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Research Article

Diamond Fuzzy Number

T. Pathinathan[1*], K. Ponnivalavan[1]

(1) P.G. and Research Departmentof Mathematics, Loyola College, Chennai –600034,India.

Abstract

In this paper we define diamond fuzzy number with the help of triangular fuzzy number. We include basic arithmetic operations like addition, subtraction of diamond fuzzy numbers with examples. We define diamond fuzzy matrix with some matrix properties. We have defined Nested diamond fuzzy number and Linked diamond fuzzy number. We have further classified Right Linked Diamond Fuzzy number and Left Linked Diamond Fuzzy number. Finally we have verified the arithmetic operations for the above mentioned types of Diamond Fuzzy Numbers.

Keywords: Fuzzy Numbers, Diamond Fuzzy Numbers, Nested Diamond Fuzzy Number, Right and Left Linked Diamond Fuzzy Numbers, Fuzzy operations, Fuzzy Matrix.

1 Introduction

L.A.Zadeh introduced fuzzy set theory in 1965. Different types of fuzzy sets[3] are defined in order to clear the vagueness of the existing problems. A fuzzy number [9], is a quantity whose values are imprecise, rather than exact as in the case with single-valued function. The concept of fuzzy numbers is the generalization of the concept of real numbers. D.Dubois and H.Prade has defined fuzzy number as a fuzzy subset of the real line [5,12]. So far fuzzy numbers like triangular fuzzy numbers [4], trapezoidal fuzzy numbers [2,10], Pentagonal fuzzy numbers[11], Hexagonal, Octagonal and pyramid fuzzy numbers have been introduced with its membership functions. These numbers have got many applications[7] like non-linear equations, risk analysis and reliability. Many operations[8,1,6] were done using fuzzy numbers. In this paper, we introduce diamond fuzzy number with its membership functions and the various types of Diamond Fuzzy Numbers. Section one presents the introduction, section two presents the basic definitions of fuzzy numbers, section three presents diamond fuzzy numbers and nested diamond fuzzy numbers, section four presents Linked diamond fuzzy numbers and finally we give conclusion on diamond fuzzy number (DFN).

* Corresponding Author. Email address: pathiloyolamaths @gmail.com, Tel: 09840929095

2 Preliminaries and notations

Definition 2.1.*(Fuzzy set):*
A fuzzy set is characterized by a membership function mapping the elements of a domain, space or universe of discourse X to the unit interval [0, 1]. A fuzzy set $\underset{\sim}{A}$ *in a universe of discourse X is defined as the following set of pairs:* $\underset{\sim}{A} = \{x, \mu_{\underset{\sim}{A}}(x);\ x \in X\}$

Here $\mu_{\underset{\sim}{A}} : X \to [0,1]$ *is a mapping called the degree of membership function of the fuzzy set* $\underset{\sim}{A}$ *and* $\mu_{\underset{\sim}{A}}(x)$ *is called the membership value of* $x \in X$ *in the fuzzy set* $\underset{\sim}{A}$. *These membership grades are often represented by real numbers ranging from [0,1].*

Definition 2.2. *(Fuzzy Number):*
A Fuzzy number $\underset{\sim}{A}$ *is a fuzzy set on the real line R, must satisfy the following conditions.*

(i) $\mu_{\underset{\sim}{A}}(x_0)$ *is piecewise continous*

(ii) There exist atleast one $x_o \in R$ *with* $\mu_{\underset{\sim}{A}}(x_0) = 1$

(iii) $\underset{\sim}{A}$ *must be normal and convex*

Definition 2.3. *(Triangular Fuzzy Number):*
Triangular Fuzzy Number is defined as $\underset{\sim}{A}$ *= {a,b,c}, where all a, b, c are real numbers and its membership function is given below.*

$$\mu_{\underset{\sim}{A}}(x) = \begin{cases} \dfrac{(x-a)}{(b-a)} & for\ a \le x \le b \\ \dfrac{(c-x)}{(c-b)} & for\ b \le x \le c \\ 0 & otherwise \end{cases}$$

Definition 2.4. *(Trapezoidal Fuzzy Number):*
A fuzzy set $\underset{\sim}{A}$ *= (a, b, c, d) is said to trapezoidal fuzzy number if its membership function is given by where* $a \le b \le c \le d$

$$\mu_{\underset{\sim}{A}}(x) = \begin{cases} 0 & for\ x < a \\ \dfrac{(x-a)}{(b-a)} & for\ a \le x \le b \\ 1 & for\ b \le x \le c \\ \dfrac{(d-x)}{(d-c)} & for\ c \le x \le d \\ 0 & for\ x > d \end{cases}$$

Definition 2.5. *(Pentagonal Fuzzy Number):*
A Pentagonal Fuzzy Number (PFN) of a fuzzy set $\tilde{A}$ is defined as $\tilde{A}_P$ = {a, b, c, d, e}, and its membership function is given by,

$$\mu_{\tilde{A}_P}(x)=\begin{cases} 0 & for\ x < a, \\ \dfrac{(x-a)}{(b-a)} & for\ a \le x \le b \\ \dfrac{(x-b)}{(c-b)} & for\ b \le x \le c \\ 1 & x = c \\ \dfrac{(d-x)}{(d-c)} & for\ c \le x \le d \\ \dfrac{(e-x)}{(e-d)} & for\ d \le x \le e \\ 0 & for\ x > e \end{cases}$$

3 Diamond Fuzzy Number

Definition 3.1. *(Diamond Fuzzy Number)*
A Diamond Fuzzy Number (DFN) of a fuzzy set $\tilde{A}$ is defined as $\tilde{A}_D$ = {a, b, c, (α_b β_b)}, and its membership function is given by,

$$\mu_{\tilde{A}_D}(x)=\begin{cases} 0 & for\ x < a, \\ \dfrac{(x-a)}{(b-a)} & for\ a \le x \le b \\ \dfrac{(c-x)}{(c-b)} & for\ b \le x \le c \\ \alpha_b \text{ - base} & \\ \dfrac{(a-x)}{(a-b)} & for\ a \le x \le b \\ \dfrac{(x-c)}{(b-c)} & for\ b \le x \le c \\ 1 & x = \beta_b \\ 0 & otherwise \end{cases}$$

where α_b is the base of the triangle a β_b c and also for the inverted reflection of the above triangle, namely abc.

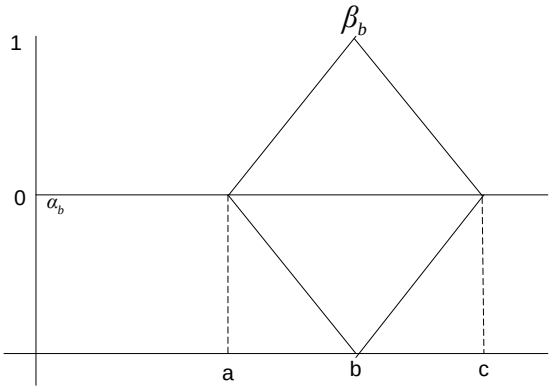

Figure 1: Graphical representation of Diamond Fuzzy Number (DFN)

Example: A diamond fuzzy number, $\mu_{\tilde{A}_D}$ *= (-4, 1, 4) and its membership function will be,*

$$\mu_{\tilde{A}_D}(x)=\begin{cases}\dfrac{x+4}{5} & -4\le x\le 1\\ \dfrac{4-x}{3} & 1\le x\le 4\\ \dfrac{-4-x}{-4-1} & -4\le x\le 1\\ \dfrac{x-4}{-3} & 1\le x\le 4\\ \beta_b=1 & \end{cases}$$

f α *- Cut of diamond fuzzy number*

$$\frac{x+4}{5}=\alpha \;\Rightarrow x=5\alpha-4$$

$$\frac{4-x}{3}=\alpha \;\Rightarrow x=-3\alpha+4$$

$$\tilde{A}_{D_\alpha}=[a^\alpha, c^\alpha]=[5\alpha-4, -3\alpha+4]$$

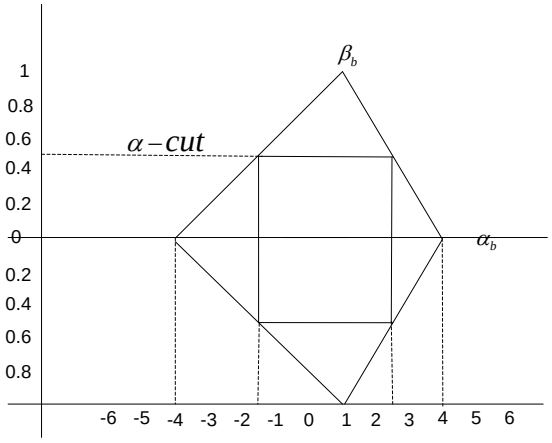

Figure 2: Graphical representation of $\alpha-cut$ of a Diamond Fuzzy Number (DFN)

When (α =0.5), we get $A_{0.5}$ *= [-1.5, 2.5],*

Also when (α =0), $\underset{\sim}{A}_{D_0}$ *=* $[a^0, c^0]$ *= [-4, 4].*

3.1. Conditions on Diamond Fuuzy Number

A diamond Fuzzy Number $\underset{\sim}{A}_D$ should satisfy the following conditions;

(i) $\mu_{\underset{\sim}{A}_D}(x)$ is a continuous function in the interval [0,1]

(ii) $\mu_{\underset{\sim}{A}_D}(x)$ is strictly increasing and continuous function on [a, b]

(iii) $\mu_{\underset{\sim}{A}_D}(x)$ is strictly decreasing and continuous function on [b,c]

3.2. Arithmetic Operations on Diamond Fuzzy Number (DFN)

3.2.1. Addition of two Diamond Fuzzy Numbers

If $\underset{\sim}{A}_D, = (a_1, b_1, c_1)$ *and* $\underset{\sim}{B}_D = (a_2, b_2, c_2);$ are two diamond fuzzy numbers then,

$\underset{\sim}{A}_D + \underset{\sim}{B}_D = (a_1 + a_2, b_1 + b_2, c_1 + c_2)$.

For example;

If, $\underset{\sim}{A}_D = (-3, 2, 3)$; $\underset{\sim}{B}_D = (-4, 1, 2)$; *Then,* $\underset{\sim}{A}_D +$ $\underset{\sim}{B}_D$ *= (- 7,3,5)*.

3.2.2. Subraction of two Diamond Fuzzy Numbers

If $\underset{\sim}{A}_D, = (a_1, b_1, c_1)$ *and* $\underset{\sim}{B}_D = (a_2, b_2, c_2);$ are two diamond fuzzy numbers then,

$\underset{\sim}{A}_D - \underset{\sim}{B}_D = (a_1 - a_2, b_1 - b_2, c_1 - c_2)$.

For example;

If , $\underset{\sim}{A}_D$ *= (- 5,2,4);* $\underset{\sim}{B}_D$ *= (- 1,0,1)* then, $\underset{\sim}{A}_D - \underset{\sim}{B}_D = (-4, 2, 3)$

3.2.3. Non Diamond Fuzzy Number

If $\underset{\sim}{A}_D = (-3, 2, 3); \underset{\sim}{B}_D = (-4, 1, 2)$ then $\underset{\sim}{A}_D - \underset{\sim}{B}_D = (1, 1, 1)$. This is not a Diamond fuzzy number,

because it should be in increasing order. (i.e, a<b<c).

3.2.4. Scalar Multiplication of Diamond Fuzzy Numbers

If $A_D = (a_1, b_1, c_1)$ then $K(A_D)$ is defined by $K(A_D) = (Ka_1, Kb_1, Kc_1)$.

Example, If $A_D = (-2, 1, 3)$,

then $2(A_D) = (-4, 2, 6)$.

3.2.5. Positive Diamond Fuzzy Number

A positive diamond Fuzzy Number (p-DFN) is defined as $\underset{\sim}{A}_D$ = (a,b,c,d), when a,b,c,d > 0.

Example; $\underset{\sim}{A}_D$ = (1,3,7)

3.2.6. Negative Diamond Fuzzy Number

A negative Diamond Fuzzy Number (n-DFN) is defined as $\underset{\sim}{A}_D$ = (a,b,c,), when a,b,c, < 0.

Example; $\underset{\sim}{A}_D$ = (-6,-3,-1)

3.3. Diamond Fuzzy Matrix (DFM)

A diamond Fuzzy Matrix (DFM) of order m×n is defined as, $A_D = (\underset{\sim}{a}_{ij})_{m\times n}$, when ijth element ($\underset{\sim}{a}_{ij}$) of is the Diamond Fuzzy Number.

3.3.1. Arithmetic Operations on Diamond Fuzzy Matrix

Let A and B be the two Diamond Fuzzy Matrix of order m×n. then we have,

(i) $\underset{\sim}{A}_D + \underset{\sim}{B}_D = (\underset{\sim}{a}_{ij} + \underset{\sim}{b}_{ij})$

(ii) $\underset{\sim}{A}_D - \underset{\sim}{B}_D = (\underset{\sim}{a}_{ij} - \underset{\sim}{b}_{ij})$

(iii) $\underset{\sim}{A}_D{}^T = (\underset{\sim}{a}_{ij})$

3.3.2. Types of Diamond Fuzzy Matrices

(i) Diamond Fuzzy Square Matrix:

A diamond Fuzzy Square Matrix is a matrix whose total number of rows and columns are equal. (i.e., m = n).

Example,

$$\underset{\sim}{A}_D = \begin{bmatrix} -4 & 1 & 4 \\ 2 & 4 & 9 \\ -2 & 0 & 1 \end{bmatrix}$$

(ii) Diamond Fuzzy Symmetric Matrix:

A diamond Fuzzy Symmetric Matrix is a matrix which is symmetric about the principal diagonal. (i.e., A^T = A).

(iii) Diamond Fuzzy Diagonal Matrix

A diamond Fuzzy diagonal Matrix is a matrix in which all the elements other than main diagonal elements are zero.

3.4. Nested Diamond Fuzzy Number

A Nested Diamond Fuzzy Number (N-DFN) is defined as $N\text{-}\ A_D = (A_{D_1}, A_{D_2}, A_{D_3})$, where $A_{D_1}, A_{D_2}, A_{D_3}$ is a set of Diamond numbers of increasing size placed adjacently with varying centre b..

Example: $N\text{-}\ A_D = (a_{1_1}, b_{1_1}, c_{1_1}, a_{1_2}, b_{1_2}, c_{1_2}, a_{1_3}, b_{1_3}, c_{1_3})$ = (-1,0,1, -2,0,3, -3,0,5).

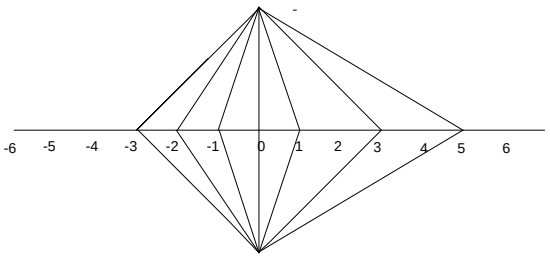

Figure 3: Graphical representation of Nested Diamond Fuzzy Number

3.4.1. Addition of two Nested Diamond Fuzzy Numbers

If $N\text{-}\ A_D = (A_{D_1}, A_{D_2}, A_{D_3})$ and $N\text{-}\ B_D = (B_{D_1}, B_{D_2}, B_{D_3})$ are two nested diamond fuzzy numbers then

$N\text{-}\ A_D + N\text{-}\ B_D = (a_{1_1} + a_{2_1}, b_{1_1} + b_{2_1}, c_{1_1} + c_{2_1}\ ,\ a_{1_2} + a_{2_2}, b_{1_2} + b_{2_2}, c_{1_2} + c_{2_2}\ ,$

$a_{1_3} + a_{2_3}, b_{1_3} + b_{2_3}, c_{1_3} + c_{2_3})$.

For example: if $N\text{-}\ A_D = (A_{D_1}, A_{D_2}, A_{D_3})$, and $N\text{-}\ B_D = (B_{D_1}, B_{D_2}, B_{D_3})$ where A_{D_1} =(-1,0,1), A_{D_2} =(-2,0,2), A_{D_3} =(-4,0,4) and B_{D_1} =(-1,1,2), B_{D_2} =(-2,1,2), B_{D_3} =(-3,1,4) then $N\text{-}\ A_D + N\text{-}\ B_D$ =(-2,1,3, -4,1,4, -7,1,8)

3.4.2. Subtraction of two Nested Diamond Fuzzy Numbers

If $N\text{-}\ A_D = (A_{D_1}, A_{D_2}, A_{D_3})$ and $N\text{-}\ B_D = (B_{D_1}, B_{D_2}, B_{D_3})$ are two nested diamond fuzzy numbers then

$(N\text{-}\ A_D)\text{-}\ (N\text{-}\ B_D) = (a_{1_1} - a_{2_1}, b_{1_1} - b_{2_1}, c_{1_1} - c_{2_1}\ , a_{1_2} - a_{2_2}, b_{1_2} - b_{2_2}, c_{1_2} - c_{2_2}\ ,$

$a_{1_3} - a_{2_3}, b_{1_3} - b_{2_3}, c_{1_3} - c_{2_3})$.

For example: if $N\text{-}\ A_D = (A_{D_1}, A_{D_2}, A_{D_3})$, and $N\text{-}\ B_D = (B_{D_1}, B_{D_2}, B_{D_3})$ where A_{D_1} =(-2,0,2), A_{D_2} = (-4,0,3), A_{D_3} =(-5,0,5) and B_{D_1} =(-1,0,1), B_{D_2} =(-2,0,2), B_{D_3} =(-2,0,4) then $(N\text{-}\ A_D)\text{-}\ (N\text{-}\ B_D)$ = (-1,0,1 -2,0,1, -3,0,1).

3.5. Right Linked Diamond Fuzzy Numbers

A Linked Diamond Fuzzy Number (RL-DFN) is defined as $N\text{-}\ A_D = (A_{D_1}, A_{D_2}, A_{D_3})$, where $A_{D_1}, A_{D_2}, A_{D_3}$ is a set of Diamond numbers, if the increase happens on the right,.

Example: $N\text{-}\ A_D$=($a_{1_1}, b_{1_1}, c_{1_1}, a_{1_2}, b_{1_2}, c_{1_2}, a_{1_3}, b_{1_3}, c_{1_3}$)=(-2,0,2, 0,2,3, 2,4,9).

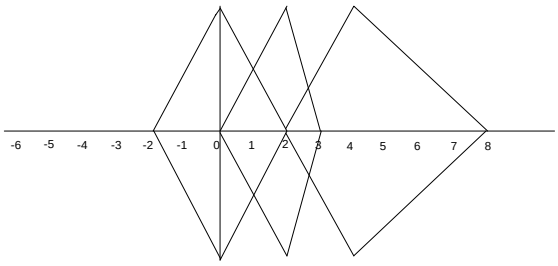

Figure 4: Graphical representation of Right Linked Diamond Fuzzy Number

3.6. Left Linked Diamond Fuzzy Numbers

A Left Linked Diamond Fuzzy Number (LL-DFN) is defined as $N\text{-}\ A_D = (A_{D_1}, A_{D_2}, A_{D_3})$, where $A_{D_1}, A_{D_2}, A_{D_3}$ is a set of Diamond numbers, if the increase happens on the left.

Example: $N\text{-}\ A_D$=($a_{1_1}, b_{1_1}, c_{1_1}, a_{1_2}, b_{1_2}, c_{1_2}, a_{1_3}, b_{1_3}, c_{1_3}$)=(-2,0,1, -5,-2,1, -8,-3,-2).

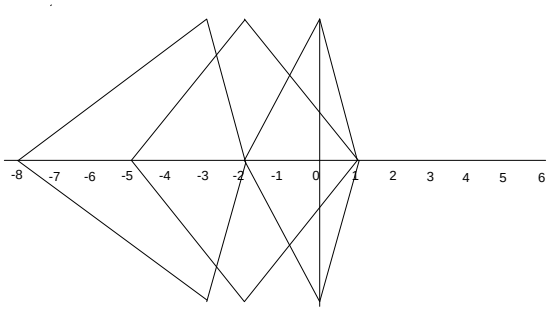

Figure 5: Graphical representation of Left Linked Diamond Fuzzy Number

4 Conclusion

In this article, the diamond Fuzzy Number has been introduced with arithmetic operations. Using a few examples,we have explained the relevant arithmetic operations. We observe that diamond fuzzy number concepts could be applied to any problem with three types of categorization or three variables which are interconnected.

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