Abstract
In the present paper, the researchers discuss the problem of weighted expected interval approximation of fuzzy numbers. This interval can be used as a crisp approximation set with respect to a fuzzy quantity. Then, by using this, the researchers propose a novel approach to ranking fuzzy numbers. The proposed model is studied for a broad class of fuzzy numbers. The calculation of this method is far simpler than the other approaches.

Keywords: Ranking; Fuzzy numbers; Defuzzification; Interval-Value; Central interval.

1 Introduction
Approximation an interval representation of a fuzzy number may have many useful applications. By utilizing this method, it is possible to apply (in fuzzy number approaches) some results derived from interval number analysis. For example, it may be applied to a comparison of fuzzy numbers by using the order relations defined on the set of interval numbers. Various authors in [6, 7, 8, 15] have studied the crisp approximation of fuzzy sets and proposed a rough theoretic definition of that crisp approximation, called the nearest interval approximation of a fuzzy set. Moreover, quite different approach to crisp approximation of fuzzy sets was applied in Chakrabarty, et al. [16], in which a rough theoretic definition of that crisp approximation, called the nearest ordinary set of a fuzzy set, was proposed and the construction of such a set suggested. Discrete fuzzy sets were discussed, however approximation of a given fuzzy set not unique. Thus this article will is not and hence, will not be discussed herein. Having reviewed the previous interval approximations, this article proposes here a method to find the weighted expected interval approximation of a fuzzy number. The main purpose of this article is to illustrate how a weighted expected interval of a fuzzy number can be utilized as a crisp set approximation of a fuzzy number. Therefore, by the employing this approximation, a new method for ranking of fuzzy numbers presented here in. In addition this method removes the ambiguities and shortcomings of previous ranking procedures addressed by others in the past. This article derive the formulae for determining the approximation interval for a fuzzy number given in a general form as well as for a fuzzy number of LR type. The paper is organized as follows: In Section 2, this article recalls some fundamental results on fuzzy numbers. In this Section, a crisp set approximation of a fuzzy number (weighted expected interval) is obtained. Proposed method for ranking fuzzy numbers is in Section 3. The paper ends with conclusions in Section 4.

*Corresponding author. Email address: srsaneifard@yahoo.com, Tel:+989149737077
2 Preliminaries

We consider the following well-known description of a fuzzy number $A$:

$$A(x) = \begin{cases} 
0 & \text{if } x \leq a_1, \\
I_a(x) & \text{if } a_1 \leq x \leq a_2, \\
1 & \text{if } a_2 \leq x \leq a_3, \\
r_a(x) & \text{if } a_3 \leq x \leq a_4, \\
0 & \text{if } a_4 \leq x. 
\end{cases} \quad (2.1)$$

where $a_1, a_2, a_3, a_4 \in R$, $I_a : [a_1, a_2] \to [0, 1]$ is a nondecreasing upper semi-continuous function, $I_a(a_1) = 0, I_a(a_2) = 1$, called the left side of the fuzzy number and $r_a : [a_3, a_4] \to [0, 1]$ is a nonincreasing upper semicontinuous function, $r_a(a_3) = 1, r_a(a_4) = 0$, called the right side of the fuzzy number. The $\alpha$-cut, $\alpha \in (0, 1]$, of a fuzzy number $A$ is a crisp set defined as

$$A_\alpha = \{x \in R : A(x) \geq \alpha\}.$$ 

The support or 0-cut $A_0$ of a fuzzy number is defined as

$$A_0 = \{x \in R : A(x) > 0\}.$$ 

Every $\alpha$-cut, $\alpha \in (0, 1]$, of a fuzzy number $A$ is a closed interval

$$A_\alpha = [A_L(\alpha), A_U(\alpha)],$$

where

$$A_L(\alpha) = \inf \{x \in R : A(x) \geq \alpha\},$$

$$A_U(\alpha) = \sup \{x \in R : A(x) \geq \alpha\},$$

for any $\alpha \in (0, 1]$. If the sides of the fuzzy number $A$ are strictly monotone then one can see easily that $A_L$ and $A_U$ are inverse functions of $I_a$ and $r_a$, respectively. We denote by $F(R)$ the set of all fuzzy numbers.

The expected interval $EI(A)$ of a fuzzy number $A, A_\alpha = [A_L(\alpha), A_U(\alpha)]$ is defined by (see [1, 2, 3])

$$EI(A) = [E_L(A), E_U(A)] = \left[ \int_0^1 A_L(\alpha)d\alpha, \int_0^1 A_U(\alpha)d\alpha \right].$$

Fuzzy numbers with simple membership functions are preferred in practice. The most used such fuzzy numbers are so-called trapezoidal fuzzy numbers. A trapezoidal fuzzy number $T$, $T_\alpha = [T_L(\alpha), T_U(\alpha)], \alpha \in [0, 1]$, is given by

$$T_L(\alpha) = t_1 + (t_2 - t_1)\alpha,$$

and

$$T_U(\alpha) = t_4 - (t_4 - t_3)\alpha,$$

where $t_1, t_2, t_3, t_4 \in R, t_1 \leq t_2 \leq t_3 \leq t_4$. When $t_2 = t_3$ we obtain a triangular fuzzy number. We denote

$$T = (t_1, t_2, t_3, t_4),$$

a trapezoidal fuzzy number and by $F^T(R)$ the set of all trapezoidal fuzzy numbers. The expected interval for a trapezoidal fuzzy number $T = (t_1, t_2, t_3, t_4)$ is

$$EI(T) = \left[ \frac{t_1 + t_2}{2}, \frac{t_3 + t_4}{2} \right].$$

Let us consider the non-negative and increasing functions $\lambda_L, \lambda_U : [0, 1] \to R$ called weighted functions with

$$\int_0^1 \lambda_L(\alpha)d\alpha > 0,$$
\[
\int_0^1 \lambda_U(\alpha) d\alpha > 0,
\]

and the weighted distance
\[
d_\lambda(A, B) = \sqrt{\int_0^1 \lambda_L(\alpha) [A_L(\alpha) - B_L(\alpha)]^2 d\alpha + \int_0^1 \lambda_U(\alpha) [A_U(\alpha) - B_U(\alpha)]^2 d\alpha},
\]

for two arbitrary fuzzy numbers \(A, B, A_\alpha = [A_L(\alpha), A_U(\alpha)]\) and \(B_\alpha = [B_L(\alpha), B_U(\alpha)]\), \(\alpha \in [0, 1]\). [13]

Taking into account [4, 5] we can use the following notations
\[
a = \int_0^1 \lambda_L(\alpha) d\alpha, \quad (2.2)
\]
\[
b = \int_0^1 \lambda_U(\alpha) d\alpha, \quad (2.3)
\]
\[
\omega_L = \frac{1}{a} \int_0^1 \alpha \lambda_L(\alpha) d\alpha, \quad (2.4)
\]
\[
\omega_U = \frac{1}{b} \int_0^1 \alpha \lambda_U(\alpha) d\alpha, \quad (2.5)
\]
\[
c = \int_0^1 (\alpha - \omega_L)^2 \lambda_L(\alpha) d\alpha, \quad (2.6)
\]
\[
d = \int_0^1 (\alpha - \omega_U)^2 \lambda_U(\alpha) d\alpha. \quad (2.7)
\]

In [9, 10, 11] the authors introduced the extended trapezoidal fuzzy number \(A = (l, u, \delta, \sigma)\) with
\[
A_\alpha = [l + \delta(\alpha - \omega_L), u - \sigma(\alpha - \omega_U)],
\]

where \(\alpha \in [0, 1]\) and \(l, u, \delta, \sigma \in R\)

\[
\delta, \sigma \geq 0,
\]

\[
\delta + \sigma \leq 2(u - l).
\]

It is immediate that
\[
l = \frac{t_1 + t_2}{2}, \quad (2.4)
\]
\[
u = \frac{t_3 + t_4}{2}, \quad (2.5)
\]
\[
\delta = t_2 - t_1, \quad (2.6)
\]
\[
\sigma = t_4 - t_3. \quad (2.7)
\]

Let \(A, B \in F(R), A_\alpha = [A_L(\alpha), A_U(\alpha)], B_\alpha = [B_L(\alpha), B_U(\alpha)], \alpha \in [0, 1]\) and \(\lambda \in R\). We consider the sum \(A + B\) by (see e.g. [12])
\[
(A + B)_\alpha = A_\alpha + B_\alpha = [A_L(\alpha) + B_L(\alpha), A_U(\alpha) + B_U(\alpha)],
\]

for every \(\alpha \in [0, 1]\). In the case of the trapezoidal fuzzy numbers \(T = (t_1, t_2, t_3, t_4)\) and \(S = (s_1, s_2, s_3, s_4)\) we obtain
\[
T + S = (t_1 + s_1, t_2 + s_2, t_3, t_4 + t_4).
\]

An extended trapezoidal fuzzy number [13] is an order pair of polynomial functions of degree less than or equal to 1. An extended trapezoidal fuzzy number may not be a fuzzy number. We denote by \(F^T(R)\) the set of all extended trapezoidal fuzzy numbers, that is
\[
F^T(R) = \{T = (l, u, \delta, \sigma) : T_L(\alpha) = l + \delta(\alpha - \frac{1}{2}), T_U(\alpha) = u - \sigma(\alpha - \frac{1}{2}), \alpha \in [0, 1], l, u, \delta, \sigma \in R\},
\]

where \(T_L\) and \(T_U\) have the same meaning as above.
Proposition 2.1. [10] Let $A(l_1, u_1, \delta_1, \sigma_1)_{\lambda}$ and $B(l_2, u_2, \delta_2, \sigma_2)_{\lambda}$ be two extended trapezoidal fuzzy numbers. Then,

$$d^2_A(A, B) = a(l_1 - l_2)^2 + b(u_1 - u_2)^2 + c(\delta_1 - \delta_2)^2 + c(\sigma_1 - \sigma_2)^2.$$ 

Proposition 2.2. [9] Let $A = (l, u, \delta, \sigma)_{\lambda}$ be extended trapezoidal fuzzy numbers. Then, (i) $A$ is trapezoidal iff

$$d \geq 0, \quad \sigma \geq 0, \quad l - u + (1 + \omega_L)\delta + (1 - \omega_U)\sigma \leq 0;$$

(ii) $A$ is triangular iff

$$d \geq 0, \quad \sigma \geq 0, \quad l - u + (1 - \omega_L)\delta + (1 - \omega_U)\sigma = 0.$$ 

Proposition 2.3. [9] Let $A$ be a fuzzy number. Then

$$d^2_A(A, B) = d^2_A(A, T_e(A)) + d^2_A(T_e(A), B),$$

for every extended trapezoidal fuzzy number $B$.

Proposition 2.4. [10] The weighted expected interval of $A \in F(R)$ is defined by

$$EI^{w}(A) = \left[ \int_{0}^{1} A_L(\alpha) \lambda_L(\alpha)d\alpha, \int_{0}^{1} A_U(\alpha) \lambda_U(\alpha)d\alpha \right],$$

where $\lambda_L$ and $\lambda_U$ are weighted functions and $a, b$ are introduced in (2.2)-(2.3).

Taking into account the above notations we obtain

$$EI^{w}(T_e(A)) = EI^{w}(A) = [l^e, u^e].$$

The Karush-Kuhn-Tucker theorem is useful in the solving of the proposed problem.

Theorem 2.1. [9] Let $f, g_1, g_2, \ldots, g_m : R_n \rightarrow R$ be convex and differentiable functions. Then $\bar{x}$ solves the convex programming problem

$$\min f(x)$$

subject to $g_i(x) \leq b_i, i \in \{1, \ldots, m\}$ if and only if there exist $\mu_i$, where $i \in \{1, \ldots, m\}$, such that

(i) $\nabla f(\bar{x}) + \sum_{i=1}^{m} \mu_i \nabla g_i(\bar{x}) = 0;$

(ii) $g_i(\bar{x}) - b_i \leq 0;$

(iii) $\mu_i \geq 0;$

(iv) $\mu_i(b_i - g_i(\bar{x})) = 0.$

Definition 2.1. [8]. Let, $A(a_1, a_2)$ is arbitrary interval number. The measure of interval number $A$ define as follows:

$$M_I(A) = \text{sign}(a_1).|a_1, a_2|. $$
Definition 2.2. Let $A$ be an arbitrary fuzzy number and $EI^0(A) = [l^*(A), u^*(A)]$ be its weighted expected interval. According to definition 2.1, the measure of $EI^0(A)$ which is an interval number is as $M_l(EI^0(A)) = \text{sign}(l^*(A)) |l^*(A)| u^*(A) |$. We define the measure of fuzzy number $A$ as follows:

$$M_{EI}(A) = \int_0^1 p(\alpha) M_l(EI^0(A)) d\alpha.$$  \hspace{1cm} (2.10)

The function $p : [0, 1] \rightarrow [0, \infty)$ denotes the distribution density of the importance of the degrees of fuzziness, where $\int_0^1 p(\alpha)d\alpha = 1$. In particular cases, it may be assumed that

$$p(\alpha) = (k + 1) \alpha^k, \quad k = 0, 1, 2, \ldots$$

Throughout this study the researchers assumed that $k = 1$, i.e. $p(\alpha) = 2\alpha$. Obviously, if a fuzzy number becomes interval numbers, then $M_l(A)$ will be the measure of the interval number which can be denoted as $M_l(A)$. For a certain fuzzy numbers, we can obtain $M_l(A)$ by definite integral. But it is not easy to compute definite integral sometimes. For trapezoid fuzzy numbers and triangular fuzzy numbers, the calculation formulas for the indices are given in the paper.

Since ever measure can be used as a crisp approximation of a fuzzy number, therefore, the resulting value is used to rank the fuzzy numbers. Thus, $M_l(A)$ is used to rank fuzzy numbers. The larger the value of $M_l(A)$, the better the ranking of $A$.

3 Ordering of fuzzy numbers by the weighted expected interval

In this section, the researchers will propose the ranking of fuzzy numbers associated with the weighted expected interval. Ever, the weighted expected interval can be used as a crisp set approximation of a fuzzy number, therefore the resulting approximation is used to rank the fuzzy numbers.

Definition 3.1. Let $A$ and $B$ be two fuzzy numbers, and $M_{EI}(A)$ and $M_{EI}(B)$ be the measure of their. Define the ranking of $A$ and $B$ by $M_{EI}(.)$ on $F$, i.e.

1. $M_{EI}(A) < M_{EI}(B)$ if only if $A < B$,
2. $M_{EI}(A) > M_{EI}(B)$ if only if $A > B$,
3. $M_{EI}(A) = M_{EI}(B)$ if only if $A \sim B$.

This article formulates the order $\geq$ and $\leq$ as $A \sim B$ if and only if $A \sim B$ or $A \sim B$, $A \sim B$ if and only if $A \sim B$ or $A \sim B$.

This study considers the following reasonable axioms that Wang and Kerre [14] proposed for fuzzy quantities ranking. Let $M$ be an ordering method, $S$ the set of fuzzy quantities for which the method $M$ can be applied, and $S'$ and $S''$ are two arbitrary finite subset of $S$. The statement “two elements $A$ and $B$ in $\mathcal{S}$ satisfy that $A$ has a higher ranking than $B$ when $M$ is applied to the fuzzy quantities in $\mathcal{S}$” will be written as “$A \sim B$ by $M$ on $\mathcal{S}$”. “$A \sim B$ by $M$ on $\mathcal{S}$”, and “$A \sim B$ by $M$ on $\mathcal{S}$” are similarly interpreted. The following proposition shows the reasonable properties of the ordering approach.

Proposition 3.1. Let $S$ be the set of fuzzy quantities for which the weighted expected interval method can be applied, and $\mathcal{S}'$ and $\mathcal{S}''$ are two arbitrary finite subsets of $S$. The following statements hold.

A1. For $u \in \mathcal{S}'$, $A \leq B$ by $M$ on $\mathcal{S}'$.
A2. For $(A, B) \in \mathcal{S}'^2$, $A \leq B$ and $B \leq A$ by $M$ on $\mathcal{S}'$, we should have $A \sim B$ by $M$ on $\mathcal{S}'$.
A3. For $(A, B, C) \in \mathcal{S}'^3$, $A \leq B$ and $B \leq C$ by $M$ on $\mathcal{S}'$, we should have $A \leq C$ by $M$ on $\mathcal{S}'$.
A4. For $(A, B) \in \mathcal{S}'^2$, $\inf \supp(B) > \sup \supp(A)$, we should have $A \leq B$ by $M$ on $\mathcal{S}'$.
A5. For $(A, B) \in (\mathcal{S}' \cap \mathcal{S}''^2)$. We obtain the ranking order $A \leq B$ by $M$ on $\mathcal{S}''$ if and only if $A \leq B$ by $M$ on $\mathcal{S}'$.
A6. Let $A$, $B$, $A + c$ and $B + c$ be elements of $S$. If $A \leq B$ by $M$ on $\{A, B\}$, then $A + c \leq B + c$ by $M$ on $\{A + c, B + c\}$.
A7. Let $A$, $B$, $A + c$ and $B + c$ be elements of $S$. If $A \leq B$ by $M$ on $\{A, B\}$, then $A + c \sim B + c$ by $M$ on $\{A + c, B + c\}$.
Figure 1: Fuzzy Numbers A, B and C of Example 3.1

Table 1: Comparative results of Example 3.1.

<table>
<thead>
<tr>
<th>fuzzy number</th>
<th>New approach</th>
<th>Sign distance (p=1)</th>
<th>Sign distance (p=2)</th>
<th>Distance Minimization</th>
<th>Chu and Tsao</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>6.2</td>
<td>3</td>
<td>2.1602</td>
<td>2.5</td>
<td>0.7407</td>
</tr>
<tr>
<td>B</td>
<td>6.5</td>
<td>3</td>
<td>2.7080</td>
<td>2.5</td>
<td>0.7407</td>
</tr>
<tr>
<td>C</td>
<td>4.8</td>
<td>3</td>
<td>2.7080</td>
<td>2.5</td>
<td>0.75</td>
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<td>Results</td>
<td>C ≺ A ≺ B</td>
<td>C ∼ A ∼ B</td>
<td>C ≺ A ∼ B</td>
<td>C ∼ A ∼ B</td>
<td>A ∼ B ≺ C</td>
</tr>
</tbody>
</table>

Remark 3.1. Ranking order \(M_{EI}\) has the axioms \(A_1, A_2, \ldots, A_5\).

Remark 3.2. If \(A \preceq B\), then \(−A \succeq −B\).

Hence, this article can infer ranking order of the images of the fuzzy numbers.

Example 3.1. Consider the three fuzzy numbers \(A = (1, 2, 5), B = (0, 3, 4)\) and \(C = (2, 2.5, 3)\). By using this new approach \(M_{EI}(A) = 6.2, M_{EI}(B) = 6.5\) and \(M_{EI}(C) = 4.83\). Hence, the ranking order is \(C \prec A \prec B\) too. To compare with some of the other methods in [1, 2, 3], the reader can refer to Table 1. (Figure 1)

Example 3.2. Consider the following sets, see Yao and Wu [17].
Set1: \(A=(0.4,0.5,1), B=(0.4,0.7,1), C=(0.4,0.9,1)\).
Set2: \(A=(0.3,0.4,0.7,0.9)\) (trapezoidal fuzzy number), \(B=(0.3,0.7,0.9)\), \(C=(0.5,0.7,0.9)\).
Set3: \(A=(0.3,0.5,0.7), B=(0.3,0.5,0.8,0.9)\) (trapezoidal fuzzy number), \(C=(0.3,0.5,0.9)\).
Set4: \(A=(0,0.3,0.7,0.8)\) (trapezoidal fuzzy number), \(B=(0.2,0.5,0.9), C=(0.1,0.6,0.8)\).
To compare with other methods authors refer the reader to Table 2. (Figures 2).
Figure 2: Fuzzy Numbers $A$, $B$ and $C$ of Example 3.2
Table 2: Comparative results of Example 3.2.

<table>
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<th>Set2</th>
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<th>Set4</th>
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<tr>
<td>Cheng distance</td>
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<td>0.7577</td>
<td>0.7071</td>
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<td>0.8149</td>
<td>0.8037</td>
<td>0.7256</td>
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<tr>
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<td>0.8602</td>
<td>0.7458</td>
<td>0.7241</td>
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<td>A ≺ C ≺ B</td>
<td>A ≺ C ≺ B</td>
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<tr>
<td>Cheng CV uniform distribution</td>
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<td>A 0.0272</td>
<td>0.328</td>
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<td>0.0095</td>
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All the above examples show that this method is more consistent with institution than the previous ranking methods.

4 Conclusion

In this article, the researchers proposed weighted expected interval of fuzzy numbers. Based on this interval, we propose new ranking index to classify fuzzy numbers and displaying several typical examples to compare the current method with some other ranking methods. We find the ranking method suggested in this paper overcomes some problems included in existing methods to some extent and possesses better efficiency of resolution and reasonability. The calculations of the proposed method are simpler than the other approaches.

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