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Research Article

Intuitionistic Fuzzy Goal Programming Technique for Solving Non-Linear Multi-objective Structural Problem

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Abstract
This paper proposes a new multi-objective intuitionistic fuzzy goal programming approach to solve a multi-objective nonlinear programming problem in context of a structural design. Here we describe some basic properties of intuitionistic fuzzy optimization. We have considered a multi-objective structural optimization problem with several mutually conflicting objectives. The design objective is to minimize weight of the structure and minimize the vertical deflection at loading point of a statistically loaded three-bar planar truss subjected to stress constraints on each of the truss members. This approach is used to solve the above structural optimization model based on arithmetic mean and compare with the solution by intuitionistic fuzzy goal programming approach. A numerical solution is given to illustrate our approach.

Keywords: Goal Programming, Structural Optimization, Non-linear optimization, intuitionistic fuzzy goal and generalized intuitionistic fuzzy goal.

1 Introduction

Over the last decades, various techniques have been used for truss optimization problems which are very popular in the field of structural optimization. Practically, the problem of structural design exhibits nonlinearity, and is aptly represented as a non-linear programming problem with both objective functions and constraints functions in fuzzy environment. The fuzzy set theory was first implemented by Zadeh[2]. Then Zimmermann [4] applied the fuzzy set theory concept with some suitable membership functions to solve linear programming problem with multiple objective functions. The fuzzy set theory also found application in Structural Model. Several researchers like Wang et al. [1], Rao [7], Yeh et.al [6], Xu [5], Shih et.al [8, 9] made distinctive implementation of the fuzzy set theory.

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In view of growing use of fuzzy set in structural problems under situations when information available is imprecise various extension of fuzzy sets immerged. In such extension, Atanassov [10] introduced Intuitionistic fuzzy set (IFS), is one of the generalizations of fuzzy set theory, is characterized by a membership function, a non membership function and a hesitancy function. IFS is very suitable for the depiction of the uncertainty and vagueness of things. The concept of IFS can be viewed as an alternative approach to define a fuzzy set in a situation where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set. In fuzzy sets the degree of acceptance is only considered but IFS is characterized by a membership function and a non-membership function so that the sum of both values is less than one. Now intuitionistic fuzzy optimization (IFO) is an open field for research work. Very little research work has been carried out on IFO in application to structural optimization. Luo.et.al [14] applied the inclusion degree of intuitionistic fuzzy set to multi criteria decision making problem. Pramanik and Roy [11] solved a vector optimization problem using an intuitionistic fuzzy goal programming. A transportation model was solved by Jana and Roy [12] using multi-objective intuitionistic fuzzy linear programming. Dey and Roy [13] use Intuitionistic fuzzy optimization technique to optimize non-linear single objective two bar truss structural model. Nan and Li [15] discussed arithmetic mean in intuitionistic fuzzy environment. Nowadays goal programming in fuzzy environment is common where as intuitionistic fuzzy goal programming is rare in context of structural design. In this study we have used arithmetic mean in intuitionistic fuzzy goal programming to solve multi-objective structural model.

The advantage of the intuitionistic fuzzy optimization technique is twofold: they give the richest apparatus for formulation of optimization problems and on the other hand, the solutions of intuitionistic fuzzy optimizations can satisfy the objective(s) with bigger degree than the analogous fuzzy optimization problem and the crisp one. This paper envisages the application of IFO in context of structural design.

The remainder of this paper is organized in the following way. In section 2, we discuss about intuitionistic fuzzy set, generalized intuitionistic fuzzy set, α-cut and (α,β)-cuts. In section 3 and in section 4, we discuss about intuitionistic fuzzy goal programming problem and generalized intuitionistic fuzzy goal programming problem respectively. In section 5, we described a multi-objective structural model of a three bar truss. In section 6 and 7, we optimize multi-objective structural model through intuitionistic fuzzy goal programming and generalized intuitionistic fuzzy goal programming respectively. Numerical solution of structural model of three bar truss is discussed in section 8. Finally we draw conclusions from the results in section 9.

2 Preliminaries

In this section some basic relevant definitions and notations are reviewed.

Definition 2.1. *[2] Let X be an universal set. Then the fuzzy set $\tilde{A}$ in X is a set of order pairs $\tilde{A}=\{x\in X : (x,\mu_{\tilde{A}}(x))\}$ where $\mu_{\tilde{A}}: X \to [0,1]$ is called the membership function which assigns a real number $\mu_{\tilde{A}}(x)$ in the interval [0, 1], to each element $x\in X$. $\tilde{A}$ is non fuzzy and $\mu_{\tilde{A}}(x)$ is identical to the characteristic function of crisp set. It is clear that the range of membership function is a subset of the non-negative real numbers.*

Definition 2.2. *The α-level set of the fuzzy set $\tilde{A}$ of X is a crisp set $\tilde{A}_\alpha$ that contains all the elements of X that have membership values greater than or equal to α i.e. $\tilde{A}_\alpha=\{x : \mu_{\tilde{A}}(x)\ge\alpha, x\in X, \alpha\in[0,1]\}$.*

Definition 2.3. *Let* $X = \{x_1, x_2, ..., x_n\}$ *be a finite universal set. An Intuitionistic fuzzy set (IFS)* $\tilde{A}^i$ *in the sense of Atanassov's [10] is given by equation* $\tilde{A}^i = \{\langle x, \mu_{\tilde{A}^i}(x_i), \upsilon_{\tilde{A}^i}(x_i)\rangle / x_i \in X\}$ *where the functions* $\mu_{\tilde{A}^i}(x_i): X \to [0,1]; \quad x_i \in X \to \mu_{\tilde{A}^i}(x_i) \in [0,1]$ *and* $\upsilon_{\tilde{A}^i}(x_i): X \to [0,1]; \quad x_i \in X \to \upsilon_{\tilde{A}^i}(x_i) \in [0,1]$ *define the degree of membership and the degree of non-membership of an element* $x_i \in X$ *to the set* $\tilde{A}^i \subseteq X$ *, respectively, such that they satisfy the condition* $0 \le \mu_{\tilde{A}^i}(x_i) + \upsilon_{\tilde{A}^i}(x_i) \le 1$ *,* $\forall x_i \in X$ *. For each IFS* $\tilde{A}^i$ *in X, the amount* $\pi_{\tilde{A}^i}(x_i) = 1 - (\mu_{\tilde{A}^i}(x_i) + \upsilon_{\tilde{A}^i}(x_i))$ *is called the degree of uncertainty (or hesitation) associated with the membership of elements* $x_i \in X$ *in* $\tilde{A}^i$*.We call it intuitionistic fuzzy index of* $\tilde{A}^i$ *with respect of element* $x_i \in X$.

Definition 2.4. *Let* $X = \{x_1, x_2, ..., x_n\}$ *be a finite universal set.*
A Generalized Intuitionistic fuzzy set (GIFS) $\tilde{A}^i$ *in the sense of Atanassov's [10] is given by equation* $\tilde{A}^i = \{\langle x, \mu_{\tilde{A}^i}(x_i), \upsilon_{\tilde{A}^i}(x_i)\rangle / x_i \in X\}$ *where the functions*
$\mu_{\tilde{A}^i}(x_i): X \to [0, w_1]; x_i \in X \to \mu_{\tilde{A}^i}(x_i) \in [0, w_1]$
and
$\upsilon_{\tilde{A}^i}(x_i): X \to [0, w_2]; x_i \in X \to \upsilon_{\tilde{A}^i}(x_i) \in [0, w_2]$.
The membership and non-membership function satisfy the condition
$0 \le \mu_{\tilde{A}^i}(x_i) + \upsilon_{\tilde{A}^i}(x_i) \le w_1 + w_2$, $\forall x_i \in X$, $0 \le w_1 + w_2 \le 1, w_1, w_2 \in [0,1]$.
w_1 *and* w_2 *are the gradations of the membership and the non-membership function respectively.*

Definition 2.5. *A set of* (α, β)*-cut, generated by an IFS* $\tilde{A}^i$ *where* $\alpha, \beta \in [0,1]$ *are fixed numbers such that* $\alpha + \beta \le 1$ *is defined as* $\tilde{A}^i_{\alpha,\beta} = \{\langle x, \mu_{\tilde{A}^i}(x), \upsilon_{\tilde{A}^i}(x)\rangle : x \in X, \mu_{\tilde{A}^i}(x) \ge \alpha, \upsilon_{\tilde{A}^i}(x) \le \beta, \alpha, \beta \in [0,1]\}$*.We define* (α, β)*-level or* (α, β)*-cut, denoted by* $\tilde{A}^i_{\alpha,\beta}$*, as the crisp set of elements* x *which belong to* $\tilde{A}^i$ *at least to the degree* α *and which belong to* $\tilde{A}^i$ *at most to the degree* β .

3 Intuitionistic Fuzzy Goal Programming Problem

Find $x = (x_1, x_2, ..., x_n)^T$
So as to
$Minimize^i$ $f_r(x)$ with target value C_r, acceptance tolerance a_r, rejection tolerance t_r, $r = 1, 2, ..., n$
Subject to

$$g_j(x) \le b_j,\ j = 1, 2, ..., m \qquad (3.1)$$
$$x > 0,$$

with membership and non-membership functions are

$$\mu_{f_r(x)}(f_r(x)) = \begin{cases} 1 & if\ f_r(x) \le C_r \\ 1 - \dfrac{f_r(x) - C_r}{a_r} & if\ C_r \le f_r(x) \le C_r + a_r \\ & if\ f_r(x) \le C_r + a_r \end{cases} \qquad (3.2)$$

$$\upsilon_{f_r(x)}(f_r(x)) = \begin{cases} 0 & if \;\; f_r(x) \le C_r \\ \dfrac{f_r(x) - C_r}{t_r} & if \; C_r \le f_r(x) \le C_r + t_r \\ 1 & if \;\; f_r(x) \le C_r + t_r \end{cases} \tag{3.3}$$

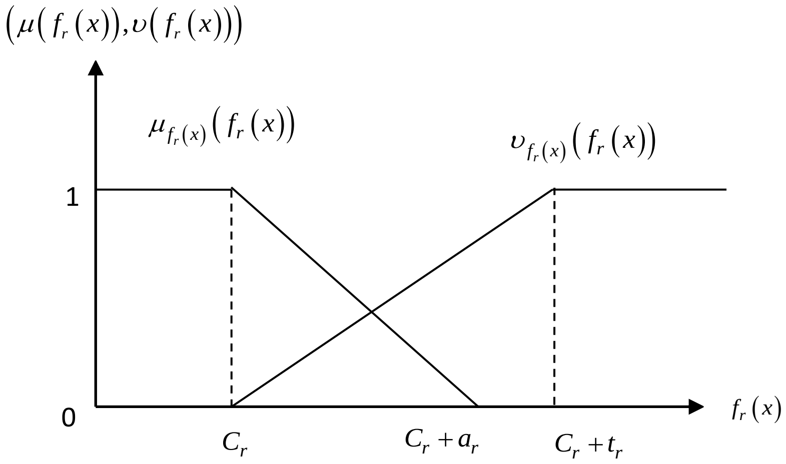

Figure 1: Membership and non-membership function for $f_r(x)$

Intuitionistic fuzzy goal programming can be transform into crisp programming model using membership and non-membership function as

$$\begin{aligned}
&Maximize \left(\mu_{f_r(x)}\left(f_r(x)\right)\right) \\
&minimize \left(\upsilon_{f_r(x)}\left(f_r(x)\right)\right) \\
&subject \;\; to \\
&0 \le \mu_{f_r(x)}\left(f_r(x)\right) + \upsilon_{f_r(x)}\left(f_r(x)\right) \le 1, r = 1, 2, \ldots, n \\
&\mu_{f_r(x)}\left(f_r(x)\right) \ge \upsilon_{f_r(x)}\left(f_r(x)\right), r = 1, 2, \ldots, n \\
&\upsilon_{f_r(x)}\left(f_r(x)\right) \ge 0, r = 1, 2, \ldots, n \\
&g_j(x) \le b_j, \; j = 1, 2, \ldots, m \\
&x > 0,
\end{aligned} \tag{3.4}$$

This is equivalent to

$$\begin{aligned}
&Maximize \; \alpha, \;\; minimize \; \beta \\
&subject \;\; to \\
&\mu_{f_r(x)}\left(f_r(x)\right) \ge \alpha, \;\; \upsilon_{f_r(x)}\left(f_r(x)\right) \le \beta, r = 1, 2, \ldots, n \\
&g_j(x) \le b_j, \; j = 1, 2, \ldots, m \\
&0 \le \alpha + \beta \le 1, \alpha \ge \beta, \alpha \in [0,1], \beta \in [0,1] \\
&x > 0,
\end{aligned} \tag{3.5}$$

Taking arithmetic mean, Equation (3.5) can be written as

$$
\begin{aligned}
& Minimize \left(\frac{\alpha+1-\beta}{2}\right)^{-1} \\
& subject\ \ to \\
& \frac{f_r(x)}{2}\left(\frac{1}{a_r}+\frac{1}{t_r}\right) \le 1+\frac{C_r}{2}\left(\frac{1}{a_r}+\frac{1}{t_r}\right)-\frac{\alpha+1-\beta}{2},\ r=1,2,...,n \qquad (3.6) \\
& g_j(x) \le b_j,\ j=1,2,...,m \\
& 0 \le \alpha+\beta \le 1, \alpha \ge \beta, \alpha \in [0,1], \beta \in [0,1] \\
& x > 0,
\end{aligned}
$$

Solve the above crisp model (3.6) by using appropriate mathematical programming algorithm to get optimal solution of objective functions.

Definition 3.1. (M-N Pareto optimal solution)
x^* is said to e a M-N Pareto optimal solution if and only if there does not exist another $x \in X$ such that $\mu_{f_r(x)}(x) \ge \mu_{f_r(x)}(x^*)$ and $\upsilon_{f_r(x)}(x) \le \upsilon_{f_r(x)}(x^*)$ for all r and strict inequality holds for at least one r.

Theorem 3.1. *$x^* \in X$ is M-N Pareto optimal solution of (3.1) if and only if x^* is Pareto optimal solution of*

Minimize $f_r(x), r=1,2,...,n$ *(3.7)*

subject to $g_j(x) \le b_j,\ j=1,2,...,m$

$x > 0$

Proof. Let $x^* \in X$ is M-N Pareto optimal solution of (3.1), then there does not exist any $x \in X$ such that $\mu_{f_r(x)}(x) \ge \mu_{f_r(x)}(x^*)$ and $\upsilon_{f_r(x)}(x) \le \upsilon_{f_r(x)}(x^*)$ for all r and strictly inequality holds for at least one r.
From the expression of membership function we have

$$1-\frac{f_r(x)-C_r}{a_r} \ge 1-\frac{f_r(x^*)-C_r}{a_r} \ \ i.e\ \ f_r(x) \le f_r(x^*)$$

Also for non-membership function

$$\frac{f_r(x)-C_r}{t_r} \le \frac{f_r(x^*)-C_r}{t_r}$$

Gives $f_r(x) \le f_r(x^*)$ with strict inequality holds for at least one r. So x^* is Pareto optimal solution of (3.7)
On the other hand, let x^* be a Pareto optimal solution of (3.7) such that $f_r(x) \le f_r(x^*)$ with strict inequality holds for at least one r.

So $f_r(x)-C_r \le f_r(x^*)-C_r$ *i.e* $\dfrac{f_r(x)-C_r}{t_r} \le \dfrac{f_r(x^*)-C_r}{t_r}$

which tells that $\upsilon_{f_r(x)}(x) \le \upsilon_{f_r(x)}(x^*)$ *and* $1-\dfrac{f_r(x)-C_r}{a_r} \ge 1-\dfrac{f_r(x^*)-C_r}{a_r}$ *which tells that* $\mu_{f_r(x)}(x) \ge \mu_{f_r(x)}(x^*)$.

Hence, x^* *is M-N Pareto optimal solution of (3.1).*

4 Generalized Intuitionistic Fuzzy Goal Programming Problem

Find $x = (x_1, x_2, \ldots.., x_n)^T$

So as to

$Minimize^i$ $f_r(x)$ with target value C_r, acceptance tolerance a_r, rejection tolerance t_r

subject to

$$g_j(x) \le b_j,\ j = 1, 2, \ldots, m \tag{4.8}$$

$x > 0$,

with membership and non-membership functions are

$$\mu_{f_r(x)}^{w_1}(f_r(x)) = \begin{cases} w_1 & if\ f_r(x) \le C_r \\ w_1\left(1 - \dfrac{f_r(x) - C_r}{a_r}\right) & if\ C_r \le f_r(x) \le C_r + a_r \\ & if\ f_r(x) \le C_r + a_r \end{cases} \tag{4.9}$$

$$\upsilon_{f_r(x)}^{w_2}(f_r(x)) = \begin{cases} 0 & if\ f_r(x) \le C_r \\ w_2\left(\dfrac{f_r(x) - C_r}{t_r}\right) & if\ C_r \le f_r(x) \le C_r + t_r \\ w_2 & if\ f_r(x) \le C_r + t_r \end{cases} \tag{4.10}$$

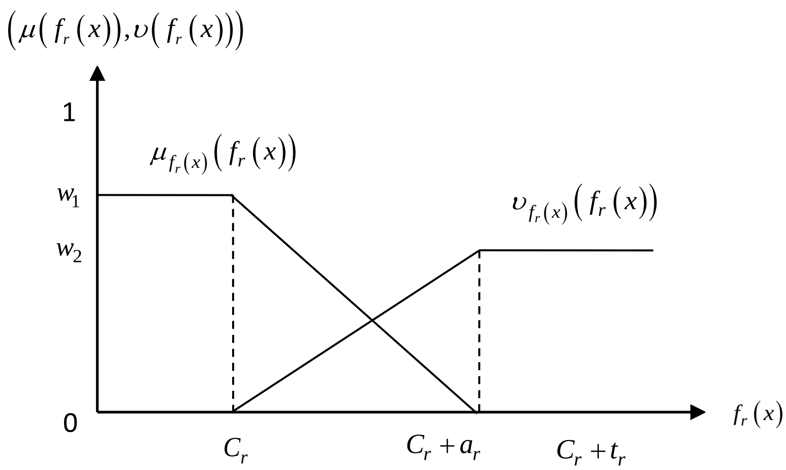

Figure 2: Membership and non-membership function for $f_r(x)$

Intuitionistic fuzzy goal programming can be transform into crisp programming model using membership and non-membership function as

$$\begin{aligned} &Maximize\left(\mu_{f_r(x)}^{w_1}(f_r(x))\right) \\ &minimize\left(\upsilon_{f_r(x)}^{w_2}(f_r(x))\right) \end{aligned} \tag{4.11}$$

$$\begin{aligned}
&subject\ \ to\\
&0 \le \mu_{f_r(x)}^{w_1}\left(f_r\left(x\right)\right) + \upsilon_{f_r(x)}^{w_2}\left(f_r\left(x\right)\right) \le 1,\\
&\mu_{f_r(x)}^{w_1}\left(f_r\left(x\right)\right) \ge \upsilon_{f_r(x)}^{w_2}\left(f_r\left(x\right)\right),\\
&\upsilon_{f_r(x)}^{w_2}\left(f_r\left(x\right)\right) \ge 0,\\
&g_j\left(x\right) \le b_j,\ j = 1,2,\ldots,m\\
&x > 0,
\end{aligned}$$

This is equivalent to

$$\begin{aligned}
&Maximize\ \alpha,\ \ minimize\ \beta \qquad (4.12)\\
&subject\ \ to\\
&\mu_{f_r(x)}^{w_1}\left(f_r\left(x\right)\right) \ge \alpha,\ \ \upsilon_{f_r(x)}^{w_2}\left(f_r\left(x\right)\right) \le \beta,\\
&g_j\left(x\right) \le b_j,\ j = 1,2,\ldots,m\\
&0 \le \alpha + \beta \le 1, \alpha \ge \beta, \alpha \in \left[0,1\right], \beta \in \left[0,1\right]\\
&x > 0, w_1, w_2 \in \left[0,1\right]
\end{aligned}$$

Taking arithmetic mean, Equation (4.12) can be written as

$$\begin{aligned}
&Minimize\left(\frac{\alpha + 1 - \beta}{2}\right)^{-1}\\
&subject\ \ to\\
&\frac{f_r\left(x\right)}{2}\left(\frac{w_1}{a_r} + \frac{w_2}{t_r}\right) \le 1 + \frac{C_r}{2}\left(\frac{w_1}{a_r} + \frac{w_2}{t_r}\right) - \frac{\alpha + 1 - \beta}{2} \qquad (4.13)\\
&g_j\left(x\right) \le b_j,\ j = 1,2,\ldots,m\\
&0 \le \alpha + \beta \le 1, \alpha \ge \beta, \alpha \in \left[0,1\right], \beta \in \left[0,1\right]\\
&x > 0, 0 \le w_1 + w_2 \le 1, w_1, w_2 \in \left[0,1\right].
\end{aligned}$$

Solve the above crisp model (4.13) by using appropriate mathematical programming algorithm to get optimal solution of objective functions.

Theorem 4.1. *For a generalized intuitionistic fuzzy goal programming model, the sum of membership and non-membership functions will lie between* 0 *and* $w_1 + w_2$.

Proof. From figure 2, we see that $0 \le \mu_{f_r(x)}^{w_1}(f_r(x)) \le w_1$ and $0 \le \upsilon_{f_r(x)}^{w_2}(f_r(x)) \le w_2$.

For $f_r\left(x\right) \le C_r$,

$\mu_{f_r(x)}^{w_1}(f_r(x)) = w_1$ and $\upsilon_{f_r(x)}^{w_2}(f_r(x)) = 0$.

Therefore $\mu_{f_r(x)}^{w_1}(f_r(x)) + \upsilon_{f_r(x)}^{w_2}(f_r(x)) = w_1 \le w_1 + w_2$.

Again $w_1 \ge 0$ gives that $\mu_{f_r(x)}^{w_1}(f_r(x)) + \upsilon_{f_r(x)}^{w_2}(f_r(x)) \ge 0$.

For $C_r < f_r\left(x\right) \le C_r + a_r$;

When $f_r\left(x\right) > C_r$, $\mu_{f_r(x)}^{w_1}(f_r(x)) + \upsilon_{f_r(x)}^{w_2}(f_r(x)) > w_1 \ge 0$.

When $f_r(x) \le C_r + a_r$; $\mu^{w_1}_{f_r(x)}(f_r(x)) + \upsilon^{w_2}_{f_r(x)}(f_r(x)) \le w_1 \frac{a_r}{t_r} < w_1 \le w_1 + w_2$, since $\frac{a_r}{t_r} < 1$ according to figure2.

For $C_r + a_r < f_r(x) \le C_r + t_r$;

When $f_r(x) > C_r + a_r$; $\mu^{w_1}_{f_r(x)}(f_r(x)) + \upsilon^{w_2}_{f_r(x)}(f_r(x)) > w_2 \frac{a_r}{t_r} > w_2 \ge 0$, since $\frac{a_r}{t_r} < 1$ according to figure 2.

When $f_r(x) \le C_r + t_r$; $\mu^{w_1}_{f_r(x)}(f_r(x)) + \upsilon^{w_2}_{f_r(x)}(f_r(x)) \le w_1 \le w_1 + w_2$.

For $f_r(x) > C_r + t_r$; $\mu^{w_1}_{f_r(x)}(f_r(x)) = 0$ and $\upsilon^{w_2}_{f_r(x)}(f_r(x)) = w_2$.

Therefore $\mu^{w_1}_{f_r(x)}(f_r(x)) + \upsilon^{w_2}_{f_r(x)}(f_r(x)) = w_2 \le w_1 + w_2$.

Again $w_2 \ge 0$ gives that $\mu^{w_1}_{f_r(x)}(f_r(x)) + \upsilon^{w_2}_{f_r(x)}(f_r(x)) \ge 0$.

Hence in all cases $0 \le \mu^{w_1}_{f_r(x)}(f_r(x)) + \upsilon^{w_2}_{f_r(x)}(f_r(x)) \le w_1 + w_2$.

5 Application to Multi-objective Structural Optimization Model of a Three-Bar Truss:

A well-known three bar [9] planar truss structure is considered. The design objective is to minimize weight of the structural $WT(A_1, A_2)$ and minimize the vertical deflection $\delta(A_1, A_2)$ at loading point of a statistically loaded three-bar planar truss subjected to stress $\sigma_i(A_1, A_2)$ constraints on each of the truss members $i = 1, 2, 3$.

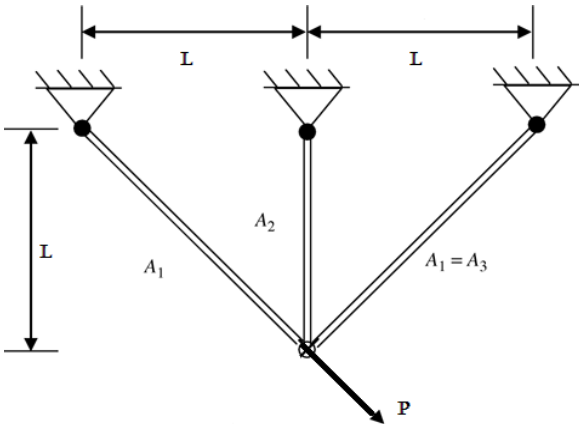

Figure 3: Design of the three-bar planar truss

The multi-objective structural optimization problem (MOSOP) can be stated as follows:

$$Minimize\ WT(A_1, A_2) = \rho L\left(2\sqrt{2}A_1 + A_2\right),$$
$$minimize\ \delta(A_1, A_2) = \frac{PL}{E\left(A_1 + \sqrt{2}A_2\right)}, \tag{5.14}$$

$$\text{subject to } \sigma_1(A_1,A_2) \equiv \frac{P\left(\sqrt{2}A_1 + A_2\right)}{2A_1A_2 + \sqrt{2}A_1^2} \le [\sigma_T],$$
$$\sigma_2(A_1,A_2) \equiv \frac{P}{A_1 + \sqrt{2}A_2} \le [\sigma_T],$$
$$\sigma_3(A_1,A_2) \equiv \frac{PA_2}{2A_1A_2 + \sqrt{2}A_1^2} \le [\sigma_C],$$
$$A_i^{\min} \le A_i \le A_i^{\max}, i = 1,2.$$

Where $P =$ applied load; ρ =material density, $L =$ Length, Allowable tensile stress = $[\sigma_T]$.Allowable compressive stress = $[\sigma_C]$, Young's modulus $= E$, A_1 = cross section of bar-1 and bar-3 A_2 =cross section of bar-2.

The multi-objective Structural model (5.14) can be expressed as an intuitionistic fuzzy model:

$$\text{Minimize } WT(A_1,A_2) = \rho L\left(2\sqrt{2}A_1 + A_2\right) \quad (5.15)$$

with target value WT_0, acceptance tolerance a_{WT}, rejection tolerance t_{WT}

$$\text{minimize } \delta(A_1,A_2) = \frac{PL}{E\left(A_1 + \sqrt{2}A_2\right)}$$

with target value δ_0, acceptance tolerance a_δ, rejection tolerance t_δ
subject to same constraints as in (5.14)

6 Intuitionistic Fuzzy goal programming to solve multi-objective Structural model

To solve MOSOP (5.15), we used article 3 Here for simplicity linear membership function $\mu_{WT}(WT(A_1,A_2))$ and non membership function $\upsilon_{WT}(WT(A_1,A_2))$ for the objective function $WT(A_1,A_2)$ are defined as

$$\mu_{WT}(WT(A_1,A_2)) = \begin{cases} 1 & \text{if } WT(A_1,A_2) \le WT_0 \\ \left(1 - \dfrac{WT(A_1,A_2) - WT_0}{a_{WT}}\right) & \text{if } WT_0 \le WT(A_1,A_2) \le WT_0 + a_{WT} \\ 0 & \text{if } WT(A_1,A_2) \ge WT_0 + a_{WT} \end{cases}$$

$$\upsilon_{WT}(WT(A_1,A_2)) = \begin{cases} 1 & \text{if } WT(A_1,A_2) \le WT_0 \\ \left(\dfrac{WT(A_1,A_2) - WT_0}{t_{WT}}\right) & \text{if } WT_0 \le WT(A_1,A_2) \le WT_0 + t_{WT} \\ 0 & \text{if } WT(A_1,A_2) \ge WT_0 + t_{WT} \end{cases}$$

Here for simplicity linear membership function $\mu_\delta(\delta(A_1,A_2))$ and non membership function $\upsilon_\delta(\delta(A_1,A_2))$ for the objective function $\delta(A_1,A_2)$ are defined as

$$\mu_\delta(\delta(A_1,A_2)) = \begin{cases} 1 & \text{if } \delta(A_1,A_2) \le \delta_0 \\ \left(1 - \dfrac{\delta(A_1,A_2) - \delta_0}{a_\delta}\right) & \text{if } \delta_0 \le \delta(A_1,A_2) \le \delta_0 + a_\delta \\ 0 & \text{if } \delta(A_1,A_2) \ge \delta_0 + a_\delta \end{cases}$$

$$\upsilon_{\delta}\left(\delta(A_1,A_2)\right)=\begin{cases}1 & if\ \ \delta(A_1,A_2)\le\delta_0\\ \left(\dfrac{\delta(A_1,A_2)-\delta_0}{t_{\delta}}\right) & if\ \ \delta_0\le\delta(A_1,A_2)\le\delta_0+t_{\delta}\\ 0 & if\ \ \delta(A_1,A_2)\ge\delta_0+t_{\delta}\end{cases}$$

According to article 3, having elicited the above membership functions crisp non-linear programming problem is formulated as follows:

$$Maximize\ \alpha,\ minimize\ \beta \tag{6.16}$$

subject to

$$\left(1-\frac{WT(A_1,A_2)-WT_0}{a_{WT}}\right)\ge\alpha\,,\ \left(\frac{WT(A_1,A_2)-WT_0}{t_{WT}}\right)\le\beta$$

$$\left(1-\frac{\delta(A_1,A_2)-\delta_0}{a_{\delta}}\right)\ge\alpha\,,\left(\frac{\delta(A_1,A_2)-\delta_0}{t_{\delta}}\right)\le\beta$$

$$\frac{P\left(\sqrt{2}A_1+A_2\right)}{2A_1A_2+\sqrt{2}A_1^2}\le\left[\sigma_T\right],$$

$$\frac{P}{A_1+\sqrt{2}A_2}\le\left[\sigma_T\right],$$

$$\frac{PA_2}{2A_1A_2+\sqrt{2}A_1^2}\le\left[\sigma_C\right],$$

$$A_i^{\min}\le A_i\le A_i^{\max},i=1,2.,$$

$$0\le\alpha+\beta\le1,\alpha\in\left[0,1\right],\beta\in\left[0,1\right]$$

Taking arithmetic mean, the problem (6.16) can be written as

$$Minimize\left(\frac{\alpha+1-\beta}{2}\right)^{-1} \tag{6.17}$$

subject to

$$\frac{1}{2}\rho L\left(2\sqrt{2}A_1+A_2\right)\left(\frac{1}{a_{WT}}+\frac{1}{t_{WT}}\right)\le1+\frac{WT_0}{2}\left(\frac{1}{a_{WT}}+\frac{1}{t_{WT}}\right)-\frac{\alpha+1-\beta}{2}$$

$$\frac{PL}{2E\left(A_1+\sqrt{2}A_2\right)}\left(\frac{1}{a_{\delta}}+\frac{1}{t_{\delta}}\right)\le1+\frac{\delta_0}{2}\left(\frac{1}{a_{\delta}}+\frac{1}{t_{\delta}}\right)-\frac{\alpha+1-\beta}{2}$$

$$\frac{P\left(\sqrt{2}A_1+A_2\right)}{2A_1A_2+\sqrt{2}A_1^2}\le\left[\sigma_T\right],$$

$$\frac{P}{A_1+\sqrt{2}A_2}\le\left[\sigma_T\right],$$

$$\frac{PA_2}{2A_1A_2+\sqrt{2}A_1^2}\le\left[\sigma_C\right],$$

$$A_i^{\min}\le A_i\le A_i^{\max},i=1,2.$$

$$0\le\alpha+\beta\le1,\alpha\in\left[0,1\right],\beta\in\left[0,1\right]$$

7 Generalized intuitionistic fuzzy goal programming to solve multi-objective structural model

To solve MOSOP (5.15), we used article 4. Here for simplicity linear membership function $\mu_{WT}^{w_1}\left(WT(A_1,A_2)\right)$ and non membership function $\upsilon_{WT}^{w_2}\left(WT(A_1,A_2)\right)$ for the objective function $WT(A_1,A_2)$ are defined as

$$\mu_{WT}^{w_1}\left(WT(A_1,A_2)\right)=\begin{cases} w_1 & \text{if } WT(A_1,A_2)\le WT_0 \\ w_1\left(1-\dfrac{WT(A_1,A_2)-WT_0}{a_{WT}}\right) & \text{if } WT_0\le WT(A_1,A_2)\le WT_0+a_{WT} \\ 0 & \text{if } WT(A_1,A_2)\ge WT_0+a_{WT} \end{cases}$$

$$\upsilon_{WT}^{w_2}\left(WT(A_1,A_2)\right)=\begin{cases} 0 & \text{if } WT(A_1,A_2)\le WT_0 \\ w_2\left(\dfrac{WT(A_1,A_2)-WT_0}{t_{WT}}\right) & \text{if } WT_0\le WT(A_1,A_2)\le WT_0+t_{WT} \\ w_2 & \text{if } WT(A_1,A_2)\ge WT_0+t_{WT} \end{cases}$$

Here for simplicity linear membership function $\mu_{\delta}^{w_1}\left(\delta(A_1,A_2)\right)$ and non membership function $\upsilon_{\delta}^{w_2}\left(\delta(A_1,A_2)\right)$ for the objective function $\delta(A_1,A_2)$ are defined as

$$\mu_{\delta}^{w_1}\left(\delta(A_1,A_2)\right)=\begin{cases} w_1 & \text{if } \delta(A_1,A_2)\le \delta_0 \\ w_1\left(1-\dfrac{\delta(A_1,A_2)-\delta_0}{a_{\delta}}\right) & \text{if } \delta_0\le \delta(A_1,A_2)\le \delta_0+a_{\delta} \\ 0 & \text{if } \delta(A_1,A_2)\ge \delta_0+a_{\delta} \end{cases}$$

$$\upsilon_{\delta}^{w_2}\left(\delta(A_1,A_2)\right)=\begin{cases} 0 & \text{if } \delta(A_1,A_2)\le \delta_0 \\ w_2\left(\dfrac{\delta(A_1,A_2)-\delta_0}{t_{\delta}}\right) & \text{if } \delta_0\le \delta(A_1,A_2)\le \delta_0+t_{\delta} \\ w_2 & \text{if } \delta(A_1,A_2)\ge \delta_0+t_{\delta} \end{cases}$$

According to article 4, having elicited the above membership functions crisp non-linear programming problem is formulated as follows:

$$\text{Maximize } \alpha,\ \text{minimize } \beta \tag{7.18}$$

subject to

$$w_1\left(1-\frac{WT(A_1,A_2)-WT_0}{a_{WT}}\right)\ge\alpha,\ w_2\left(\frac{WT(A_1,A_2)-WT_0}{t_{WT}}\right)\le\beta$$

$$w_1\left(1-\frac{\delta(A_1,A_2)-\delta_0}{a_{\delta}}\right)\ge\alpha,\ w_2\left(\frac{\delta(A_1,A_2)-\delta_0}{t_{\delta}}\right)\le\beta$$

$$\frac{P\left(\sqrt{2}A_1+A_2\right)}{2A_1A_2+\sqrt{2}A_1^2}\le[\sigma_T],$$

$$\frac{P}{A_1+\sqrt{2}A_2}\le[\sigma_T],$$

$$\frac{PA_2}{2A_1A_2+\sqrt{2}A_1^2}\le[\sigma_C],$$

$$A_i^{\min} \le A_i \le A_i^{\max}, i = 1, 2.,$$
$$0 \le \alpha + \beta \le 1, \alpha \in [0,1], \beta \in [0,1]$$
$$0 \le w_1 + w_2 \le 1, w_1, w_2 \in [0,1].$$

Taking arithmetic mean, the problem (7.18) can be written as

$$Minimize \left(\frac{\alpha + 1 - \beta}{2} \right)^{-1} \tag{7.19}$$

subject to

$$\frac{1}{2}\rho L\left(2\sqrt{2}A_1 + A_2\right)\left(\frac{w_1}{a_{WT}} + \frac{w_2}{t_{WT}}\right) \le 1 + \frac{WT_0}{2}\left(\frac{w_1}{a_{WT}} + \frac{w_2}{t_{WT}}\right) - \frac{\alpha + 1 - \beta}{2}$$
$$\frac{PL}{2E\left(A_1 + \sqrt{2}A_2\right)}\left(\frac{w_1}{a_\delta} + \frac{w_2}{t_\delta}\right) \le 1 + \frac{\delta_0}{2}\left(\frac{w_1}{a_\delta} + \frac{w_2}{t_\delta}\right) - \frac{\alpha + 1 - \beta}{2}$$
$$\frac{P\left(\sqrt{2}A_1 + A_2\right)}{2A_1A_2 + \sqrt{2}A_1^2} \le [\sigma_T],$$
$$\frac{P}{A_1 + \sqrt{2}A_2} \le [\sigma_T],$$
$$\frac{PA_2}{2A_1A_2 + \sqrt{2}A_1^2} \le [\sigma_C],$$
$$A_i^{\min} \le A_i \le A_i^{\max}, i = 1, 2.$$
$$0 \le \alpha + \beta \le 1, \alpha \in [0,1], \beta \in [0,1]$$
$$0 \le w_1 + w_2 \le 1, w_1, w_2 \in [0,1].$$

8 Numerical Solution of a Multi-objective Structural Optimization Problem (MOSOP) of a Three-Bar Truss:

The input data for MOSOP (5.14) is given as follows

Table 1: Input data for crisp model (5.14)

Applied load P (KN)	Material density ρ (KN/m^3)	Length L (m)	Maximum allowable tensile stress σ_T (KN/m^2)	Maximum allowable compressive stress σ_C (KN/m^2)	Young's modulus E (KN/m^2)	$A_i^{\min}$ and $A_i^{\max}$ of cross section of bars $(10^{-4} m^2)$
20	100	1	20	15	2×10^8	$A_1^{\min} = 0.1$ $A_1^{\max} = 5$ $A_2^{\min} = 0.1$ $A_2^{\max} = 5$

The intuitionistic fuzzy goal multi-objective structural optimization problem (IFGMOSOP) is $Minimize^i\ WT(A_1, A_2) = \rho L\left(2\sqrt{2}A_1 + A_2\right)$ with target $4\times10^2 KN$, acceptance tolerance $2\times10^2 KN$ and rejection tolerance $4.5\times10^2 KN$.

$minimize^{i}\ \delta\left(A_1,A_2\right)=\dfrac{PL}{E\left(A_1+\sqrt{2}A_2\right)}$ with target $2.5\times10^{-7}m$, acceptance tolerance $2.5\times10^{-7}m$ and rejection tolerance $4.5\times10^{-7}m$.

$$subject\ to\ \ \sigma_1\left(A_1,A_2\right)\equiv\frac{P\left(\sqrt{2}A_1+A_2\right)}{2A_1A_2+\sqrt{2}A_1^2}\le\left[\sigma_T\right],$$

$$\sigma_2\left(A_1,A_2\right)\equiv\frac{P}{A_1+\sqrt{2}A_2}\le\left[\sigma_T\right],$$

$$\sigma_3\left(A_1,A_2\right)\equiv\frac{PA_2}{2A_1A_2+\sqrt{2}A_1^2}\le\left[\sigma_C\right],$$

$$A_i^{\min}\le A_i\le A_i^{\max},i=1,2.$$

Solution: According to fuzzy goal programming and intuitionitic fuzzy goal programming we get optimal solution of three bar truss as shown in table 2 (I, II) as follows

Table 2 (I): Optimal solutions MOSOP by IFGP and GIFGP based on arithmetic mean

Method	**Decision variables**	**Optimal objective functions**
IFGP	$A_1\times10^{-4}m^2=0.5922241$ $A_2\times10^{-4}m^2=3.331468$	$WT^*\left(A_1,A_2\right)=5.006530\times10^2KN$ $\delta\left(A_1,A_2\right)=3.7710001\times10^{-7}m$
GIFGP $\left[w_1=.15\ \ w_2=.8\right]$	$A_1\times10^{-4}m^2=0.5921178$ $A_2\times10^{-4}m^2=3.325522$	$WT^*\left(A_1,A_2\right)=5.000284\times10^2KN$ $\delta\left(A_1,A_2\right)=3.777065\times10^{-7}m$

Table 2 (II): Optimal solutions MOSOP by IFGP and GIFGP based on arithmetic mean

Method	α^*,β^*	**Membership and non-membership**	**Sum of membership and non-membership**
IFGP	$\alpha^*=0.6045773$ $\beta^*=0.3954227$	$\mu_{WT}\left(A_1,A_2\right)=0.4967348$ $\upsilon_{WT}\left(A_1,A_2\right)=0.2875801$ $\mu_\delta\left(A_1,A_2\right)=0.4915994$ $\upsilon_\delta\left(A_1,A_2\right)=0.2824448$	$\mu_{WT}\left(A_1,A_2\right)+\upsilon_{WT}\left(A_1,A_2\right)=0.7843149$ $\mu_\delta\left(A_1,A_2\right)+\upsilon_\delta\left(A_1,A_2\right)=0.7740442$
GIFGP $\left[w_1=.15\ \ w_2=.8\right]$	$\alpha^*=0.8481711$ $\beta^*=0.1518289$	$\mu_{WT}\left(A_1,A_2\right)=0.4998579$ $\upsilon_{WT}\left(A_1,A_2\right)=0.2857955$ $\mu_\delta\left(A_1,A_2\right)=0.4891739$ $\upsilon_\delta\left(A_1,A_2\right)=0.2837923$	$\mu_{WT}\left(A_1,A_2\right)+\upsilon_{WT}\left(A_1,A_2\right)=0.7856534$ $\mu_\delta\left(A_1,A_2\right)+\upsilon_\delta\left(A_1,A_2\right)=0.7729662$

From the above table it is clear that all objective functions are attain their goals as well as all restrictions of membership and non-membership function in both IFGP and GIFGP. The sum of membership and non-membership function for each objective is less to sum of gradations $\left(w_1+w_2\right)$. Hence all the criteria of generalized intuitionistic fuzzy set are satisfied.

9 Conclusions

The main purpose of this paper is to describe goal programming in intuitionistic fuzzy environment and also in generalized intuitionistic environment. A comparative study for intuitionistic fuzzy goal programming and generalized intuitionistic fuzzy goal programming on multi-objective structural model has been discussed. We Numerical example is given to illustrate our approach. This method presented is quite simple and can be applied to other areas of engineering science.

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