A Approach for Ranking of Exponential Fuzzy Number With TRD distance

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Abstract
In this paper, we want represented a method for ranking of two exponential trapezoidal fuzzy numbers with TRD distance. The main advantage of the proposed approach is that the proposed approach provide the correct ordering of exponential trapezoidal fuzzy numbers and also the proposed approach is very simple and easy to apply in the real life problems.

Keywords: Exponential Trapezoidal Fuzzy Numbers, Ranking Method, TRD distance.

1 Introduction
In most of cases in our life, the data obtained for decision making are only approximately known. In 1965, Zadeh [19] introduced the concept of fuzzy set theory to meet those problems. In 1987, Dubois and Prade defined any of the fuzzy numbers as a fuzzy subset of the real line [6]. Fuzzy numbers allow us to make the mathematical model of linguistic variable or fuzzy environment. Bortolan and Degani [2] reviewed some of these ranking methods for ranking fuzzy subsets. Chen [3] presented ranking fuzzy numbers with maximizing set and minimizing set. Abbasbandy and Hajjari [1] introduced a new approach for ranking of trapezoidal fuzzy numbers based on the left and right spreads at some α-levels of trapezoidal fuzzy numbers and S. Rezvani ([9]-[16]) evaluated the system of ranking fuzzy numbers. Some of the interesting Approach Ranking Of Trapezoidal Fuzzy Number can be found in Amit Kumar [7] and Nasibove [8] proposed Fuzzy least squares regression model based of weighted distance between fuzzy numbers.
In this paper, we want represented a method for ranking of two exponential trapezoidal fuzzy numbers with TRD distance. The main advantage of the proposed approach is that the proposed approach provide the correct ordering of exponential trapezoidal fuzzy numbers and also the proposed approach is very simple and easy to apply in the real life problems.

2 Preliminaries
Generally, a generalized fuzzy number $A$ is described as any fuzzy subset of the real line $R$, whose membership function $\mu_A$ satisfies the following conditions,
(i) \( \mu_A \) is a continuous mapping from \( \mathbb{R} \) to the closed interval \([0,1] \),

(ii) \( \mu_A(x) = 0, -\infty < x \leq a \),

(iii) \( \mu_A(x) = L(x) \) is strictly increasing on \([a,b]\),

(iv) \( \mu_A(x) = w, b \leq x \leq c \),

(v) \( \mu_A(x) = R(x) \) is strictly decreasing on \([c,d]\),

(vi) \( \mu_A(x) = 0, d \leq x < \infty \)

Where \( 0 < w \leq 1 \) and \( a,b,c \), and \( d \) are real numbers. We call this type of generalized fuzzy number a trapezoidal fuzzy number, and it is denoted by \( A = (a,b,c,d)_E \).

When \( w = 1 \), this type of generalized fuzzy number is called normal fuzzy number and is represented by \( A = (a,b,c,d)_{LR} \).

However, these fuzzy numbers always have a fix range as \([c,d]\) . Here, we define its general from as follows:

\[
f_A(x) = \begin{cases}
we^{-[(b-x)/(b-a)]} & a \leq x \leq b, \\
w & b \leq x \leq c, \\
we^{-[(x-c)/(d-c)]} & c \leq x \leq d,
\end{cases}
\]

where \( 0 < w \leq 1 \), \( a,b \) are real numbers, and \( c,d \) are positive real numbers. We denote this type of generalized exponential fuzzy number as \( A = (a,b,c,d; w)_E \). Especially, when \( w = 1 \), we denote it as \( A = (a,b,c,d)_E \).

We define the representation of generalized exponential fuzzy number based on the integral value of graded mean h-level as follow. Let the generalized exponential fuzzy number \( A = (a,b,c,d)_E \), where \( 0 < w \leq 1 \), and \( c,d \) are positive real numbers, \( a,b \) are real numbers as in formula (2.1). Now, let two monotonic functions be

\[
L(x) = we^{-[(b-x)/(b-a)]}, R(x) = we^{-[(x-c)/(d-c)]}
\]

**Definition 2.1.** The following values constitute the weighted averaged representative and weighted width, respectively, of the fuzzy number \( A \):

\[
I(A) = \int_0^1 (CL_A(\alpha) + (1-C)R_A(\alpha)) \, d\alpha \tag{2.3}
\]

and

\[
D(A) = \int_0^1 (R_A(\alpha) - L_A(\alpha)) \, f(\alpha) \, d\alpha \tag{2.4}
\]

Here \( 0 \leq c \leq 1 \) denotes an “optimism/pessimism” coefficient in conducting operations on fuzzy numbers. The function \( f(\alpha) \) nonnegative and increasing function on \([0,1]\) with \( f(0) = 0 \), \( f(1) = 1 \) and \( \int_0^1 f(\alpha) \, d\alpha \) The function is also called weighting function. In actual applications, function can be chosen according to the actual situation. In practical cases, it may be assume that

\[
f(\alpha) = \frac{k+1}{2} \alpha^k, \quad k = 1,2,... \tag{2.5}
\]

**Definition 2.2.** For arbitrary fuzzy numbers and the quantity

\[
TRD(A,B) = \sqrt{[I(A) - I(B)]^2 + [D(A) - D(B)]^2} \tag{2.6}
\]

is called the TRD distance between the fuzzy numbers \( A \) and \( B \).
3 Ranking Function

An efficient approach for comparing the fuzzy numbers is by the use of a ranking function, $\mathcal{R} : F(R) \to R$, where $F(R)$ is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists,

\begin{align*}
(1) \; A > B & \iff \mathcal{R}(A) > \mathcal{R}(B). \\
(2) \; A < B & \iff \mathcal{R}(A) < \mathcal{R}(B). \\
(3) \; A = B & \iff \mathcal{R}(A) = \mathcal{R}(B).
\end{align*}

Remark 3.1. [10]. For all fuzzy numbers $A$, $B$, $C$ and $D$, we have

\begin{align*}
(1) \; A > B & \Rightarrow A \oplus C > B \oplus C. \\
(2) \; A > B & \Rightarrow A \ominus C > B \ominus C. \\
(3) \; A \sim B & \Rightarrow A \oplus C \sim B \oplus C. \\
(4) \; A > B, C > D & \Rightarrow A \oplus C > B \oplus D.
\end{align*}

4 Shortcomings of Chen and Chen [4] Approach

In this section, the shortcomings of Chen and Chen approach [4], on the basis of reasonable properties of fuzzy quantities and on the basis of height of fuzzy numbers, are pointed out. Let $A$ and $B$ be any two fuzzy numbers. Then

$A > B \Rightarrow A \ominus B > B \ominus B$ (Using Remark 2.),

That is

$\mathcal{R}(A) > \mathcal{R}(B) \Rightarrow \mathcal{R}(A \ominus B) > \mathcal{R}(B \ominus B).$

In this subsection, several examples are choosen to prove that the ranking function proposed by Chen and Chen does not satisfy the reasonable property,

\[ A > B \Rightarrow A \ominus B > B \ominus B \quad \text{(4.7)} \]

for the ordering of fuzzy quantities i.e., according to Chen Chen approach

\[ A > B, \; \text{not result} \; A \ominus B > B \ominus B \]

Example 4.1. Let $A = (0.1,0.3,0.3,0.5;1)$ and $B = (0.2,0.3,0.3,0.4;1)$ be two generalized trapezoidal fuzzy numbers. Then according to Chen and Chen approach $B > A$, but

$B \ominus A < A \ominus A$ that is , $B > A$ not result $B \ominus A > A \ominus A$.

Example 4.2. Let $A = (0.1,0.3,0.3,0.5;0.8)$ and $B = (0.1,0.3,0.3,0.5;1)$ be two generalized trapezoidal fuzzy numbers. Then according to Chen and Chen approach $B > A$, but

$B \ominus A < A \ominus A$ that is , $B > A$ not result $B \ominus A > A \ominus A$. 
Example 4.3. Let $A = (-0.8, -0.6, -0.4, -0.2, 0.35)$ and $B = (-0.4, -0.3, -0.2, -0.1, 0.7)$ be two generalized trapezoidal fuzzy numbers. Then according to Chen and Chen approach $A > B$, but

$$A \odot B < B \odot B$$

that is, $A > B$ not result $A \odot B > B \odot B$.

Example 4.4. Let $A = (0.2, 0.4, 0.6, 0.8, 0.35)$ and $B = (0.1, 0.2, 0.3, 0.4, 0.7)$ be two generalized trapezoidal fuzzy numbers. Then according to Chen and Chen approach $B > A$, but

$$B \odot A < A \odot A$$

that is, $B > A$ not result $B \odot A > A \odot A$.

5 On the Basis of Height of Fuzzy Numbers

In this section, it is proved that, in some cases, Chen and Chen approach [4] states that the ranking of fuzzy numbers depends upon height of fuzzy numbers while in several cases the ranking does not depend upon the height of fuzzy numbers.

Let $A_1 = (a_1, b_1, \gamma_1, \beta_1; w_1)$ and $A_2 = (a_2, b_2, \gamma_2, \beta_2; w_2)$ be two generalized trapezoidal fuzzy numbers. Then according to Chen and Chen [4] there may be two cases.

Case(1) If $(a_1 + b_1 + \gamma_1 + \beta_1) \neq 0$, then

$$\begin{align*}
A < B & \text{ if } w_1 < w_2 \\
A > B & \text{ if } w_1 > w_2 \\
A \sim B & \text{ if } w_1 \sim w_2.
\end{align*}$$

(5.8)

Case(2) If $(a_1 + b_1 + \gamma_1 + \beta_1) = 0$, then $A \sim B$ for all values of $w_1$ and $w_2$.

According to Chen and Chen [4] in first case ranking of fuzzy numbers depends upon height and in second case ranking does not depend upon the height which is contradiction.

Example 5.1. Let $A = (1, 1, 1, 1; w_1)$ and $B = (1, 1, 1, 1; w_2)$ be two generalized trapezoidal fuzzy numbers. Then according to Chen and Chen approach $A < B$ if $w_1 < w_2$, $A > B$ if $w_1 > w_2$ and $A = B$ if $w_1 = w_2$.

Example 5.2. Let $A = (-0.4, -0.2, -0.1, 0.7; w_1)$ and $B = (-0.4, -0.2, -0.1, 0.7; w_2)$ be two generalized triangular fuzzy numbers. Then $A = B$ for all values of $w_1$ and $w_2$.

6 Proposed Approach

In this section some important results, that are useful for the proposed approach, are proved.

Theorem 6.1. weighted averaged representative of the fuzzy number $A$

$$I(A) = wC(b - a)e^{\frac{1-2b}{\alpha}} + w(1 - C)(c - d)e^{\frac{1}{\alpha}}$$

(6.9)

Proof.

$$I(A) = \int_0^1 (CL_A(\alpha) + (1 - C)R_A(\alpha)) \, d\alpha$$

$$= \int_0^1 [Cw e^{-((b - a)/(b - a))} + (1 - C)we^{-((\alpha - c)/(d - c))}] \, d\alpha$$

$$= wC(b - a)e^{\frac{1-2b}{\alpha}} + w(1 - C)(c - d)e^{\frac{1}{\alpha}}$$
Theorem 6.2. weighted width representative of the fuzzy number $A$

$$D(A) = w(c - d)e^{\frac{1}{c-a}} - w(b - a)e^{\frac{1}{b-d}}$$ (6.10)

Proof.

$$D(A) = \int_{0}^{1} (R_{A}(\alpha) - L_{A}(\alpha)) f(\alpha) \, d\alpha$$

$$= \int_{0}^{1} \left[ w e^{-[(x-c)/(d-c)]} - we^{-[(b-x)/(b-a)]} \right] d\alpha = w(c - d)e^{\frac{1}{c-a}} - w(b - a)e^{\frac{1}{b-d}}$$

Theorem 6.3. TRD distance between the fuzzy numbers $A$ and $B$

$$TRD(A, 0) = \sqrt{|I(A) - 0|^2 + |D(A) - 0|^2}$$ (6.11)

Theorem 6.4. Let $A = (a_{A}, b_{A}, c_{A}, d_{A}; w_{A})_E$ and $B = (a_{B}, b_{B}, c_{B}, d_{B}; w_{B})_E$ be two generalized exponential fuzzy numbers then

* If $TRD(A, 0) < TRD(B, 0)$ then $A < B$

* If $TRD(A, 0) > TRD(B, 0)$ then $A > B$

* If $TRD(B, 0) \sim TRD(A, 0)$ then $A \sim B$

6.1 Method to Find the $TRD(A)$ and $TRD(B)$

Let $A = (a_{A}, b_{A}, c_{A}, d_{A}; w_{A})_E$ and $B = (a_{B}, b_{B}, c_{B}, d_{B}; w_{B})_E$ be two generalized exponential fuzzy numbers then use the following steps to find the values of $TRD(A)$ and $TRD(B)$

* Step 1. Find $I(A)$ and $I(B)$

* Step 2. Find $D(A)$ and $D(B)$

* Step 3. Find $TRD(A)$ and $TRD(B)$

* Step 4. Use to the Theorem 6.4.

7 Examples and Results

In this section, the correct ordering of fuzzy numbers, are obtained. Also, in the Table 1, it is shown that proposed ranking function satisfies the all reasonable properties of fuzzy quantities proposed by Wang and Kerre [18].

Example 7.1. Let $A = (0.1, 0.3, 0.3, 0.5; 1)$ and $B = (0.2, 0.3, 0.3, 0.4; 1)$ be two generalized trapezoidal fuzzy numbers. Since

* Step 1.

$$I(A) = C(0.3 - 0.1)e^{1.0/0.8} + (1 - C)(0.3 - 0.5)e^{1.0/0.8} = 31.26C - 29.78$$
and 
\[ I(B) = C(0.3 - 0.2)e^{1.06} + (1 - C)(0.3 - 0.4)e^{1.06} = 2222.08C - 2216.61 \]

* Step 2.
\[ D(A) = (0.3 - 0.5)e^{1.06} - (0.3 - 0.1)e^{1.06} = -31.26 \]

and 
\[ D(B) = (0.3 - 0.4)e^{1.06} - (0.3 - 0.2)e^{1.06} = -2222.08 \]

* Step 3.
\[ TRD(A,0) = \sqrt{[31.26C - 29.78]^2 + [-31.26]^2} \]

and 
\[ TRD(B,0) = \sqrt{[2222.08C - 2216.61]^2 + [-2222.08]^2} \]

* Step 4. With \( C = 1 \), \( TRD(A,0) = 31.3 \) and \( TRD(B,0) = 2222.1 \), then \( A < B \).

With \( C = 0 \), \( TRD(A,0) = 43.2 \) and \( TRD(B,0) = 3138.7 \), then \( A < B \).

With \( C = \frac{1}{2} \), \( TRD(A,0) = 31.7 \) and \( TRD(B,0) = 2222.58 \), then \( A < B \).

**Example 7.2.** Let \( A = (0.1,0.3,0.3,0.5;0.8) \) and \( B = (0.1,0.3,0.3,0.5;1) \) be two generalized trapezoidal fuzzy numbers. Since

* Step 1.
\[ I(A) = C(0.8)(0.3 - 0.1)e^{1.06} + (1 - C)(0.8)(0.3 - 0.5)e^{1.06} = 25.01C - 23.82 \]

and 
\[ I(B) = C(0.3 - 0.1)e^{1.06} + (1 - C)(0.3 - 0.5)e^{1.06} = 31.26C - 29.78 \]

* Step 2.
\[ D(A) = 0.8(0.3 - 0.5)e^{1.06} - 0.8(0.3 - 0.1)e^{1.06} = -25.01 \]

and 
\[ D(B) = (0.3 - 0.5)e^{1.06} - (0.3 - 0.1)e^{1.06} = -31.26 \]

* Step 3.
\[ TRD(A,0) = \sqrt{[25.01C - 23.82]^2 + [-25.01]^2} \]

and 
\[ TRD(B,0) = \sqrt{[31.26C - 29.78]^2 + [-31.26]^2} \]

* Step 4. With \( C = 1 \), \( TRD(A,0) = 25.04 \) and \( TRD(B,0) = 31.3 \), then \( A < B \).

With \( C = 0 \), \( TRD(A,0) = 34.54 \) and \( TRD(B,0) = 43.2 \), then \( A < B \).

With \( C = \frac{1}{2} \), \( TRD(A,0) = 27.4 \) and \( TRD(B,0) = 31.7 \), then \( A < B \).
Example 7.3. Let \( A = (-0.8, -0.6, -0.4, -0.2; 0.35) \) and \( B = (-0.4, -0.3, -0.2, -0.1; 0.7) \) be two generalized trapezoidal fuzzy numbers. Since

\[ I(A) = C(0.35)(-0.6 + 0.8)e^{-\frac{1+1.2}{(0.6+0.8)}} + (1-C)(0.35)(-0.4 + 0.2)e^{-\frac{1}{(0.4+0.2)}} = 4230.8C - 10.42 \]
and
\[ I(B) = C(0.7)(-0.3 + 0.4)e^{-\frac{1+0.6}{(0.3+0.4)}} + (1-C)(0.7)(-0.2 + 0.1)e^{-\frac{1}{(0.2+0.1)}} = 629900C - 1551.6 \]

\* Step 2.

\[ D(A) = (0.35)(-0.4 + 0.2)e^{-\frac{1}{(0.4+0.2)}} - (0.35)(-0.6 + 0.8)e^{-\frac{1+1.2}{(0.6+0.8)}} = -4230.8 \]
and
\[ D(B) = (0.7)(-0.2 + 0.1)e^{-\frac{1}{(0.2+0.1)}} - (0.7)(-0.3 + 0.4)e^{-\frac{1+0.6}{(0.3+0.4)}} = -629900 \]

\* Step 3.

\[ TRD(A, 0) = \sqrt{[4230.8C - 10.42]^2 + [-4230.8]^2} \]
and
\[ TRD(B, 0) = \sqrt{[629900C - 1551.6]^2 + [-629900]^2} \]

\* Step 4. With \( C = 1 \), \( TRD(A, 0) = 4231.8 \) and \( TRD(B, 0) = 889716.65 \), then \( A \prec B \).

With \( C = 0 \), \( TRD(A, 0) = 4230.8 \) and \( TRD(B, 0) = 629901.91 \), then \( A \prec B \).

With \( C = \frac{1}{7} \), \( TRD(A, 0) = 4231.05 \) and \( TRD(B, 0) = 703557.08 \), then \( A \prec B \).

Example 7.4. Let \( A = (0.2, 0.4, 0.6, 0.8; 0.35) \) and \( B = (0.1, 0.2, 0.3, 0.4; 0.7) \) be two generalized trapezoidal fuzzy numbers. Since

\[ I(A) = C(0.35)(0.4 - 0.2)e^{\frac{1-0.4}{0.4-0.2}} + (1-C)(0.35)(0.6 - 0.8)e^{\frac{1}{0.6-0.8}} = 0.3C - 0.11 \]
and
\[ I(B) = C(0.7)(0.2 - 0.1)e^{\frac{1-0.4}{0.2-0.1}} + (1-C)(0.7)(0.3 - 0.4)e^{\frac{1}{0.3-0.4}} = 1579.9C - 1551.6 \]

\* Step 2.

\[ D(A) = (0.35)(0.6 - 0.8)e^{\frac{1-0.4}{0.6-0.8}} - (0.35)(0.4 - 0.2)e^{\frac{1}{0.4-0.2}} = -0.3 \]
and
\[ D(B) = (0.7)(0.3 - 0.4)e^{\frac{1}{0.3-0.4}} - (0.7)(0.2 - 0.1)e^{\frac{1}{0.2-0.1}} = -1579.9 \]

\* Step 3.

\[ TRD(A, 0) = \sqrt{[0.3C - 0.11]^2 + [-0.3]^2} \]
and

$$TRD(B,0) = \sqrt{[1579.9C - 1551.6]^2 + [1579.9]^2}$$

* Step 1. \[ I(A) = C(w_A)(1 - 1)e^{\frac{1-\delta}{\mu_A}} + (1 - C)(w_A)(1 - 1)e^{\frac{1-\delta}{\mu_A}} = 0 \]

and

$$I(B) = C(w_B)(1 - 1)e^{\frac{1-\delta}{\mu_B}} + (1 - C)(w_B)(1 - 1)e^{\frac{1-\delta}{\mu_B}} = 0$$

* Step 2.

$$D(A) = (w_A)(1 - 1)e^{\frac{1-\delta}{\mu_A}} - (w_A)(1 - 1)e^{\frac{1-\delta}{\mu_A}} = 0$$

and

$$D(B) = (w_B)(1 - 1)e^{\frac{1-\delta}{\mu_B}} - (w_B)(1 - 1)e^{\frac{1-\delta}{\mu_B}} = 0$$

* Step 3.

$$TRD(A,0) = 0$$

and

$$TRD(B,0) = 0$$

* Step 4. \( A \sim B \).

**Example 7.5.** Let \( A = (1,1,1,1;w_1) \) and \( B = (1,1,1,1;w_2) \) be two generalized fuzzy numbers. Since

\[ TRD(A,0) = 0 \]

and

\[ TRD(B,0) = 0 \]

**Example 7.6.** Let \( A = (-0.4,-0.2,-0.1,0.7;w_1) \) and \( B = (-0.4,-0.2,-0.1,0.7;w_2) \) be two generalized trapezoidal fuzzy numbers. Since

* Step 1.

$$I(A) = C(w_A)(-0.2 + 0.4)e^{\frac{1-0.4}{\mu_A}} + (1 - C)(w_A)(-0.1 - 0.7)e^{\frac{1}{\mu_A}} = w_A(223.09C - 2.79)$$

and

$$I(B) = C(w_B)(-0.2 + 0.4)e^{\frac{1-0.4}{\mu_B}} + (1 - C)(w_B)(-0.1 - 0.7)e^{\frac{1}{\mu_B}} = w_B(223.09C - 2.79)$$

* Step 2.

$$D(A) = (w_A)(-0.1 - 0.7)e^{\frac{1}{\mu_A}} - (w_A)(-0.2 + 0.4)e^{\frac{1-0.4}{\mu_A}} = -223.09w_A$$

and

$$D(B) = (w_B)(-0.1 - 0.7)e^{\frac{1}{\mu_B}} - (w_B)(-0.2 + 0.4)e^{\frac{1-0.4}{\mu_B}} = -223.09w_B$$
* Step 3.

\[ TRD(A,0) = \sqrt{w_A(223.09C - 2.79)^2 + [-223.09w_A]^2} \]

and

\[ TRD(B,0) = \sqrt{w_B(223.09C - 2.79)^2 + [-223.09w_B]^2} \]

* Step 4. With \( C = 1 \), \( TRD(A,0) = 313.5w_A \) and \( TRD(B,0) = 313.5w_B \).

With \( C = 0 \), \( TRD(A,0) = 223.11w_A \) and \( TRD(B,0) = 223.11w_B \).

With \( C = \frac{1}{2} \), \( TRD(A,0) = 248.19w_A \) and \( TRD(B,0) = 248.19w_B \).

7.1 Testimony of the Results

In the above examples it can be easily checked that, for a particular value of \( \alpha \)

(I) \( A \sim B \Rightarrow A \ominus B \sim B \ominus B \), That is

\[ RM((A \ominus B) \ominus (B \ominus B)) \sim RM((A \ominus B) \ominus (B \ominus B)) \],

(II) \( A > B \Rightarrow A \ominus B > B \ominus B \), That is

\[ RM((A \ominus B) \ominus (B \ominus B)) > RM((A \ominus B) \ominus (B \ominus B)) \],

(III) \( A < B \Rightarrow A \ominus B < B \ominus B \), That is

\[ RM((A \ominus B) \ominus (B \ominus B)) < RM((A \ominus B) \ominus (B \ominus B)) \].

7.2 Testimony of Proposed Ranking Function

Table 1, it is shown that proposed ranking function satisfies the all reasonable properties of fuzzy quantities proposed by Wang and Kerre [18]
Table (1): Fulfilment of the axioms for the ordering in the first and second class [18]

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8 Conclusion

The main advantage of this paper is to propose a new approach for the ranking of generalized exponential trapezoidal fuzzy numbers with TRD distance. In this method, we used of weighted averaged representative and weighted width representative in TRD distance. Also the proposed approach is very simple and easy to apply in the real life problems.

References

http://dx.doi.org/10.1016/j.camwa.2008.10.090

http://dx.doi.org/10.1016/0165-0114(85)90012-0

http://dx.doi.org/10.1016/0165-0114(85)90050-8

http://dx.doi.org/10.1016/j.eswa.2008.08.015

http://dx.doi.org/10.1016/S0898-1221(01)00277-2

http://dx.doi.org/10.1016/0165-0114(87)90028-5
http://dx.doi.org/10.1007/s12543-010-0036-7

http://dx.doi.org/10.3103/S0146411607010026


http://dx.doi.org/10.5121/ijfls.2013.3102


http://dx.doi.org/10.1016/S0165-0114(99)00062-7

http://dx.doi.org/10.1016/S0019-9958(65)90241-X