A few more on intuitionistic fuzzy set

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Abstract
Besides the various basic operations for intuitionistic fuzzy sets already available in literature, a new operation is introduced in this paper. Some properties of this operation are discussed and some new relations are also established. At the same time the behavior of modal operators over these operations are rigorously studied. A problem regarding the rank of the students in a certain examination is discussed and the solution is obtained with the help of this new operation.

Keywords: Fuzzy sets, Intuitionistic fuzzy sets, Modal operators, Average operation.

1 Introduction
In 1983, Atanassov [1] has done an excellent job by introducing the concept of intuitionistic fuzzy set as an extension of fuzzy set earlier invented by L.A.Zadeh [9] in 1965. Since then many authors and researchers are giving much attention as well as concentration for developing intuitionistic fuzzy sets. In recent past, some results on algebraic laws in intuitionistic fuzzy sets [3, 5, 7, 8] and some basic relation among modal operators [6] are discussed. It is also well known to us that every fuzzy set is intuitionistic fuzzy set but the reverse is not true. But more importantly there exist some operators by which we can transform intuitionistic fuzzy sets into fuzzy sets easily. As discussing the past, present and future of intuitionistic fuzzy sets, Atanassov [4] has remarkably mentioned about the importance of modal operators which are analogous of the modal logic operators ‘necessity’ and ‘possibility’. Here we concentrate deeply upon these operators and try to solve a real life problem related to examination where the percentage of hesitation margin is known well ahead.

2 Preliminaries
Throughout this paper, intuitionistic fuzzy set and fuzzy set are denoted by IFS and FS respectively.
Definition 2.1. [9].
Let X be a nonempty set. A fuzzy set A drawn from X is defined as \( A = \{ <x, \mu_A(x) : x \in X \} \), where \( \mu_A(x) : x \to [0,1] \) is the membership function of the fuzzy set A. Fuzzy set is a collection of objects with graded membership i.e. having degrees of membership.

Definition 2.2. [2].
Let X be a nonempty set. An intuitionistic fuzzy set A in X is an object having the form \( A = \{ <x, \mu_A(x), \nu_A(x) : x \in X \} \), where the functions \( \mu_A(x), \nu_A(x) : X \to [0,1] \) define respectively, the degree of membership and degree of non-membership of the element \( x \in X \) to the set A, which is a subset of X, and for every element \( x \in X \), \( 0 \leq \mu_A(x) + \nu_A(x) \leq 1 \).
Furthermore, we have \( \pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \) called the intuitionistic fuzzy set index or hesitation margin of \( x \) in A. \( \pi_A(x) \) is the degree of indeterminacy of \( x \in X \) to the IFS A and \( \pi_A(x) \in [0,1] \) i.e., \( \pi_A(x) : X \to [0,1] \) and \( 0 \leq \pi_A(x) \leq 1 \) for every \( x \in X \).
\( \pi_A(x) \) expresses the lack of knowledge of whether \( x \) belongs to IFS A or not.

Definition 2.3. [2].
Let \( A, B \) be two IFSs in X. The basic operations are defined as follows:
1. [ inclusion] \( A \subseteq B \iff \mu_A(x) \leq \mu_B(x) \) and \( \nu_A(x) \geq \nu_B(x) \) \( \forall x \in X \).
2. [ equality] \( A = B \iff \mu_A(x) = \mu_B(x) \) and \( \nu_A(x) = \nu_B(x) \) \( \forall x \in X \).
3. [ complement] \( A^c = \{ <x, \mu_A(x), \nu_A(x) : x \in X \} \).
4. [ union] \( A \cup B = \{ <x, \max(\mu_A(x), \mu_B(x)), \min(\nu_A(x), \nu_B(x)) : x \in X \} \).
5. [ intersection] \( A \cap B = \{ <x, \min(\mu_A(x), \mu_B(x)), \max(\nu_A(x), \nu_B(x)) : x \in X \} \).
6. [ addition] \( A \oplus B = \{ <x, \mu_A(x) + \mu_B(x) - \mu_A(x) \mu_B(x), \nu_A(x) \nu_B(x) : x \in X \} \).
7. [ multiplication] \( A \otimes B = \{ <x, \mu_A(x) \mu_B(x), \nu_A(x) \nu_B(x) - \nu_A(x) \nu_B(x) : x \in X \} \).
8. [ difference] \( A - B = \{ <x, \min(\mu_A(x), \nu_B(x)), \max(\nu_A(x), \mu_B(x)) : x \in X \} \).
9. [ symmetric difference] \( A \Delta B = \{ <x, \max[\min(\mu_A(x), \nu_B(x)), \min(\mu_B(x), \nu_A(x))], \min[\max(\nu_A(x) \mu_B(x), \max(\nu_B(x), \mu_A(x))) : x \in X \} \).

Definition 2.4. [3].
Let \( A, B \) and \( C \) be IFSs in X. The algebraic laws are as follows:
1. [ complementary law] \( A^c = A \).
2. [ idempotent law] \( i) A \cup A = A \) \( ii) A \cap A = A \).
3. [ commutative law] \( i) A \cup B = B \cup A \) \( ii) A \cap B = B \cap A \).
4. [ associative law] \( i) (A \cup B) \cup C = A \cup (B \cup C) \) \( ii) (A \cap B) \cap C = A \cap (B \cap C) \).
5. [ distributive law] \( i) A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \).
6. [ De Morgan’s law] \( i) A^c = A \cup B \) \( ii) A \cap B = A \cap B^c \).
7. [ absorption laws] \( i) A \cap (A \cup B) = A \) \( ii) A \cup (A \cap B) = A \).
8. \( i) A \oplus B = B \oplus A \) \( ii) A \otimes B = B \otimes A \).
9. \( i) A \oplus (B \oplus C) = (A \oplus B) \oplus C \) \( ii) A \otimes (B \otimes C) = (A \otimes B) \otimes C \).
10. \( i) (A \oplus B) = A^c \oplus B^c \) \( ii) (A \otimes B) = A^c \otimes B^c \).
11. \( i) A \oplus (B \cup C) = (A \oplus B) \cup (A \oplus C) \) \( ii) A \otimes (B \cap C) = (A \otimes B) \cap (A \otimes C) \).
12. \( i) A \otimes (B \cup C) = (A \otimes B) \cup (A \otimes C) \) \( ii) A \oplus (B \cap C) = (A \oplus B) \cap (A \oplus C) \).

Distribution laws hold for both right and left distributions.
Definition 2.5. [6].
A is said to be a proper subset of B i.e. $A \subset B$ if $A \subseteq B$ and $A \neq B$. It means $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ but $\mu_A(x) \neq \mu_B(x)$ and $\nu_A(x) \neq \nu_B(x)$ for $x \in X$.

Definition 2.6. [4]
Let $X$ be a nonempty set. If $A$ is an IFS drawn from $X$, then,
(i) $\Box A = \{ <x, \mu_A(x), 1-\mu_A(x) > : x \in X \}$
(ii) $\Diamond A = \{ <x, 1-\nu_A(x), \nu_A(x) > : x \in X \}$
For a proper IFS, $\Box A \subset A \subset \Diamond A$ and $\Box A \neq A \neq \Diamond A$

Theorem 2.1. [6].
Let $X$ be nonempty. For every IFS $A$ in $X$,
\begin{enumerate}
  \item $\Box \Box A = \Box A = A$
  \item $\Diamond \Diamond A = \Diamond A$
  \item $\Box \Diamond A = \Diamond A$
  \item $\Diamond \Box A = \Box A = A$
\end{enumerate}

Theorem 2.2. [6].
Let $X$ be a nonempty set. If $A$ and $B$ be two IFSs drawn from $X$, then,
\begin{enumerate}
  \item $\Box (A \cap B) = \Box A \cap \Box B = A \cap B$
  \item $\Diamond (A \cap B) = \Diamond A \cap \Diamond B$
  \item $\Box (A \cup B) = \Box A \cup \Box B = A \cup B$
  \item $\Diamond (A \cup B) = \Diamond A \cup \Diamond B$
  \item $\Box (A \oplus B) = \Box A \oplus \Box B$
  \item $\Diamond (A \oplus B) = \Diamond A \oplus \Diamond B$
  \item $\Box (A \otimes B) = \Box A \otimes \Box B$
  \item $\Diamond (A \otimes B) = \Diamond A \otimes \Diamond B$
\end{enumerate}

Definition 2.7. [4].
Let $\alpha \in [0,1]$ and $A \in$ IFS $X$. Then the operator $D_\alpha (A)$ can be defined as $D_\alpha (A) = \{ <x, \mu_A(x) + \alpha \pi_A(x), \nu_A(x) + (1-\alpha) \pi_A(x) > : x \in X \}$

Definition 2.8. [4].
Let $\alpha, \beta \in [0,1]$ and $A \in$ IFS $X$. Then the operator $F_{\alpha,\beta} (A)$ can be defined as $F_{\alpha,\beta} (A) = \{ <x, \mu_A(x) + \alpha \pi_A(x), \nu_A(x) + \beta \pi_A(x) > : x \in X \}$, where $\alpha + \beta \leq 1$.

3 Modal operators in IFS
Here $A, B \in$ IFSs means $A = \{ <x, \mu_A(x), \nu_A(x) > : x \in X \}$ and $B = \{ <x, \mu_B(x), \nu_B(x) > : x \in X \}$.

Theorem 3.1.
If $X$ be a nonempty set and two IFSs $A, B$ of $X$ such that $A \subseteq B$, then,
\begin{enumerate}
  \item $\Box A \subseteq \Box B$
  \item $\Diamond A \subseteq \Diamond B$
\end{enumerate}
Proof.
Obvious.
Theorem 3.2. Let X be a nonempty set. If A and B be two IFSs drawn from X, then,

(a) □ (A - B) = ◊A - ◊B
(b) ◊ (A - B) = □A - □B

Proof.
(a) □(A - B) = □ {< x, min(μA(x), νB(x)), max (νA(x), μB(x)) >}
    = {< x, min(μA(x), νB(x)), 1 - min (μA(x), νB(x)) >}
Again, ◊A - ◊B = {< x, 1 - νA(x), νA(x) > - < x, 1 - νB(x), νB(x) >}
    = {< x, min (μA(x), νA(x)), max (μB(x), νB(x)) >}

Hence the result. The proof of (b) is similar to (a).

Theorem 3.3. Let A and B be two IFSs in a nonempty set X. Then
(i) (A Δ B) = A - B iff B ⊆ A.
(ii) (A Δ B) = B - A iff A ⊆ B.

Proof. The proof is straightforward from the definition.

Remark 3.1. If X be nonempty and A and B be two IFSs of X, then,
(a) □(A - B) ≠ □A - □B
(b) ◊(A - B) ≠ ◊A - ◊B

Example: Let A= < .7, .2, .1> and B= < .6, .3, .1>
(a) Now (A - B) = < .3, .6 > and □(A - B) = < .3, .7 > but □A - □B = < .4, .6 >
So □(A - B) ≠ □A - □B
(b) Again ◊(A - B) = < .4, .6 > and ◊A - ◊B = < .3, .7 >
So ◊(A - B) ≠ ◊A - ◊B

Theorem 3.4. Let X be a nonempty set. If A and B be two IFSs drawn from X with B ⊆ A or A ⊆ B, then
(a) (□A) Δ (□ B) = ◊(A Δ B)
(b) (◊A) Δ (◊ B) = □ (A Δ B)

Proof.
(a) When B ⊆ A, (□A) Δ (□ B) = (□A) - (□ B) = ◊ (A - B) = ◊(A Δ B)
and when A ⊆ B, (□A) Δ (□ B) = (□B) - (□ A) = ◊ (B - A) = ◊(A Δ B) [ by 3.2 & 3.3]
Similarly (b) can be proved.
It can be noted that the above theorem does not hold if B ⊈ A or A ⊈ B.
Suppose A= < .3, .6, .1 > and B= < .2, .1, .7 > so that B ⊈ A.
Here (□A) Δ (□ B) = < .3, .7 >, ◊(A Δ B) = < .7, .3 >, (◊A) Δ (◊ B) = < .6, .4 >, and
□ (A Δ B) = < .2, .8 >.

Remark 3.2. If X be nonempty and A and B be two IFSs of X, then,
(a) (□A) Δ (□ B) ≠ □ (A Δ B)
(b) (◊A) Δ (◊ B) ≠ ◊ (A Δ B)

Example: Let A= < .7, .2, .1> and B= < .6, .3, .1>
(a) (□A) Δ (□ B) = < max (.4, .3), min (.6 , .7) > = < .4, .6 >
Again, □ (A Δ B) = < .3, .7 >
 $$\text{So } (\Box A) \Delta (\Box B) \neq \Box (A \Delta B)$$  
(b)  $$\Diamond (A) \Delta (\Diamond B) = < .3, .7 > \text{ and } \Diamond (A \Delta B) = < .4, .6 >$$ 
So $$\Diamond (A) \Delta (\Diamond B) \neq \Diamond (A \Delta B)$$

Corollary 3.1. If X be nonempty and A and B be two proper IFSs of X, then,

(a) $$\Box (A - B) \subset A - B \subset \Diamond (A - B)$$
(b) $$\Box (A - B) \neq A - B \neq \Diamond (A - B)$$
(c) $$\Box (A \Delta B) \subset (A \Delta B) \subset \Diamond (A \Delta B)$$
(d) $$\Box (A \Delta B) \neq (A \Delta B) \neq \Diamond (A \Delta B)$$

(a) and (c) can be proved by using the definitions and 2.6. For (b) and (d), we consider an example.

Let $$A = < .7, .2, .1> \text{ and } B = < .6, .3, .1>$$
Here, $$(A - B) = < .3, .6, .1>$$, $$\Box (A - B) = < .3, .7 >$$ and $$\Diamond (A - B) = < .4, .6 >$$

Also $$(A \Delta B) = < .3, .6, .1>$$, $$\Box (A \Delta B) = < .3, .7 >$$ and $$\Diamond (A \Delta B) = < .4, .6 >$$

4 A new operation in IFS

Definition 4.1. Let A and B be two IFSs in a nonempty set X. We define the average operation denoted by $$A \oplus B$$ as

$$A \oplus B = < x, \frac{1}{2}[ \mu_A(x) + \mu_B(x)], \frac{1}{2}[ \nu_A(x) + \nu_B(x)] >.$$ 

Theorem 4.1. For IFSs A and B of the universe X, we have $$A \oplus B$$ is IFS.

Proof.

Here $$A \oplus B = < x, \frac{1}{2}[ \mu_A(x) + \mu_B(x)], \frac{1}{2}[ \nu_A(x) + \nu_B(x)] >.$$ 
So $$\mu_{A \oplus B}(x) + \nu_{A \oplus B}(x) = \frac{1}{2}[ \mu_A(x) + \mu_B(x) + \nu_A(x) + \nu_B(x)] \leq 1.$$ 
Hence the proof.

Definition 4.2. Let $$A_1, A_2, A_3, \ldots, A_n$$ be n IFSs of the universe X. Then the average operation over these IFSs can be defined as

$$A_1 \oplus A_2 \oplus A_3 \oplus \ldots \oplus A_n = < x, \frac{1}{n}[ \mu_{A_1}(x) + \mu_{A_2}(x) \ldots + \mu_{A_n}(x)], \frac{1}{n}[ \nu_{A_1}(x) + \nu_{A_2}(x) \ldots + \nu_{A_n}(x)] >.$$ 

Theorem 4.2. For IFSs $$A_1, A_2, A_3, \ldots, A_n$$ of the universe X, we have $$A_1 \oplus A_2 \oplus A_3 \oplus \ldots \oplus A_n$$ is IFS.

Proof. The proof is similar to the above theorem 4.1.

Theorem 4.3. Let A, B, and C be IFSs in X, then the following properties are valid.

(i) $$A \oplus A = A$$
(ii) $$A \oplus B = B \oplus A$$
(iii) $$A \oplus (B \oplus C) = (A \oplus B) \oplus C$$
(iv) $$(A \oplus B)^c = A^c \oplus B^c$$
(v) $$A \oplus (B \cup C) = (A \oplus B) \cup (A \oplus C)$$
(vi) $$A \oplus (B \cap C) = (A \oplus B) \cap (A \oplus C)$$

Proof.

The proof of (i), (ii) and (iii) follows from the definition.

iv) $$A^c \oplus B^c = < x, \nu_A(x), \mu_A(x) > \oplus < x, \nu_B(x), \mu_B(x) >$$
$$= < x, \frac{1}{2}[ \nu_A(x) + \nu_B(x), \frac{1}{2}[ \mu_A(x) + \mu_B(x)] > = (A \oplus B)^c$$
Thus the proof.

v) L.H.S = $$A \oplus (B \cup C) = < x, \mu_A(x), \nu_A(x) > \oplus < x, \max (\mu_B(x), \mu_C(x)), \min (\nu_B(x), \nu_C(x)) >$$
Thus the proof. Similarly (vi) can be proved.

**Theorem 4.** For two IFSs $A$ and $B$ in $X$, we have

(i) $\square (A \ominus B) = \square A$

(ii) $\Diamond (A \ominus B) = \Diamond A$

(iii) $\square (A \ominus B) = \square A \ominus \square B$

(iv) $\Diamond (A \ominus B) = \Diamond A \ominus \Diamond B$

(v) $\square (A \ominus B) = \Diamond (A \ominus B)$

Proof.

(i) and (ii) are obvious.

(iii) L.H.S. $= \square (A \ominus B) = \{x, \frac{1}{2}[\mu_A(x) + \mu_B(x)], 1 - \frac{1}{2}[\mu_A(x) + \mu_B(x)] \}$

R.H.S. $= \square A \ominus \square B = \{x, \frac{1}{2}[\mu_A(x) + \mu_B(x)], 1 - \frac{1}{2}[\mu_A(x) + \mu_B(x)] \}$

Hence the proof.

Similarly (iv) can be proved.

(v) L.H.S. $= \square (A \ominus B) = \Diamond (A \ominus B)$

Proof.

$\Diamond (A \ominus B) = \Diamond (A \ominus B)$

Thus the proof. Similarly (vi) can be proved.

**Corollary 4.** If $X$ be nonempty and $A$ and $B$ be two proper IFSs of $X$, then,

(a) $\square (A \ominus B) \subset (A \ominus B) \subset \Diamond (A \ominus B)$

(b) $\square (A \ominus B) \neq (A \ominus B) \neq \Diamond (A \ominus B)$

Proof.

Similar to Corollary 3.1.

**Theorem 4.5.** Let $X$ be nonempty and $A, B, C \in X$ are IFSs such that $A \subseteq B$, then $A \ominus C \subseteq A \ominus B$

Proof.

Follows from the definition.

**Theorem 4.6.** For every IFS $A$, and for any real number $\alpha, \beta \in [0, 1]$, we have

(i) $D_{\alpha} (A') \in IFS$
(ii) \( D_\alpha (A \cup B) = D_\alpha (A) \cup D_\alpha (B) \)

(iii) \( D_\alpha (A \cap B) = D_\alpha (A) \cap D_\alpha (B) \)

(iv) \( D_\alpha (A \bigoplus B) = D_\alpha (A) \bigoplus D_\alpha (B) \)

(v) \( F_{a,\beta} (A \bigoplus B) = F_{a,\beta} (A) \bigoplus F_{a,\beta} (B) \)

(vi) \( D_0 (A \bigoplus B) = F_{0,1} (A \bigoplus B) \)

Proof.

(i) We have, \( D_\alpha (A^c) = \{ < x, \nu_A(x) + (1-\alpha) \pi_A(x), \mu_A(x) + \alpha \pi_A(x) > : x \in X \} \)

Now \( \mu_{D_\alpha (A^c)} + \nu_{D_\alpha (A^c)} = \nu_A(x) + (1-\alpha) \pi_A(x) + \mu_A(x) + \alpha \pi_A(x) \leq 1 \)

(ii) and (iii) are straightforward.

(iv) L.H.S = \( D_\alpha (A \bigoplus B) = < x, \frac{1}{2}[ \mu_A(x) + \pi_A(x)] , \frac{1}{2}[ \nu_A(x) + (1-\alpha) \pi_A(x)] > \)

= \( < x, \frac{1}{2}[ (\mu_A(x) + \mu_B(x)) + \alpha(\pi_A(x) + \pi_B(x)) ] , \frac{1}{2}[ (\nu_A(x) + \nu_B(x)) + (1-\alpha) (\pi_A(x) + \pi_B(x))] > \)

R.H.S = \( D_\alpha (A \bigoplus D_\alpha (B)) \)

= \( < x, \mu_A(x) + \alpha \pi_A(x) , \nu_A(x) + (1-\alpha) \pi_A(x) > \bigoplus < x, \mu_B(x) + \alpha \pi_B(x) , \nu_B(x) + (1-\alpha) \pi_B(x) > \)

= \( < x, \frac{1}{2}[ (\mu_A(x) + \mu_B(x)) + \alpha(\pi_A(x) + \pi_B(x)) ] , \frac{1}{2}[ (\nu_A(x) + \nu_B(x)) + (1-\alpha) (\pi_A(x) + \pi_B(x))] > \)

Hence the proof.

(v) same as (iv).

(vi) L.H.S = \( D_0 (A \bigoplus B) = < x, \frac{1}{2}[ \mu_A(x) , \frac{1}{2}[ \nu_A(x) + \pi_A(x)] > \)

R.H.S = \( F_{0,1} (A \bigoplus B) = < x, \frac{1}{2}[ \mu_A(x) , \frac{1}{2}[ \nu_A(x) + \pi_A(x)] > \)

This completes the proof.

5 Application of IFS with the help of average operation

Problem

Five students appeared in a Mathematical Olympiad test in which the question paper consists of four different section namely Logical Reasoning, Mathematical Reasoning, Everyday Mathematics and Achiever’s Section containing 25 marks each. When the examination is over the answer key is uploaded in the website and the students have confirmed that in the hesitation or uncertain part of their answer 60%, 70%, 60%, 50%, and 40% respectively are correct in each section. Find the rank of the students in the examination.

Solution

Here we use the concept of IFS for representing the marks. The membership degree represents the marks for the correct answer, the non-membership degree represents the marks which is not obtained for incorrect answers and the hesitation degree means the marks which is not confirmed by the students i.e, the students are in hesitation whether the answer is correct or not. Table-1 represents the marks of the students.

<table>
<thead>
<tr>
<th>Student</th>
<th>Logical Reasoning</th>
<th>Mathematical Reasoning</th>
<th>Everyday Mathematics</th>
<th>Achiever’s Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anu</td>
<td>&lt;.7, .2, .1 &gt;</td>
<td>&lt;.8, 0, .2 &gt;</td>
<td>&lt;.6, 1, .3 &gt;</td>
<td>&lt;.4, .2, .4 &gt;</td>
</tr>
<tr>
<td>Gagan</td>
<td>&lt;.9, .1, 0 &gt;</td>
<td>&lt;.7, .2, .1 &gt;</td>
<td>&lt;.6, 3, 1 &gt;</td>
<td>&lt;.5, .4, 1 &gt;</td>
</tr>
<tr>
<td>Jayee</td>
<td>&lt;.8, .1, .2 &gt;</td>
<td>&lt;.8, 2, 0 &gt;</td>
<td>&lt;.5, 1, 4 &gt;</td>
<td>&lt;.7, 1, 2 &gt;</td>
</tr>
<tr>
<td>Hena</td>
<td>&lt;.8, 0, .2 &gt;</td>
<td>&lt;.6, 1, 3 &gt;</td>
<td>&lt;.7, 0, 3 &gt;</td>
<td>&lt;.6, .2, 2 &gt;</td>
</tr>
<tr>
<td>Dipika</td>
<td>&lt;.9, 0, .1 &gt;</td>
<td>&lt;.7, 1, 1 &gt;</td>
<td>&lt;.8, 1, 1 &gt;</td>
<td>&lt;.5, .2, .3 &gt;</td>
</tr>
</tbody>
</table>

Since the percentage of the hesitation part is given, we now calculate the membership degree and non-membership degree using the operator \( F_{a,\beta} (A) \) [by 2.8] and get the table below.
Table 2

<table>
<thead>
<tr>
<th>Student</th>
<th>Logical Reasoning</th>
<th>Mathematical Reasoning</th>
<th>Everyday Mathematics</th>
<th>Achiever’s Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anu</td>
<td>&lt;.76, .24 &gt;</td>
<td>&lt;.92, .08 &gt;</td>
<td>&lt;.78, .22 &gt;</td>
<td>&lt;.64, .36 &gt;</td>
</tr>
<tr>
<td>Gagan</td>
<td>&lt;.9, .1 &gt;</td>
<td>&lt;.77, .23 &gt;</td>
<td>&lt;.67, .33 &gt;</td>
<td>&lt;.57, .43 &gt;</td>
</tr>
<tr>
<td>Jayee</td>
<td>&lt;.86, .14 &gt;</td>
<td>&lt;.8, .2 &gt;</td>
<td>&lt;.74, .26 &gt;</td>
<td>&lt;.82, .18 &gt;</td>
</tr>
<tr>
<td>Hena</td>
<td>&lt;.9, .1 &gt;</td>
<td>&lt;.75, .25 &gt;</td>
<td>&lt;.85, .15 &gt;</td>
<td>&lt;.7, .3 &gt;</td>
</tr>
<tr>
<td>Dipika</td>
<td>&lt;.94, .06 &gt;</td>
<td>&lt;.78, .22 &gt;</td>
<td>&lt;.84, .16 &gt;</td>
<td>&lt;.62, .38 &gt;</td>
</tr>
</tbody>
</table>

These marks consist only the membership degree and non-membership degree. Now using the average operation we determine the required rank of the students.

Table 3

<table>
<thead>
<tr>
<th>Student</th>
<th>Average</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anu</td>
<td>&lt;.775, .225 &gt;</td>
<td>4th</td>
</tr>
<tr>
<td>Gagan</td>
<td>&lt;.7275, .2725 &gt;</td>
<td>5th</td>
</tr>
<tr>
<td>Jayee</td>
<td>&lt;.805, .195 &gt;</td>
<td>1st</td>
</tr>
<tr>
<td>Hena</td>
<td>&lt;.8, .2 &gt;</td>
<td>2nd</td>
</tr>
<tr>
<td>Dipika</td>
<td>&lt;.795, .205 &gt;</td>
<td>3rd</td>
</tr>
</tbody>
</table>

6 Conclusion

Among the various operations, the average operation is very simple and useful on IFSs. The advantage of this operation is that it is too easy to handle over a finite number of IFSs. The application based on IFSs for determining the rank or performance of the students in an examination is shown. The very idea is attractive and innovative as the percentage of hesitation degree is already given. At the end a clear picture of the student’s performance has been visualized.

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