrix solution $\tilde{x}_{kj} = x_{kj} + y_{kj}E$ if and only if $y_{kj} \geq 0, k = 1, 2, ..., n,\ j = 1, 2, ..., l$.*

Proof. *From the definition of linear formed fuzzy number by structured element [22], if fuzzy solution $\tilde{x}_{kj} = x_{kj} + y_{kj}E$, we can obtain that $y_{kj} \geq 0$, that is, the necessary condition is true. The sufficient condition is obvious, according to formed method of (3.9) and operation properties of Theorem (2.3), when $y_{kj} \geq 0$, $\tilde{x}_{kj} = x_{kj} + y_{kj}E$ makes the equality of fuzzy linear system established.* □

Theorem 3.3. *If the solution x_{kj}, y_{kj} exists in (3.9) and $(A')^{+}_{ij} \geq 0$, then fuzzy matrix equation (3.1) has fuzzy matrix solution $\tilde{x}_{kj} = x_{kj} + y_{kj}E$.*

Proof. *Apparently, since $Y = (A')^{+}C$, when $(A')_{ij} \geq 0$, we have $Y \geq 0$, that is, it meets the sufficient condition of Theorem (2.3).* □

4 Examples

Example 4.1. *Consider the fuzzy matrix equation*

$$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \tilde{x}_{11} & \tilde{x}_{12} \\ \tilde{x}_{21} & \tilde{x}_{22} \end{pmatrix} = \begin{pmatrix} 1+E & 2+3E \\ 1+2E & 10+5E \end{pmatrix}.$$

Suppose that E is a triangle structured element, unknown fuzzy solution $\tilde{x}_{kj} = x_{kj} + y_{kj}E$ for $k, j = 1, 2$, by Eq.(3.9), we have

$$\begin{cases} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 1 & 10 \end{pmatrix}, \\ \\ \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix}, \end{cases}$$

The pseudo-inverse of A and A' are

$$A^{+} = \frac{1}{4}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, \quad A'^{+} = \frac{1}{4}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \geq 0,$$

therefore, we get the minimal solution $\tilde{x}_{11} = 0.75E, \tilde{x}_{12} = -2 + 2E, \tilde{x}_{21} = 0.75E$ *and* $\tilde{x}_{22} = 2 + 2E$ *and their membership function*

$$\mu_{\tilde{x}_{11}} = \begin{cases} \frac{x+0.75}{0.75}, & x \in [-0.75, 0], \\ \frac{0.75-x}{0.75}, & x \in [0, 0.75], \\ 0, & other, \end{cases} \qquad \mu_{\tilde{x}_{12}} = \begin{cases} \frac{x+4}{2}, & x \in [-4, -2], \\ \frac{-x}{2}, & x \in [-2, 0], \\ 0, & other, \end{cases}$$

$$\mu_{\tilde{x}_{21}} = \begin{cases} \frac{x+0.75}{0.75}, & x \in [-0.75, 0], \\ \frac{0.75-x}{0.75}, & x \in [0, 0.75], \\ 0, & other, \end{cases} \qquad \mu_{\tilde{x}_{22}} = \begin{cases} \frac{x}{2}, & x \in [0, 2], \\ \frac{4-x}{2}, & x \in [2, 4], \\ 0, & other. \end{cases}$$

Example 4.2. *Consider the fuzzy matrix equation*

$$\begin{pmatrix} 1 & 1 \\ -1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} \tilde{x}_{11} & \tilde{x}_{12} \\ \tilde{x}_{21} & \tilde{x}_{22} \end{pmatrix} = \begin{pmatrix} -1+E & 2+3E \\ 1+2E & -4+5E \\ 3+E & 2+2E \end{pmatrix}.$$

Suppose that E is a triangle structured element, unknown fuzzy solution $\tilde{x}_{kj} = x_{kj} + y_{kj}E$ *for* $k, j = 1, 2$*, by Eq.(3.9), we have*

$$\begin{cases} \begin{pmatrix} 1 & 1 \\ -1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 1 & -4 \\ 3 & 2 \end{pmatrix}, \\ \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 2 & 5 \\ 1 & 2 \end{pmatrix}, \end{cases}$$

We get the minimal solution $\tilde{x}_{11} = 0.6667E, \tilde{x}_{12} = 2.5 + 1.6667E, \tilde{x}_{21} = -1 + 0.6667E$ *and* $\tilde{x}_{22} = -0.5 + 1.6667E$.

5 Conclusion

In this paper, we proposed a general model for solving a class of inconsistent fuzzy matrix equation $A\tilde{X} = \tilde{B}$ which A is a $m \times n$ crisp matrix and the right-hand side matrix $\tilde{B}$ is an $m \times l$ arbitrary fuzzy number matrix, which is linear formed by the same symmetrical fuzzy structured element E. In this paper we the original system was converted to two crisp matrix equation.

References

[1] S. Abbasbandy and M. Alavi, A method for solving fuzzy linear systems, Iranian J. fuzzy systems 2 (2005) 37-43.

[2] S. Abbasbandy and M. Alavi, A new method for solving symmetric fuzzy linear systems, Mathematics Scientific Journal, Islamic Azad University of Arak, 1 (2005) 55-62.

[3] S. Abbasbandy, A. Jafarian and R. Ezzati, Conjugate gradient method for fuzzy symmetric positive definite system of linear equations, Appl. Math. Comput. 171 (2005) 1184-1191.
http://dx.doi.org/10.1016/j.amc.2005.01.110

[4] S. Abbasbandy, R. Ezzati and A. Jafarian, LU decomposition method for solving fuzzy system of linear equations, Appl. Math. Comput. 172 (2006) 633-643.
http://dx.doi.org/10.1016/j.amc.2005.02.018

[5] S. Abbasbandy, J.J. Nieto and M. Alavi, Tuning of reachable set in one dimensional fuzzy differential inclusions, Chaos, Solitons & Fractals 26 (2005) 1337-1341.

[6] S. Abbasbandy, M. Otadi and M. Mosleh, Minimal solution of general dual fuzzy linear systems, Chaos Solitons & Fractals 37 (2008) 1113-1124.

[7] T. Allahviranloo, Numerical methods for fuzzy system of linear equations, Appl. Math. Comput. 155 (2004) 493-502.
http://dx.doi.org/10.1016/S0096-3003(03)00793-8

[8] T. Allahviranloo, Successive over relaxation iterative method for fuzzy system of linear equations, Appl. Math. Comput. 162 (2005) 189-196.
http://dx.doi.org/10.1016/j.amc.2003.12.085

[9] T. Allahviranloo, M. Afshar Kermani, Solution of a fuzzy system of linear equation, Applied Mathematics and Computation 175 (2006) 519-531.
http://dx.doi.org/10.1016/j.amc.2005.07.048

[10] T. Allahviranloo, S. Salahshour, Fuzzy symmetric solutions of fuzzy linear systems, J. Comp. Math. 235 (2011) 4545-4553.
http://dx.doi.org/10.1016/j.cam.2010.02.042

[11] B. Asady, S. Abbasbandy and M. Alavi, Fuzzy general linear systems, Appl. Math. Comput. 169 (2005) 34-40.
http://dx.doi.org/10.1016/j.amc.2004.10.042

[12] S. Barnet, Matrix Methods and Applications, Clarendon Press, Oxford, 1990.

[13] M. Caldas and S. Jafari, θ-Compact fuzzy topological spaces, Chaos Solitons & Fractals 25 (2005) 229-232.

[14] D. Dubois and H. Prade, Operations on fuzzy numbers, J. Systems Sci. 9 (1978) 613-626.
http://dx.doi.org/10.1080/00207727808941724

[15] MS. Elnaschie, On a fuzzy Kãhler manifold which is consistent with the two slit experiment, Int. J. Nonlinear Science and Numerical Simulation 6 (2005) 95-98.

[16] MS. Elnaschie, Elementary number theory in superstrings, loop quantum mechanics, twistors and E-infinity high energy physics, Chaos, Solitons & Fractals 27 (2006) 297-330.

[17] MS. Elnaschie, Superstrings, entropy and the elementary particles content of the standard model, Chaos, Solitons & Fractals 29 (2006) 48-54.

[18] G. Feng and G. Chen, Adaptive control of discrete-time chaotic systems: a fuzzy control approach, Chaos Solitons & Fractals 23 (2005) 459-467.

[19] M. Friedman, Ma Ming and A. Kandel, Fuzzy linear systems, Fuzzy Sets and Systems 96 (1998) 201-209.
http://dx.doi.org/10.1016/S0165-0114(96)00270-9

[20] M. Friedman, Ma Ming and A. Kandel, Duality in fuzzy linear systems, Fuzzy Sets and Systems 109 (2000) 55-58.
http://dx.doi.org/10.1016/S0165-0114(98)00102-X

[21] S.Z. Guo, Commonly express method of fuzzy-valued function based on structured element, Fuzzy systems and mathematics 19 (2005) 82-86.

[22] S.Z. Guo, Method of structuring element in fuzzy analysis, Journal of Liaoning Technical University 21 (2002) 670-673.

[23] W. Jiang and Q. Guo-Dong and D. Bin, H_∞ Variable universe adaptive fuzzy control for chaotic system, Chaos Solitons & Fractals 24 (2005) 1075-1086.

[24] A. Kaufmann and M.M. Gupta, Introduction Fuzzy Arithmetic, Van Nostrand Reinhold, New York, 1985.

[25] D. Kincaid and W. Cheney, Numerical Analysis, Mathematics of scientific computing, Second Edition. Brooks/Cole Publishing Co., Pacific Grove, CA, 1996.

[26] S. Muzzioli and H. Reynaerts, Fuzzy linear systems of the form $A_1x + b_1 = A_2x + b_2$, Fuzzy Sets and Systems 157 (2006) 939-951.

[27] K. Nozari and B. Fazlpour B, Some consequences of spacetime fuzziness, Chaos, Solitons & Fractals 34 (2007) 224-234.

[28] M. Otadi and M. Mosleh, Simulation and evaluation of dual fully fuzzy linear systems by fuzzy neural network, Applied Mathematical Modelling, 35 (2011) 5026-5039.

[29] M. Otadi, M. Mosleh and S. Abbasbandy, Numerical solution of fully fuzzy linear systems by fuzzy neural network, Soft Computing, 15 (2011) 1513-1522.

[30] J.H. Park, Intuitionistic fuzzy metric spaces, Chaos Solitons & Fractals 22 (2004) 1039-1046.

[31] X.D. Sun and S.Z. Guo, Linear formed general fuzzy linear systems, System Engineering theory & practice, 29 (2009) 92-98.

[32] Y. Tanaka, Y. Mizuno and T. Kado, Chaotic dynamics in the Friedman equation, Chaos, Solitons & Fractals 24 (2005) 407-422.

[33] X. Wang, Z. Zhong and M. Ha, Iteration algorithms for solving a system of fuzzy linear equations, Fuzzy Sets and Systems 119 (2001) 121-128.
http://dx.doi.org/10.1016/S0165-0114(98)00284-X

[34] Sun X.D. and Guo S.Z, Linear formed general fuzzy linear systems, Systems Engineering - Theory & Practice, 29 (2009) 92-98.

[35] L.A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning, Inform. Sci. 8 (1975) 199-249.
http://dx.doi.org/10.1016/0020-0255(75)90036-5