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Research Article

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Ranking Exponential Trapezoidal Fuzzy Numbers by Median Value

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Abstract

In this paper, we want represented a method for ranking of two exponential trapezoidal fuzzy numbers. A median value is proposed for the ranking of exponential trapezoidal fuzzy numbers. For the validation the results of the proposed approach are compared with different existing approaches.

Keywords: Exponential Trapezoidal Fuzzy Numbers, Median Value, Ranking Method.

1 Introduction

In most of cases in our life, the data obtained for decision making are only approximately known. In1965, Zadeh [23] introduced the concept of fuzzy set theory to meet those problems. In 1987, Dubois and Prade defined any of the fuzzy numbers as a fuzzy subset of the real line [9]. Fuzzy numbers allow us to make the mathematical model of linguistic variable or fuzzy environment. Ranking fuzzy numbers were first proposed by Jain [10] for decision making in fuzzy situations by representing the ill-defined quantity as a fuzzy set. Bortolan and Degani [2] reviewed some of these ranking methods [9],[10],[13] for ranking fuzzy subsets. Chen [4] presented ranking fuzzy numbers with maximizing set and minimizing set. [11] and Wang and Lee [22] also used the centroid concept in developing their ranking index. Chen and Chen [5] presented a method for ranking generalized trapezoidal fuzzy numbers. Abbasbandy and Hajjari [1] introduced a new approach for ranking of trapezoidal fuzzy numbers based on the left and right spreads at some -levels of trapezoidal fuzzy numbers. Bodjanova [3] proposed Median value and median interval of a fuzzy number. S. Rezvani [13]-[19] evaluated the system of ranking fuzzy numbers. Moreover, Rezvani [18] proposed a new method for ranking with Euclidean distance by the incentre of centroids.
In this paper, we want represented a method for ranking of two exponential trapezoidal fuzzy numbers. A median value is proposed for the ranking of exponential trapezoidal fuzzy numbers. For the validation the results of the proposed approach are compared with different existing approaches.

2 Preliminaries

Definition 2.1. *Generally, a generalized fuzzy number A is described as any fuzzy subset of the real line R, whose membership function* μ_A *satisfies the following conditions,*

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(i) μ_A *is a continuous mapping from R to the closed interval [0,1],*

(ii) $\mu_A(x) = 0, -\infty < u \leq a,$

(iii) $\mu_A(x) = L(x)$ *is strictly increasing on* $[a,b]$,

(iv) $\mu_A(x) = w, b \leq x \leq c,$

(v) $\mu_A(x) = R(x)$ *is strictly decreasing on* $[c,d]$,

(vi) $\mu_A(x) = 0, d \leq x < \infty.$

Where $0 < w \leq 1$ *and* $a,b,c,$ *and* d *are real numbers. We call this type of generalized fuzzy number a trapezoidal fuzzy number, and it is denoted bye* $A = (a,b,c,d;w)_{LR}$.

When $w = 1$*, this type of generalized fuzzy number is called normal fuzzy number and is represented by* $A = (a,b,c,d)_{LR}$.

However, these fuzzy numbers always have a fix range as $[c,d]$ *. Here, we define its general from as follows:*

$$f_A(x) = \begin{cases} we^{-[(b-x)/(b-a)]} & a \leq x \leq b, \\ w & b \leq x \leq c, \\ we^{-[(x-c)/(d-c)]} & c \leq x \leq d, \end{cases} \tag{2.1}$$

where $0 < w \leq 1$*,* a,b *are real numbers, and* c,d *are positive real numbers. we denote this type of generalized exponential fuzzy number as* $A = (a,b,c,d;\ w)_E$*. Especially, when* $w = 1$*, we denote it as* $A = (a,b,c,d)_E$.

we define the representation of generalized exponential fuzzy number based on the integral value of graded mean h-level as follow. Let the generalized exponential fuzzy number $A = (a,b,c,d)_E$, where $0 < w \leq 1$, and c,d are positive real numbers, a,b are real numbers as in formula (2.1). Now, let two monotonic functions be

$$L(x) = we^{-[(b-x)/(b-a)]},\ R(x) = we^{-[(x-c)/(d-c)]} \tag{2.2}$$

3 Proposed Approach

In this section some important results, that are useful for the proposed approach, are proved.

Definition 3.1. *Cardinality of a fuzzy number A is the value of the integral [3]*

$$card\ A = \int_a^b A(x)\ dx = \int_0^1 (b_\alpha - a_\alpha)\ d\alpha \tag{3.3}$$

Definition 3.2. *The median value of a fuzzy number A is the real number* m_A *from the support of A such that [3]*

$$\int_a^{m_A} A(x)\ dx = \int_{m_A}^d A(x)\ dx \tag{3.4}$$

Proposition 3.1. *If* $A = (a,b,c,d)$ *is a fuzzy number with light tails then [3]*

$$m_A = \frac{a+b}{2} + 0.5\ (\int_c^d A(x)\ dx - \int_a^b A(x)\ dx) \tag{3.5}$$

We have all the above definitions apply to exponential trapezoidal fuzzy numbers.

Theorem 3.1. *Cardinality of a exponential trapezoidal fuzzy number A characterized by (2.1) is the value of the integral*

$$card\ A = w(c-b) + \frac{w}{e}((b-a)(e-1) + (c-d)(1-e)) \tag{3.6}$$

Proof.

$$card\ A = \int_a^b A(x)\ dx = \int_a^b we^{-[(b-x)/(b-a)]}\ dx + \int_b^c w\ dx + \int_c^d we^{-[(x-c)/(d-c)]}\ dx$$

$$= w(b-a)(1-\frac{1}{e}) + w(c-b) + w(c-d)(\frac{1}{e}-1) = w(c-b) + \frac{w}{e}((b-a)(e-1)+(c-d)(1-e))$$

□

Now the article will study location of the median value m_A in the support of A. The article will also identify the fuzziness of m_A determined by its membership grade $A(m_A)$.

Theorem 3.2. *If A is a exponential trapezoidal fuzzy number with light tails then*

$$m_A = \frac{w(b+c)}{2} + \frac{w}{2e}[(c-d)(1-e)-(b-a)(e-1)] \tag{3.7}$$

Proof.

$$m_A = \frac{w(b+c)}{2} + \frac{1}{2}\ [\int_c^d A(x)\ dx - \int_a^b A(x)\ dx]$$

$$= \frac{w(b+c)}{2} + \frac{1}{2}\ [\int_c^d we^{-[(x-c)/(d-c)]}\ dx - \int_a^b we^{-[(b-x)/(b-a)]}\ dx]$$

$$= \frac{w(b+c)}{2} + \frac{1}{2}\ [w(c-d)(\frac{1}{e}-1) - w(b-a)(1-\frac{1}{e})] = \frac{w(b+c)}{2} + \frac{w}{2e}[(c-d)(1-e)-(b-a)(e-1)]$$

□

So we can Define the ranking of median value in exponential trapezoidal fuzzy number.

Theorem 3.3. *If $A = (a,b,c,d)_E$ is a exponential trapezoidal fuzzy number, and m_A the median value of them, So*

(i) If $m_A < m_B$ then $A < B$,

(ii) If $m_A > m_B$ then $A > B$,

(iii) If $m_A \sim m_B$ then $A \sim B$.

4 Results

Example 4.1. *Let $A = (0.2, 0.4, 0.6, 0.8; 0.35)$ and $B = (0.1, 0.2, 0.3, 0.4; 0.7)$ be two generalized trapezoidal fuzzy number, then*

$$m_A = \frac{w_A(b_A+c_A)}{2} + \frac{w_A}{2e}[(c_A-d_A)(1-e)-(b_A-a_A)(e-1)]$$

$$= \frac{0.35(0.4+0.6)}{2} + \frac{0.35}{2\times 2.72}[(0.6-0.8)(1-2.72)-(0.4-0.2)(2.72-1)]$$

$$= 0.175 + 0.064[0.344-0.344] = 0.175$$

and

$$m_B = \frac{w_B(b_B+c_B)}{2} + \frac{w_B}{2e}[(c_B-d_B)(1-e)-(b_B-a_B)(e-1)]$$

$$= \frac{0.7(0.2+0.3)}{2} + \frac{0.7}{2\times 2.72}[(0.3-0.4)(1-2.72)-(0.2-0.1)(2.72-1)]$$

$$= 0.175 + 0.13[0.172-0.172] = 0.175$$

So with use of theorem 3.3, we have $m_A \sim m_B$ then $A \sim B$.

Example 4.2. *Let* $A = (0.1, 0.2, 0.4, 0.5; 1)$ *and* $B = (0.1, 0.3, 0.3, 0.5; 1)$ *be two generalized trapezoidal fuzzy number, then*

$$m_A = \frac{w_A(b_A + c_A)}{2} + \frac{w_A}{2e}[(c_A - d_A)(1-e) - (b_A - a_A)(e-1)]$$
$$= \frac{(0.2+0.4)}{2} + \frac{1}{2\times 2.72}[(0.4-0.5)(1-2.72) - (0.2-0.1)(2.72-1)]$$
$$= 0.3 + 0.184[0.172 - 0.172] = 0.3$$

and

$$m_B = \frac{w_B(b_B + c_B)}{2} + \frac{w_B}{2e}[(c_B - d_B)(1-e) - (b_B - a_B)(e-1)]$$
$$= \frac{(0.3+0.3)}{2} + \frac{1}{2\times 2.72}[(0.3-0.5)(1-2.72) - (0.3-0.1)(2.72-1)]$$
$$= 0.3 + 0.184[0.344 - 0.344] = 0.3$$

So with use of theorem 3.3, we have $m_A \sim m_B$ *then* $A \sim B$ *.*

Example 4.3. *Let* $A = (0.1, 0.2, 0.4, 0.5; 1)$ *and* $B = (1, 1, 1, 1; 1)$ *be two generalized trapezoidal fuzzy number, then*

$$m_A = \frac{w_A(b_A + c_A)}{2} + \frac{w_A}{2e}[(c_A - d_A)(1-e) - (b_A - a_A)(e-1)]$$
$$= \frac{(0.2+0.4)}{2} + \frac{1}{2\times 2.72}[(0.4-0.5)(1-2.72) - (0.2-0.1)(2.72-1)]$$
$$= 0.3 + 0.184[0.172 - 0.172] = 0.3$$

and

$$m_B = \frac{w_B(b_B + c_B)}{2} + \frac{w_B}{2e}[(c_B - d_B)(1-e) - (b_B - a_B)(e-1)]$$
$$= \frac{(1+1)}{2} + \frac{1}{2\times 2.72}[(1-1)(1-2.72) - (1-1)(2.72-1)]$$
$$= 1 + 0.184[0-0] = 1$$

So with use of theorem 3.3, we have $m_A < m_B$ *then* $A < B$ *.*

Example 4.4. *Let* $A = (-0.5, -0.3, -0.3, -0.1; 1)$ *and* $B = (0.1, 0.3, 0.3, 0.5; 1)$ *be two generalized trapezoidal fuzzy number, then*

$$m_A = \frac{w_A(b_A + c_A)}{2} + \frac{w_A}{2e}[(c_A - d_A)(1-e) - (b_A - a_A)(e-1)]$$
$$= \frac{(-0.3-0.3)}{2} + \frac{1}{2\times 2.72}[(-0.3+0.1)(1-2.72) - (-0.3+0.5)(2.72-1)]$$
$$= -0.3 + 0.184[0.344 - 0.344] = -0.3$$

and

$$m_B = \frac{w_B(b_B + c_B)}{2} + \frac{w_B}{2e}[(c_B - d_B)(1-e) - (b_B - a_B)(e-1)]$$
$$= \frac{(0.3+0.3)}{2} + \frac{1}{2\times 2.72}[(0.3-0.5)(1-2.72) - (0.3-0.1)(2.72-1)]$$
$$= 0.3 + 0.184[0.344 - 0.344] = 0.3$$

So with use of theorem 3.3, we have $m_A < m_B$ *then* $A < B$ *.*

Example 4.5. *Let* $A = (0.3, 0.5, 0.5, 1; 1)$ *and* $B = (0.1, 0.6, 0.6, 0.8; 1)$ *be two generalized trapezoidal fuzzy number, then*

$$m_A = \frac{w_A(b_A + c_A)}{2} + \frac{w_A}{2e}[(c_A - d_A)(1-e) - (b_A - a_A)(e-1)]$$
$$= \frac{(0.5+0.5)}{2} + \frac{1}{2\times 2.72}[(0.5-1)(1-2.72) - (0.5-0.3)(2.72-1)]$$
$$= 0.5 + 0.184[0.86 - 0.344] = 0.5 + 0.095 = 0.595$$

and

$$m_B = \frac{w_B(b_B + c_B)}{2} + \frac{w_B}{2e}[(c_B - d_B)(1-e) - (b_B - a_B)(e-1)]$$
$$= \frac{(0.6+0.6)}{2} + \frac{1}{2\times 2.72}[(0.6-0.8)(1-2.72) - (0.6-0.1)(2.72-1)]$$
$$= 0.6 + 0.184[0.344 - 0.86] = 0.6 - 0.095 = 0.505$$

So with use of theorem 3.3, we have $m_A > m_B$ *then* $A > B$ *.*

Example 4.6. *Let* $A = (0, 0.4, 0.6, 0.8; 1)$ *and* $B = (0.2, 0.5, 0.5, 0.9; 1)$ *and* $C = (0.1, 0.6, 0.7, 0.8; 1)$ *be three generalized trapezoidal fuzzy number, then*

$$m_A = \frac{w_A(b_A + c_A)}{2} + \frac{w_A}{2e}[(c_A - d_A)(1-e) - (b_A - a_A)(e-1)]$$
$$= \frac{(0.4+0.6)}{2} + \frac{1}{2\times 2.72}[(0.6-0.8)(1-2.72) - (0.4-0)(2.72-1)]$$
$$= 0.5 + 0.184[0.344 - 0.688] = 0.5 - 0.06 = 0.44$$

and

$$m_B = \frac{w_B(b_B + c_B)}{2} + \frac{w_B}{2e}[(c_B - d_B)(1-e) - (b_B - a_B)(e-1)]$$
$$= \frac{(0.5+0.5)}{2} + \frac{1}{2\times 2.72}[(0.5-0.9)(1-2.72) - (0.5-0.2)(2.72-1)]$$
$$= 0.5 + 0.184[0.688 - 0.516] = 0.5 + 0.03 = 0.53$$

and

$$m_C = \frac{w_B(b_B + c_B)}{2} + \frac{w_B}{2e}[(c_B - d_B)(1-e) - (b_B - a_B)(e-1)]$$
$$= \frac{(0.6+0.7)}{2} + \frac{1}{2\times 2.72}[(0.7-0.8)(1-2.72) - (0.6-0.1)(2.72-1)]$$
$$= 0.65 + 0.184[0.172 - 0.86] = 0.65 - 0.13 = 0.52$$

So with use of theorem 3.3, we have $m_A < m_C < m_B$ *then* $A < C < B$ *.*

Example 4.7. *Let* $A = (0.1, 0.2, 0.4, 0.5; 1)$ *and* $B = (-2, 0, 0, 2; 1)$ *be two generalized trapezoidal fuzzy number, then*

$$m_A = \frac{w_A(b_A + c_A)}{2} + \frac{w_A}{2e}[(c_A - d_A)(1-e) - (b_A - a_A)(e-1)]$$
$$= \frac{(0.2+0.4)}{2} + \frac{1}{2\times 2.72}[(0.4-0.5)(1-2.72) - (0.2-0.1)(2.72-1)]$$
$$= 0.3 + 0.184[0.172 - 0.172] = 0.3$$

and

$$m_B = \frac{w_B(b_B + c_B)}{2} + \frac{w_B}{2e}[(c_B - d_B)(1-e) - (b_B - a_B)(e-1)]$$
$$= \frac{(0+0)}{2} + \frac{1}{2\times 2.72}[(0-2)(1-2.72) - (0+2)(2.72-1)]$$
$$= 0 + 0.184[3.44 - 3.44] = 0$$

So with use of theorem 3.3, we have $m_A > m_B$ *then* $A > B$ *.*

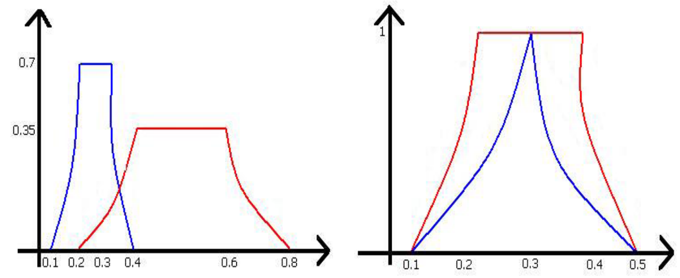

Figure 1: Example 1, Example 2.

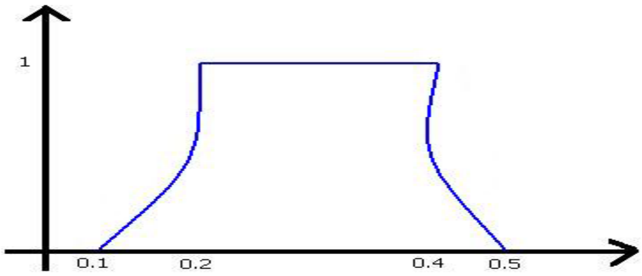

Figure 2: Example 3.

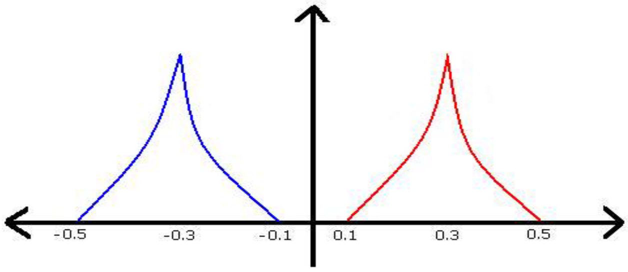

Figure 3: Example 4.

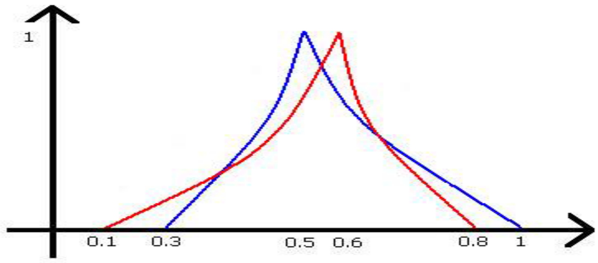

Figure 4: Example 5.

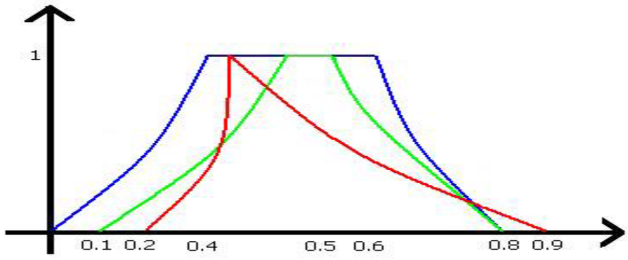

Figure 5: Example 6.

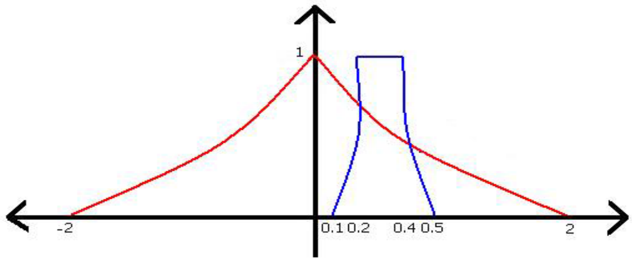

Figure 6: Example 7.

Table (1): A comparison of the ranking results for different approaches

Approaches	Ex.1	Ex.2	Ex.3	Ex.4	Ex.5	Ex.6	Ex.7
Cheng[7]	$A < B$	$A \sim B$	Error	$A \sim B$	$A > B$	$A < B < C$	Error
Chu[8]	$A < B$	$A \sim B$	Error	$A < B$	$A > B$	$A < B < C$	Error
Chen[5]	$A < B$	$A < B$	$A < B$	$A < B$	$A > B$	$A < C < B$	$A > B$
Abbasbandy[1]	Error	$A \sim B$	$A < B$	$A \sim B$	$A < B$	$A < B < C$	$A > B$
Chen[21]	$A < B$	$A < B$	$A < B$	$A < B$	$A > B$	$A < B < C$	$A > B$
Kumar[12]	$A > B$	$A \sim B$	$A < B$	$A < B$	$A > B$	$A < B < C$	$A > B$
Rezvani[18]	$A > B$	$A > B$	$A < B$	$A < B$	$A < B$	$A < B < C$	$A > B$
Proposed approach	$A \sim B$	$A \sim B$	$A < B$	$A < B$	$A > B$	$A < C < B$	$A > B$

5 Conclusion

It is clear from Table 1 that the results of the proposed approach are same as obtained by using the existing apparoch (Chen and Chen, 2009). The main advantage of the proposed approach is that the proposed approach provides the correct ordering of generalized and normal trapezoidal fuzzy numbers and also the proposed approach is very simple and easy to apply in the real life problems.

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