Fuzzy multi-item inventory model with time varying stock dependent demand under partial backlogging

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Abstract

In this paper we have formulated a multi-item fuzzy inventory model with stock dependent demand rate and shortages are allowed with partial backlogging. The multi-item inventory model with demand dependent unit cost has been formulated alone with storage space constraints. The partial backlogging rate of unsatisfied demand is assumed to be decreasing exponential function for waiting time. Here the cost parameters, the objective function and constraint are imposed in fuzzy nature. The model has been solved by Fuzzy Goal Programming (FGP) method. Finally the model is expressed by numerical example.

Keywords: Inventory models, Storage space, Partial backlogging, Multi-item, Goal programming, Trapezoidal fuzzy number.

1 Introduction

The research works on inventory problems are designed by considering that the demand rate of an item is variable with time. When the demand of the item is high, an item is produced in large numbers. The GP (Goal Programming) technique was first used by Charnes and Cooper in 1960s and they [12] have also discussed the basic concept on GP multi-objective optimization. The GP technique has become a widely used approach and its variants have been applied to solve large-scale multi criteria decision making problems. The GP is an improved method for solving multi-objective problems. In 1970, R. E. Bellman and L. A. Zadeh [2] introduced decision-making in fuzzy environment. S. Kumar and U.S. Rajput [3] gave concept an inventory model for perishable items with time varying stock dependent demand and trade credit under inflation. In 2010, C. K. Tripathy and L. M. Pradhan developed [10] an EOQ model for weibull deteriorating items with power demand and partial backlogging. S. Islam and T. K. Roy [5] introduced multi-objective inventory model of deteriorating items under investment and order constraints. N. K Mandal, T. K Roy and M. Maity [6] introduced multi-objective fuzzy inventory model

In this paper we have formulated multi-item multi-objective inventory problem alone with constraints storage space. Here the shortages are permitted and partial backlogging. The model is solved by FGP (Fuzzy Goal Programming) technique. Finally the model is illustrated by numerical examples.

2 Assumptions and Notations:

The mathematical model is based on the following notation and assumptions.

Notation:

\( A_i \): Ordering cost per order for i-th item.

\( h_i \): The inventory holding cost per order for i-th item.

\( s_i \): Shortages cost per unit time for i-th item.

\( p_i \): Purchase cost per unit time for i-th item.

\( k_i \): Production cost per unit time for i-th item.

\( N_i \): Opportunity cost per unit time for i-th item due to lost sales.

\( w_i \): Store space for i-th item.

\( W \): The goal associated to store space.

\( t_w \): The tolerance of W.

\( TC_i \): The total cost for i-th item.

\( G_i \): The goal of the objective function \( TC_i \) for i-th item.

\( Q_{0i} \): The tolerances for the goal \( G_i \) for i-th item.

\( I_i(t_i) \): The level of positive and negative inventory in the interval \([0, t_i] \) and \([t_i, T_i] \) respectively for i-th item.

Assumption:

1. The inventory system deals with multi-item.
2. The planning horizon is infinite.
3. The demand rate \( d_i(t_i) = a_i + b_i \ t_i + k_i \ I_i(t_i) \), \( a_i \geq 0, 0 \leq b_i \leq 1, k_i > 0 \).
4. Shortages are allowed and partial backlogging.
5. The unsatisfied demand is backlogged at the rate of \( e^{-\sigma_i t_i} \), where \( t_i \) is the waiting time and \( \sigma_i \) is backlogged parameter during the shortages interval.
3 Mathematical Model

Suppose an inventory consist \( Q_i \) units of product in the beginning of each cycle. Due to demand, the inventory level decreases in \([0, t_{1i}]\) and becomes zero at \( t_{1i} \) and also the interval \([t_{1i}, T_i]\) is the shortages interval.

Therefore, the inventory level at any instant of time during \([0, t_{1i}]\) is described by the differential equations,

\[
\frac{di(t_i)}{dt} = -d_i, \quad 0 \leq t_i \leq t_{1i} 
\]

and the boundary conditions, \( I_i(t_{1i}) = 0, I_i(0) = Q_i \). Then solving the differential equation (3.1) we get,

\[
I_i(t_i) = \{ a_i(t_{1i} - t_i) + (a_i k_i + b_i) \left( \frac{t_i^2}{2} - \frac{t_{1i}^2}{2} \right) + \frac{b_i k_i}{3} (t_{1i}^3 - t_i^3) \} (1-k_i t_i), \quad 0 \leq t_i \leq t_{1i} \tag{3.2}
\]

using \( I_i(0) = Q_i \) we have

\[
Q_i = a_i t_{1i} + (a_i k_i + b_i) \left( \frac{t_{1i}^2}{3} + \frac{b_i k_i}{3} t_{1i}^3 \right) \tag{3.3}
\]

Again, during the shortages interval the unsatisfied demand is backlogged at the rate of \( e^{-\sigma_i t_i} \), where \( t_i \) is the waiting time & \( \sigma_i \) is backlogged parameter. Therefore the inventory can be represented in \([t_{1i}, T_i]\) by differential equation,

\[
\frac{di(t_i)}{dt} = -d_i e^{-\sigma_i t_i}, \quad t_{1i} \leq t_i \leq T_i \tag{3.4}
\]

Solving (3.4) and using boundary condition \( I_i(t_{1i}) = 0 \) we have

\[
I_i(t_i) = \left\{ \{ a_i t_{1i} + a_i k_i \left( \frac{t_{1i}^2}{2} - \frac{a_i}{6} t_{1i}^3 \right) + \left( \frac{b_i - a_i \sigma_i}{2} \right) t_{1i}^3 + (b_i - a_i \sigma_i) k_i \left( \frac{t_{1i}^3}{3} - \frac{a_i}{8} t_{1i}^4 \right) - \frac{b_i \sigma_i}{3} t_{1i}^3 - b_i \sigma_i k_i \left( \frac{t_{1i}^4}{4} - \frac{a_i}{10} t_{1i}^5 \right) \} - \{ a_i t_{1i} + a_i k_i \left( \frac{t_{1i}^2}{2} - \frac{a_i}{6} t_{1i}^3 \right) + \left( \frac{b_i - a_i \sigma_i}{2} \right) t_{1i}^3 + (b_i - a_i \sigma_i) k_i \left( \frac{t_{1i}^3}{3} - \frac{a_i}{8} t_{1i}^4 \right) - \frac{b_i \sigma_i}{3} t_{1i}^3 - b_i \sigma_i k_i \left( \frac{t_{1i}^4}{4} - \frac{a_i}{10} t_{1i}^5 \right) \} \right\}, \quad t_{1i} \leq t_i \leq T_i \tag{3.5}
\]

The ordering cost per cycle for i-th item

\[
OC_i = A_i \tag{3.6}
\]
Inventory holding cost per cycle for i-th item

\[ IH_i = \int_{t_{i+1}}^{T_i} h_i \cdot I_i(t_i) \, dt_i = h_i \cdot I_i \cdot \frac{a_i k_i + b_i}{3} \cdot t_{i+1}^3 + \frac{b_i k_i}{4} \cdot t_{i+1}^4 + (1 - k_i t_{i+1}) \]

= \[ h_i \cdot I_i \]

\[ (3.7) \]

Shortages cost per cycle for i-th item

\[ SC_i = s_i \cdot (t_{i+1} - I_i(t_i)) \cdot dt_i = \]

\[ = -s_i \cdot \left[ a_i k_i \left( \frac{t_{i+1}^3}{6} - \frac{k_i^2}{3} T_i - \frac{k_i^2}{3} t_{i+1}^3 \right) + a_i k_i \left( \frac{t_{i+1}^2}{2} / 6 - \frac{k_i^2}{3} T_i - \frac{k_i^2}{3} t_{i+1}^2 \right) \right. \]

\[ + a_i k_i \left( \frac{t_{i+1}^2}{2} / 6 - \frac{k_i^2}{3} T_i - \frac{k_i^2}{3} t_{i+1}^2 \right) \]

\[ + \left. \frac{b_i k_i}{3} \right] \cdot \frac{t_{i+1}^5}{4} - \frac{k_i^2}{3} t_{i+1}^5 \left] \cdot \left( t_{i+1} - k_i t_{i+1} \right) \cdot \left( t_{i+1} - k_i t_{i+1} \right) \cdot \left( t_{i+1} - k_i t_{i+1} \right) \]

\[ = -s_i \cdot I_i \]

\[ (3.8) \]

The purchase cost per cycle in \([0, t_{i+1})\) for i-th item

\[ PC_{i+t} = p_i \cdot Q_i = q_i \cdot \left( a_i \cdot t_{i+1} + (a_i \cdot k_i + b_i) \cdot \frac{t_{i+1}^2}{2} + \frac{a_i k_i}{3} \cdot t_{i+1}^3 \right) \]

\[ (3.9) \]

The purchase cost per cycle in \([t_{i+1}, T_i]\) for the i-th item

\[ PC_{i+t} = p_i \cdot \int_{t_{i+1}}^{T_i} \left( a_i + b_i \cdot t_i + k_i \cdot I(t_i) \right) \cdot e^{-\sigma_i t_i} \cdot dt_i = q_i \cdot \left( a_i \cdot T_i + (a_i \cdot k_i + b_i) \cdot \frac{T_i^2}{2} + \frac{a_i k_i}{3} \cdot T_i^3 \right) \]

\[ + k_i \cdot \left( \frac{T_i^3}{6} - \frac{k_i^2}{3} T_i - \frac{k_i^2}{3} t_{i+1}^3 \right) + \]

\[ + \frac{b_i k_i}{3} \] \[ \cdot \frac{T_i^5}{4} - \frac{k_i^2}{3} t_{i+1}^5 \left] \cdot \left( t_{i+1} - k_i t_{i+1} \right) \cdot \left( t_{i+1} - k_i t_{i+1} \right) \cdot \left( t_{i+1} - k_i t_{i+1} \right) \]

\[ = p_i \cdot H_i \]

\[ (3.10) \]

Due to lost sales the opportunity cost per cycle in \([t_{i+1}, T_i] \) for i-th item

\[ LS_i = N_i \cdot \int_{t_{i+1}}^{T_i} \left( a_i + b_i \cdot t_i + k_i \cdot I(t_i) \right) \cdot e^{-\sigma_i t_i} \cdot dt_i = \]

\[ = N_i \left( a_i \cdot T_i + (a_i \cdot k_i + b_i) \cdot \frac{T_i^2}{2} + \frac{a_i k_i}{3} \cdot T_i^3 \right) + a_i \cdot k_i \left( \frac{T_i^3}{6} - \frac{k_i^2}{3} T_i - \frac{k_i^2}{3} t_{i+1}^3 \right) + \]

\[ + k_i \cdot \left( \frac{T_i^3}{6} - \frac{k_i^2}{3} T_i - \frac{k_i^2}{3} t_{i+1}^3 \right) + \frac{b_i k_i}{3} \] \[ \cdot \frac{T_i^5}{4} - \frac{k_i^2}{3} t_{i+1}^5 \left] \cdot \left( t_{i+1} - k_i t_{i+1} \right) \cdot \left( t_{i+1} - k_i t_{i+1} \right) \cdot \left( t_{i+1} - k_i t_{i+1} \right) \]

\[ = N_i \cdot H_i \]

\[ (3.11) \]
\[
\begin{align*}
\frac{T_i^6}{6} + \frac{b_i \sigma_i}{2} T_i^4 - \frac{T_i^2}{2} + \frac{k_i \sigma_i}{4} T_i^2 - \frac{k_i \sigma_i}{6} - \frac{k_i \sigma_i}{70} T_i^4 + \left(\frac{b_i - a_i \sigma_i}{2} T_i^2 + (b_i - a_i \sigma_i) \right) k_i \\
\left(\frac{T_i^3}{3} - \frac{a_i \sigma_i}{6} T_i^2 + \frac{b_i \sigma_i}{10} T_i^2 - b_i \sigma_i \right) \sum_{i=1}^{n} \left(\frac{T_i^3}{3} - \frac{a_i \sigma_i}{6} T_i^2 + \frac{b_i \sigma_i}{10} T_i^2 - b_i \sigma_i \right) \sum_{i=1}^{n} \left(\frac{T_i^3}{3} - \frac{a_i \sigma_i}{6} T_i^2 + \frac{b_i \sigma_i}{10} T_i^2 - b_i \sigma_i \right)
\end{align*}
\]

\[
\frac{\mu_{\tilde{A}}(x)}{\mu_{\tilde{B}}(x)} = w \left(\frac{x-a}{b-a} \right) \text{ for } a \leq x \leq b
\]

\[
\frac{\mu_{\tilde{A}}(x)}{\mu_{\tilde{B}}(x)} = w \left(\frac{d-x}{d-c} \right) \text{ for } c \leq x \leq d
\]

\[
\frac{\mu_{\tilde{A}}(x)}{\mu_{\tilde{B}}(x)} = \begin{cases} 
\mu_{L_{\tilde{A}}}(x) = w \left(\frac{x-a}{b-a} \right) \text{ for } a \leq x \leq b \\
\mu_{R_{\tilde{A}}}(x) = w \left(\frac{d-x}{d-c} \right) \text{ for } c \leq x \leq d \\
0 & \text{for otherwise}
\end{cases}
\]

Where \(a < b < c < d\) and \(w \in (0,1)\)

If \(w = 1\), the generalized fuzzy number \(\tilde{A}\) is called a trapezoidal fuzzy number (TrFN) denoted \(\tilde{A} \equiv (a, b, c, d; w_1)\) and \(\tilde{B} \equiv (a, b, c, d; w_2)\) which denote two different decision maker’s opinions. The values \(w_1\) and \(w_2\) represent the degrees of confidence of the opinions of the decision makers \(\tilde{A}\) and \(\tilde{B}\), respectively, where \(w_1 = 0.8\) and \(w_2 = 1\).
Because of traditional fuzzy arithmetic operations we can any deal with normalized fuzzy numbers, they not only change the type of membership function of fuzzy number after arithmetical operations, but also have a drawback of requiring troublesome and tedious arithmetical operations.

Let \( \lambda \in [0, 1] \) be a pre-assigned parameter, called the degree of optimism. The graded mean value (or total \( \lambda \)-integral value) of \( \tilde{A} \) is defined as

\[
I_\lambda(\tilde{A}) = \lambda I_R(\tilde{A}) + (1 - \lambda) I_L(\tilde{A})
\]

where \( I_R(\tilde{A}) \) and \( I_L(\tilde{A}) \) are the left and right interval values of \( \tilde{A} \) defined as

\[
I_L(w)(\tilde{A}) = \int_0^1 (\mu_L(\tilde{A})^{-1}) - 1 \alpha d\alpha
\]

and

\[
I_R(w)(\tilde{A}) = \int_0^1 (\mu_R(\tilde{A})^{-1}) - 1 \alpha d\alpha
\]

Now

\[
(\mu_L^{-1})^{-1} = a + \frac{a}{w} (b - a)
\]

and

\[
(\mu_R^{-1})^{-1} = d + \frac{a}{w} (d - c)
\]

Therefore the left and right integral values are

\[
I_L(w)(\tilde{A}) = w \left( \frac{a + b}{2} \right) \quad \text{and} \quad I_R(w)(\tilde{A}) = w \left( \frac{c + d}{2} \right)
\]

Hence the total \( \lambda \)-integral value of \( \tilde{A} \) is

\[
I_\lambda(w)(\tilde{A}) = [\lambda w \left( \frac{c + d}{2} \right) + (1 - \lambda) w \left( \frac{a + b}{2} \right)]
\]

The left integral value is used to reflect the pessimistic viewpoint and the right integral value is used to reflect the optimistic viewpoint of the decision-maker. The total \( \lambda \)-integral value is a convex combination of right and left integral values through the degree of optimism.
Some Properties:

Property 1: If $\bar{U} = (u_1, u_2, u_3, u_4 ; w)$ and $\bar{y} = k\bar{u}$ is a fuzzy number $(ku_1, ku_2, ku_3, ku_4 ; w)$

Property 2: If $y = ku$, $k < 0$ then $\bar{y} = k\bar{u}$ is a fuzzy number $(ku_1, ku_2, ku_3, ku_4 ; w)$.

Property 3: If $\bar{A}_1 = (a_1, b_1, c_1, d_1 ; w_1)$ and $\bar{A}_2 = (a_2, b_2, c_2, d_2 ; w_2)$ then $\bar{A}_1 \oplus \bar{A}_2$ is a fuzzy number $(a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2 ; \min (w_1, w_2))$.

Fuzzy Inventory Model with Imprecise Costs:

In a multi-item inventory system, the warehouse capacity $W$ m$^2$ becomes uncertain in non-stochastic sense and the storage area can be expressed by a fuzzy set. In business, he/she takes the advantage of special discount or minimum transportation cost to depending upon different aspects, inventory cost parameters fluctuate. So, ordering cost, holding cost, shortages cost, purchase cost and opportunity costs are also assumed as trapezoidal fuzzy numbers. Hence the cost parameters $A_i, h_i, s_i, p_i, N_i$ are fuzzy numbers, the above crisp model reduces to

Min $\bar{T}\mathcal{C}_i = \frac{1}{T_i} \left( \bar{A}_i + \bar{h}_i \cdot E_i - \bar{s}_i \cdot F_i + \bar{p}_i \cdot Q_i + \bar{H}_i + \bar{N}_i \cdot J_i \right)$

Subject to

$$S(Q) = \sum_{i=1}^{n} w_i Q_i \leq W$$ (4.15)

where $\bar{A}_i, \bar{h}_i, \bar{s}_i, \bar{p}_i, \bar{N}_i$ is trapezoidal fuzzy number i.e. $\bar{A}_i \equiv (A_{1i}, A_{12}, A_{13}, A_{14}), \bar{h}_i \equiv (h_{1i}, h_{12}, h_{13}, h_{14}), \bar{s}_i \equiv (s_{1i}, s_{12}, s_{13}, s_{14}), \bar{p}_i \equiv (p_{1i}, p_{12}, p_{13}, p_{14}), \bar{N}_i \equiv (N_{1i}, N_{12}, N_{13}, N_{14})$ and also the imprecise constraints which may linear and/or non-linear are defined by membership functions. We assume the membership function $\mu_s(Q)$ for one constraints in fuzzy system as follows

$$\mu_s(Q) = \begin{cases} 0 & \text{for } \sum_{i=1}^{n} w_i Q_i > W + t_w \\ 1 - \frac{t_w}{\sum_{i=1}^{n} w_i Q_i - W} & \text{for } W \leq \sum_{i=1}^{n} w_i Q_i \leq W + t_w \\ \frac{1}{\sum_{i=1}^{n} w_i Q_i} & \text{for } \sum_{i=1}^{n} w_i Q_i < W \end{cases}$$ (4.16)

where $t_w$ is the admissible violations of total space.

Fuzzy Goal Programming Technique to solve Fuzzy Inventory Model:

If fuzzy cost components are taken as TrFNs ( $(A_{1i}, A_{12}, A_{13}, A_{14}), (h_{1i}, h_{12}, h_{13}, h_{14}), (s_{1i}, s_{12}, s_{13}, s_{14}), (p_{1i}, p_{12}, p_{13}, p_{14}), (N_{1i}, N_{12}, N_{13}, N_{14})$) then $\bar{T}\mathcal{C}_i = (TC_{1i}, TC_{12}, TC_{13}, TC_{14})$ for $i = 1, 2, ..., n$ is a TrFN where

$$TC_{ik} = \frac{1}{T_i} \left( A_{ik} + h_{ik} \cdot E_i - s_{ik} \cdot F_i + p_{ik} \cdot Q_i + p_{ik} \cdot H_i + N_{ik} \cdot J_i \right), \text{ where } k = 1, 2, 3, 4.$$  

so, $I_k (\bar{T}\mathcal{C}_i) = \left( I_k (\bar{A}_i) + I_k (\bar{h}_i) E_i - I_k (\bar{s}_i) F_i + I_k (\bar{p}_i) Q_i + I_k (\bar{p}_i) H_i + I_k (\bar{N}_i) J_i \right).$

According to Werner [13] the objective functions fuzzy in nature. So, $\lambda \in [0,1]$ then the following fuzzy goal programming problem

Find $t_{1i}, T_i$

$$I_k (TC_i (t_{1i}, T_i)) \leq G_i \text{ for } i = 1, 2, ..., n.$$ (4.17)

with the fuzzy constraints.

Now the formulation for first item, we assume that $G_1$ is the target of expenditure of the manufacturer. It may happen that in course of business, he/she may be compelled to augment some more capital invest say $Q_{01}$ for the first item to take some advantages of the business. Similar cases may also happen for other items.

Here, we assume that the objective goals are imprecise having the minimum targets $G_1, ..., G_n$ with positive
tolerances \( Q_0, Q_0, \ldots, Q_n \), for \( \lambda \in [0, 1] \). The target value \((G_i, i = 1, 2, \ldots, n)\) may be obtained the Werners [13] method.

Now, the imprecise objectives may be linear or nonlinear which are defined by their membership functions. For \( n \)-objective we assume that \( \mu_i(I_{\lambda}(\overline{T}C_i)) \) be the linear membership function then,

\[
\mu_i(I_{\lambda}(\overline{T}C_i)) = \begin{cases} 
0 & \text{for } I_{\lambda}(\overline{T}C_i) > G_i + Q_{0i} \\
1 - \frac{I_{\lambda}(\overline{T}C_i) - G_i}{Q_{0i}} & \text{for } G_i \leq I_{\lambda}(\overline{T}C_i) \leq G_i + Q_{0i} \\
1 & \text{for } I_{\lambda}(\overline{T}C_i) < G_i 
\end{cases}
\]  

(4.18)

For \( i = 1, 2, \ldots, n \).

and graphically represented as

![Figure 3: Membership function of \( I_{\lambda}(\overline{T}C_i) \)](image)

So, Belman and Zadeh's [2] max-min operator or convex combination operator the fuzzy goal programming problem (4.15) can be formulated as follows

\[
\text{Max } V_i(t_{1i}, T_i) = \sum_{i=1}^{n} \delta_i \mu_i(I_{\lambda}(\overline{T}C_i)) + \delta_s \mu_s(Q)
\]

subject to

\[
\mu_i(I_{\lambda}(\overline{T}C_i)) = 1 - \frac{I_{\lambda}(\overline{T}C_i) - G_i}{Q_{0i}}
\]

(4.19)

\[
\mu_s(Q) = 1 - \frac{\sum_{i=1}^{n} w_i Q_i - W}{w}
\]

where \( \mu_i(I_{\lambda}(\overline{T}C_i)), \mu_s(Q) \in [0, 1] \), for \( i = 1, 2, \ldots, n \). and also \( \delta_i \) and \( \delta_s \) may be taken as positive normalized preference of objective function and store space respectively i.e. \( \sum_{i=1}^{n} \delta_i + \delta_s = 1 \).

5 Numerical Example

Consider an inventory system the storage area is 1000 sq. m. A manufacturing company produced two types of machines 1 and 2 in lots. The manager of company decides to place order 3 times per month of products. He also allowed one extra orders, if necessary. Now the holding cost of the machine-1 is near about Rs. 1 but never less than Rs. 0.6 and above Rs. 1.2 (i.e. \( \h_1 \equiv (0.6, 0.8, 1.1, 1.2) \)). Similarly, holding cost of second machine are \( \h_2 \equiv \text{Rs.}(0.7, 0.9, 1.0, 1.1) \). Similarly, the ordering costs of first and second machine are \( \lambda_1 \equiv \text{Rs.}(90, 95, 100, 105) \) and \( \lambda_2 \equiv \text{Rs.}(85, 90, 95, 100) \) respectively. Similarly, the shortages cost of first and second machine are \( s_1 \equiv \text{Rs.}(0.3, 0.5, 0.6, 0.7) \) and \( s_2 \equiv \text{Rs.}(0.3, 0.6, 0.8, 0.9) \) respectively. Similarly, purchase cost of first and second machine are \( \p_1 \equiv \text{Rs.}(7, 10, 12, 14) \) and \( \p_2 \equiv \text{Rs.}(6, 8, 9, 10) \) respectively. Similarly, opportunity cost of second machine are \( \n_1 \equiv \text{Rs.}(1.5, 1.7, 2, 2.3) \) and \( \n_2 \equiv \text{Rs.}(1.6, 1.8, 2, 2.2) \) respectively and we input the data in table-1
Table 1: Input Data

<table>
<thead>
<tr>
<th>Item</th>
<th>A_i (Rs.)</th>
<th>a_i (Rs.)</th>
<th>b_i (Rs.)</th>
<th>s_i (Rs.)</th>
<th>p_i (Rs.)</th>
<th>k_i (Rs.)</th>
<th>N_i (Rs.)</th>
<th>G_i (Rs.)</th>
<th>Q_{0i} (Rs.)</th>
<th>σ_i (Rs.)</th>
<th>t_i</th>
<th>w_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>100</td>
<td>40</td>
<td>0.5</td>
<td>1</td>
<td>0.5</td>
<td>10</td>
<td>1.2</td>
<td>2</td>
<td>190</td>
<td>0.01</td>
<td>0.2</td>
<td>0.02</td>
</tr>
<tr>
<td>II</td>
<td>90</td>
<td>35</td>
<td>0.6</td>
<td>0.9</td>
<td>0.6</td>
<td>8</td>
<td>1</td>
<td>2</td>
<td>185</td>
<td>0.02</td>
<td>0.1</td>
<td>0.01</td>
</tr>
<tr>
<td>III</td>
<td>211.71</td>
<td>205.41</td>
<td>0.6</td>
<td>0.9</td>
<td>0.6</td>
<td>8</td>
<td>1</td>
<td>2</td>
<td>185</td>
<td>0.02</td>
<td>0.1</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Now we have shown the target expenditure of total cost which is flexible to the objective goals in table-2

Table 2: (Target Expenditure of total cost i.e. G_k, k = 1, 2, …,n.)

<table>
<thead>
<tr>
<th>k</th>
<th>m_k (Minimum values of TC_k without tolerance) (Rs.)</th>
<th>M_k (Minimum values of TC_k with tolerance) (Rs.)</th>
<th>G_k = min{m_k, M_k} (Rs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>197.41</td>
<td>190.31</td>
<td>190.31</td>
</tr>
<tr>
<td>II</td>
<td>201.42</td>
<td>197.27</td>
<td>197.27</td>
</tr>
<tr>
<td>III</td>
<td>211.71</td>
<td>205.41</td>
<td>205.41</td>
</tr>
</tbody>
</table>

For change of total cost for various weights in table-3 as follows

Table 3

<table>
<thead>
<tr>
<th>Type</th>
<th>Weights</th>
<th>Item</th>
<th>t_{1i}</th>
<th>T_{1i}</th>
<th>TC_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Equal importance to the objective(δ_i = 0.5 = δ_s)</td>
<td>1 t_{11} = 0.97</td>
<td>T_{1i} = 1.26</td>
<td>TC_i = 204.56</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 t_{12} = 1.03</td>
<td>T_{2i} = 1.27</td>
<td>TC_i = 191.21</td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>More importance to the cost goal(δ_i = 0.6, δ_s = 0.4)</td>
<td>1 t_{11} = 1.12</td>
<td>T_{1i} = 1.49</td>
<td>TC_i = 220.21</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 t_{12} = 1.23</td>
<td>T_{2i} = 1.55</td>
<td>TC_i = 201.51</td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>More importance to the profit goal (δ_i = 0.4, δ_s = 0.6)</td>
<td>1 t_{11} = 0.90</td>
<td>T_{1i} = 1.10</td>
<td>TC_i = 195.42</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2 t_{12} = 0.96</td>
<td>T_{2i} = 1.14</td>
<td>TC_i = 186.21</td>
<td></td>
</tr>
</tbody>
</table>

It is observed from the table-3 we see that if δ_i < δ_s then the problem faces the better performance for minimum value of total cost.

Sensitivity Analysis:

The total λ-integral value is a convex combination of the right and left integral values through the degree of optimism. A large value of λ specifies the higher degree of optimism. When λ = 0 then the total λ-integral value is I_0^w (A) = w (a+b)/2 = I_L^w (A) represents a pessimistic viewpoint. When λ = 1 then the total λ-integral value is I_1^w (A) = w (c+d)/2 = I_R^w (A) represents optimistic viewpoint and when λ = 0.5 then the total λ-integral value is I_{0.5}^w (A) = w/4 (a + b + c + d) = 1/2 [I_L^w (A) + I_R^w (A)] represents a moderately optimistic decision-maker’s viewpoint and it is same approximated value of the fuzzy number.
Table 4

<table>
<thead>
<tr>
<th>Change of ( \lambda )</th>
<th>FGP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( TC_1 ) (Rs.)</td>
</tr>
<tr>
<td>0.1</td>
<td>229.21</td>
</tr>
<tr>
<td>0.2</td>
<td>231.33</td>
</tr>
<tr>
<td>0.3</td>
<td>233.49</td>
</tr>
<tr>
<td>0.4</td>
<td>235.79</td>
</tr>
<tr>
<td>0.5</td>
<td>237.97</td>
</tr>
<tr>
<td>0.6</td>
<td>240.31</td>
</tr>
</tbody>
</table>

\( \lambda \)-integral value

Figure 4: Graph of \( \lambda \)-integral value and total average cost of items \( TC_1 \) and \( TC_2 \)

6 Conclusion

In this work we have proposed a concept of optimal solution of inventory problem with fuzzy coefficients and imprecise constraints functions and also we have developed an inventory model with time varying stock dependent demand with shortages are allowed and partial backlogged. The different value of optimism (\( \lambda \in [0, 1] \)) and weights(\( \delta_i \), \( \delta_j \)) has been illustrated through the numerical example. The GP (Goal Programming) appears to be an appropriate powerful and flexible technique for multi-item multi-objective function. The model can be easily extended to generic inventory problem with constraints and this method can be applied to the real inventory problems faced in industry or other areas. This method may be applied to several type of fuzzy model in mathematical optimization.

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