Qualitative Model of the Input Impedance of Rectangular Microstrip Antenna

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Abstract
In this paper, a fuzzy-based approach is proposed so as to predict the input impedance of the rectangular microstrip antenna. In the proposed approach, at first, behavior of single microstrip antenna is represented as simple and unchanged membership functions, and the feed probe effect on the input impedance is then extracted as simple curves so that the input impedance of microstrip antenna in despite of other existing models is efficiently predicted.

Keywords: Microstrip antenna; input impedance; Fuzzy inference.

1 Introduction
Microstrip antennas due to light weight, compatible with solid state devices and inexpensive to fabricate are widely used from phased antenna arrays to biomedical diagnostics [1, 2]. Exact analysis of such structures, i.e., input impedance, mutual coupling and radiation pattern, is thus of importance.

Basically, there are two models for analysis of such structure including exact and approximate models. The first ones such as method of moments (MoM) [3] on one hand lead to accurate predictions, but they are complex and time consuming specially over wide frequency band, the second ones such as transmission line model (TLM) [4-6] on the other hand is simple, but leads to considerably errors in wide frequency band.

To remove the above drawbacks, intelligent models such as neural networks (N.N) and neural-fuzzy systems (N.FS) [7-9] can be used. It is however well known that these models require too many initial input-output data to create the model and also training process is too long especially when the number of inputs is increased.

In contrast with these intelligent models, the fuzzy-based approach proposed only for modeling electromagnetic problems [10-12] can be used. This model has two main advantageous against the other ones such as N.N that makes it more efficient. The first is extracting behavior of the problem as a moving
circle using a few input-output data, and the second is dividing multi input-multi output system to single input-multi output sub systems, then extracting knowledge base of each sub system separately as simple curves and finally combining them through spatial membership functions [13]. General idea of modeling process is schematically illustrated in figure 1. In the introduced model, a few basic circles making overall circular movement are fuzzy inputs, and behavior of the problem is extracted based on how to fit them on the circular movement. Finally changing the problem parameters on the output is related to changing the radius and displacement of these basic circles in polar plane. Further information about this approach is in the next section through applying it to rectangular microstrip antenna.

Figure 1: Schematic diagram of the proposed model.

Up to now, this approach has been applied to a few applications [14-16] by author successfully. In this study, as another application, this intelligent model is applied to rectangular microstrip antenna to predict input impedance over wide frequency band.

The paper is organized as follows: In section II, behavior of the problem is qualitatively expressed and saved as simple membership functions. The effect of the feed probe on the input impedance is extracted as simple curves in section III. Finally conclusion is given in section IV.

2 Behavior of rectangular microstrip antenna

A rectangular microstrip antenna of length L, width W, embedded on the grounded dielectric of thickness h and relative dielectric constant εr is shown in figure 2.
Assume a rectangular microstrip antenna of parameters \( L = 2.86 \text{Cm} \), \( W = 3.44 \text{Cm} \), \( h = 0.7874 \text{mm} \), \( x_f = L/2 \), \( y_f = W/2 \) and \( \varepsilon_r = 2.33 \). To extract the behavior of the antenna, at first the amplitude versus phase of the input impedance of the microstrip antenna is computed by MoM and shown in polar plane in figure 3. As it is seen, decreasing guided wavelength \( \lambda_g \) (increasing frequency) is led to a circular movement where \( \lambda_g = \frac{\lambda_0}{\sqrt{\varepsilon_{eff}}} \) and \( \lambda_0 \) is wave length in free space and \( \varepsilon_{eff} \) is effective relative dielectric constant calculated as following:

\[
\varepsilon_{eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2} \left( 1 + 10 \frac{h}{W} \right)^{-1/2}
\]  

(2.1)

Figure 3: Amplitude versus phase of the input impedance in polar plane, as well as three input-output pairs around \( \lambda_g = 3.4, 4.4, \text{and} 5.4 \) (stars) for defining fitted circles (dotted circles).

In figure 3, three basic circles are distinguishable which are moving from one to another smoothly. These basic circles can be defined via choosing three input-output pairs around \( \lambda_g = 3.4, 4.4, \text{and} 5.4 \) (stars in figure 3). After defining these fitted circles, this circular movement can be represented as the following if-then rules:

\[
\begin{align*}
\text{If } \lambda_g \text{ is short then first fitted circle} \\
\text{If } \lambda_g \text{ is medium then second fitted circle} \\
\text{If } \lambda_g \text{ is long then third fitted circle}
\end{align*}
\]  

(2.2)

Each fitted circle can be represented as a membership function of following belongingness value:

\[
a(\lambda_g) = \begin{cases} 
\frac{1}{2}(1 + \cos \pi \frac{\lambda_g - a_1}{a_2 - a_1}) & \lambda_g : a_1 \rightarrow a_2 \\
\frac{1}{2}(1 - \cos \pi \frac{\lambda_g - a_1}{a_2 - a_1}) & \lambda_g : a_1 \rightarrow a_2 
\end{cases}
\]  

(2.3)

where \( \beta_1 \) and \( \beta_2 \) represents optimizations parameters and \( \lambda_g \) is input variable. Also \( a_1 \) and \( a_2 \) ar points where the circular movement are completely separated from the fitted circles. For the problem under consideration, the membership functions are shown in figure 4. As seen in this figure, these membership
functions, have belongingness one on the fitted circles and are smoothly decreasing to zero on the neighbor fitted circles.

![Image of membership functions](image)

**Figure 4:** Three membership functions have belongingness one on the fitted circles and are smoothly decreasing to zero on the neighbor fitted circles.

Through the above membership functions and the fitted circles, a circle is inferred for each wavelength value (called fuzzy circle) based on Takagi-Sugeno method [17], that is

\[
\begin{align*}
  x(\lambda_g) &= \sum_{i=1}^{3} x_i \alpha_i(\lambda_g) \\
  y(\lambda_g) &= \sum_{i=1}^{3} y_i \alpha_i(\lambda_g) \\
  r(\lambda_g) &= \sum_{i=1}^{3} r_i \alpha_i(\lambda_g)
\end{align*}
\]

(2.4)

in which \(x_i, y_i, r_i\) \(i=1,2,3\) are center coordinates and radius of fitted circles, and \(x, y, r\) are center coordinates and radius of the fuzzy circles for each value \(\lambda_g\). Also \(\alpha_i\)'s are membership functions in figure 3.

Now, the exact position of the input impedance on each fuzzy circle can be found through modeling partial phase (phase with respect to the center of inferred circles for each wavelength value). This is carried out in figure 5. Figure 5 illustrates a smoothly increasing curve including four linear parts (dashed-dotted lines). These lines can be defined through choosing four input-output pairs around \(\lambda_g = 3,4,4,4,5,4,\ and\ 5.7\). The same as circular movement, this curve is easily modeled using following inference equations:

\[
\begin{align*}
  m(\lambda_g) &= \frac{\sum_{i=0}^{4} m_i \alpha'_i(\lambda_g)}{\sum_{i=0}^{4} \alpha'_i(\lambda_g)} \\
  n(\lambda_g) &= \frac{\sum_{i=0}^{4} n_i \alpha'_i(\lambda_g)}{\sum_{i=0}^{4} \alpha'_i(\lambda_g)}
\end{align*}
\]

(2.5)

in which \(m_i, n_i, \ i=1,2,...n\) are slope and bias of fitted lines, and \(m, n\) are slope and bias of inferred fuzzy lines for each wavelength value. Also \(\alpha'_i\)'s membership functions in figure 5 (dotted lines) having belongingness one on the fitted lines and decreasing to zero on the neighbor fitted lines.
Finally, the real part (resistance) and imaginary part (reactance) of the input impedance versus $\lambda_g$ is computed through the proposed model (method of fuzzy, MoF) as follows:

$$\text{input impedance } (\lambda_g) = x + jy + r \exp(j\theta)$$

$$\theta = m \times \lambda_g + n$$

(2.6)

Where $(x, y, r)$ and $(m,n)$ are computed by equations (2.4) and (2.5) respectively and $\theta$ is in radians.

Figure 6 compares the modeled input impedance by MoM and MoF. As it is seen, an excellent agreement is achieved.

As a result, the membership functions in figure 4, and 5 are considered as behavior of the single microstrip antenna.

3 Feed position effect

3.1. Feed position effect along patch width

In the previous sections, the behavior of microstrip antenna was extracted as simple membership functions. As we know, the position of feed probe both along width, $y_f$, and length, $x_f$, changes the input impedance considerably. Hence, in this section, extracting feed probe on the input impedance is qualitatively investigated.
Figure 7: The amplitude versus phase of the input impedance for different values of $y_{f_n} = y_f/W$. 

(a) $y_{f_n} = 0.2$

(b) $y_{f_n} = 0.3$

(c) $y_{f_n} = 0.1$
At first the effects of $x_f$ and $y_f$ on the input impedance are separately extracted. To extract the effect $y_f$ only, it is assumed that $x_f = L/2$, and the amplitude versus phase of the input impedance for a few values of $y_f = y_f/W = 0.1, 0.2, 0.3$ is computed by MoM and shown in figure 7. Figure 7 shows that when $y_f$ is changed, the circular movement is kept, and the center coordinates and radius of fitted circles are only changed. Hence, the characteristics of fitted circles as well as fitted lines for a few values of $y_f$ are computed by MoM and shown in figure 8 and 9 respectively. In Figures 8, simple curves on center coordinates and radius of three fitted circles can be fitted. Similarly simple curves can be fitted on slop and bias of fitted lines in figure 9. It means that to compute the characteristics of the fitted circles and lines for other values of $y_f$, it is not needed to use MoM anymore. In fact, using these simple curves as fuzzy inputs and the extracted behavior of the microstrip antenna, the input impedance for an arbitrary value of $y_f$, is efficiently predicted.

Figure 8: The center coordinates and radius of fitted circles versus $y_f$.

Figure 9: The slope and bias of four fitted lines (as defined in figure 5) versus $y_f$.

3.2. Feed position effect along patch length
In the previous sub-section, the effect of feed probe along patch width was extracted. Similarly to extract feed position effect along patch length only, it is assumed that $y_f = W/2$ ($y_f = 0.5$), and $x_f = x_f/L$ is variable. This is carried out and the characteristics of the fitted circles and lines versus
\( x_{fa} = x_f / L \) are shown in figures 10, and 11 respectively. Now, fitting simple curves on the data computed by MoM, the effect of \( x_{fa} \) is extracted. For instance, figure 12, shows the input impedance for \( x_{fa} = 0.35 \). As it is seen, an excellent agreement is achieved; in addition run-time is vanishingly short.

Figure 10: The center coordinates and radius of fitted circles versus \( x_{fa} \).

Figure 11: The slope and bias of fitted lines versus \( x_{fa} \).

Figure 12: Comparing the input impedance computed by MoM and MoF for \( x_{fa} = 0.35 \).
3.3. Complete model

In this sub-section, using spatial membership functions [12], the simultaneous effect of feed probe along both length and width is achieved. The spatial membership functions here used are as follows:

\[
\alpha_i(x_{fi}, y_{fi}) = \begin{cases} 
\frac{1}{2} \left[ 1 - \cos \pi \frac{\phi_i - \phi_2}{\phi_1 - \phi_2} \right]^\beta_i & \text{for } \phi \in [\phi_1, \phi_2] \\
\frac{1}{2} \left[ 1 + \cos \pi \frac{\phi_i - \phi_2}{\phi_1 - \phi_2} \right]^\beta_i & \text{for } \phi \in [\phi_1, \phi_2] 
\end{cases} 
\]  

(3.7)

in which \( \phi = \tan^{-1}(x_{fi} / y_{fi}) \), \( \beta_1, \beta_2 = \text{optimizing parameters} \) and \( i = 1, 2 \). The spatial membership functions for the problem under consideration are shown in figure 13. In this figure, two fuzzy sets for two independent parameters \( x_{fi} \) and \( y_{fi} \) are seen. Each one has belongingness value of one at its individual axis and it is smoothly decreasing to zero at the other axis.

![Spatial membership functions](image)

Figure 13: Spatial membership functions for combining effects of \( x_{fi} \) and \( y_{fi} \).

To extract the simultaneous effects of feed probe along both length and width, the following inferred equations can be used:

\[
x_f(x_{fi}, y_{fi}) = \frac{x_f(x_{fi})\alpha_1(x_{fi}, y_{fi}) + x_f(y_{fi})\alpha_2(x_{fi}, y_{fi})}{\alpha_1(x_{fi}, y_{fi}) + \alpha_2(x_{fi}, y_{fi})}
\]

\[
y_f(x_{fi}, y_{fi}) = \frac{y_f(x_{fi})\alpha_1(x_{fi}, y_{fi}) + y_f(y_{fi})\alpha_2(x_{fi}, y_{fi})}{\alpha_1(x_{fi}, y_{fi}) + \alpha_2(x_{fi}, y_{fi})}
\]

\[
r_f(x_{fi}, y_{fi}) = \frac{r_f(x_{fi})\alpha_1(x_{fi}, y_{fi}) + r_f(y_{fi})\alpha_2(x_{fi}, y_{fi})}{\alpha_1(x_{fi}, y_{fi}) + \alpha_2(x_{fi}, y_{fi})}
\]

(3.8)

Where \( x_f(i), y_f(i), r_f(i), \ i = x_{fi}, y_{fi}, \ j = 1, 2, 3 \) are center coordinates and radii of fitted circles obtained in figure 8 and 10, and \( x_f(x_{fi}, y_{fi}), y_f(x_{fi}, y_{fi}), r_f(x_{fi}, y_{fi}), \ j = 1, 2, 3 \) are center coordinates and radius of fitted circles when both \( x_f \) and \( y_f \) are variable. Finally, \( \alpha_1 \) and \( \alpha_2 \) are spatial membership functions as shown figure 13.

Similarly, the slope and bias of fitted lines are as following:
\[ m_j(x_{f_r}, y_{f_r}) = \frac{m_j(x_{f_r})\alpha'_l(x_{f_r}, y_{f_r}) + m_j(y_{f_r})\alpha'_l(x_{f_r}, y_{f_r})}{\alpha'_l(x_{f_r}, y_{f_r}) + \alpha'_l(x_{f_r}, y_{f_r})} \]
\[ n_j(x_{f_r}, y_{f_r}) = \frac{n_j(x_{f_r})\alpha'_l(x_{f_r}, y_{f_r}) + n_j(y_{f_r})\alpha'_l(x_{f_r}, y_{f_r})}{\alpha'_l(x_{f_r}, y_{f_r}) + \alpha'_l(x_{f_r}, y_{f_r})} \]

in which \( m_j(i), n_j(i) \) \( i = x_{f_r}, y_{f_r} \), \( j = 1, 2, 3, 4 \) are the slope and biases of fitted lines obtained in figure 9 and 11 respectively. For instance, the radius of first circle inferred from (3.8) is shown in figure (14).

Figure 14: The radius of first circle versus \( x_{f_r} \) and \( y_{f_r} \).

From now on, using these inferred spatial fuzzy inputs and the behavior of rectangular microstrip antenna, the input admittance for arbitrary value of \( (x_{f_r}, y_{f_r}) \) is efficiently predicted.

Table 1 compares the run-time of the two models for computing the input impedance. Meanwhile, the time of MoF is valid, when the proposed model is created.

<table>
<thead>
<tr>
<th>Method</th>
<th>MoM</th>
<th>MoF</th>
<th>TLM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run-time</td>
<td>35min</td>
<td>0.5sec</td>
<td>1.6min</td>
</tr>
</tbody>
</table>

4 Conclusion

In this paper, analysis of rectangular microstrip antenna was investigated based upon qualitative concepts proposed by Tayarani et al [10-12]. At the first step of analysis, the behavior of the microstrip antenna was well qualitatively extracted. Then the effect of feed probe on the input impedance was easily extracted as simple curves. As a result, the input impedance of rectangular microstrip antenna is computed accurately; moreover the executing time is considerably reduced. From now on, the achieved model can be used in designing rectangular antennas, analyzing active microstrip antennas [18] and adaptive microstrip antennas [19]. This approach can be extended in analyzing rectangular microstrip antenna array including mutual coupling effects [5].
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