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Research Article

Data Envelopment Analysis with Fixed Inputs, Undesirable Outputs and Negative Data

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Abstract

In Data Envelopment Analysis (DEA), different models have been measured to evaluate the performance of decision making units with multiple inputs and outputs. Revised model of Slack-based measures known as MBSM of collective models family has been introduced by Sharp et al. Slack-based measure has been introduced by Ton. In this study, a model is proposed that is able to estimate the efficiency when a number of outputs of decision making units are undesirable, inputs are fixed and some of outputs and inputs are negative. So that, level of undesirable output is reduced at the constant level of inputs in the evaluation unit and by conserving the efficiency.

Keywords: Data envelopment analysis, Fixed Inputs, Undesirable Outputs, Negative data, Evaluating the efficiency.

1 Introduction

Data Envelopment Analysis is a mathematical programming approach used to evaluate the efficiency of Decision Making Units with similar activities and multiple inputs and outputs. If, there are undesirable inputs or outputs among the inputs and outputs of the decision making units, rate of these inputs or outputs should be increased and decreased respectively. The most famous models of Data Envelopment Analysis include: CCR by Charnes and Cooper and Rodez [3] (1978), BCC by Banker, Charnes, Cooper [1] (1984) and collective model proposed by Charnes et al [2] (1985) and … all these models have been formulated for desirable inputs and outputs.

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The most famous models of Data Envelopment Analysis include: CCR by Charnes and Cooper and Rodez [3] (1978), BCC by Banker, Charnes, Cooper [1] (1984) and collective model proposed by Charnes et al [2] (1985) and … all these models have been formulated for desirable inputs and outputs. When inputs and outputs are undesirable, data envelopment analysis is concerned and these studies were scattered and are limited to some of specific applications. The analysis of efficiency in economic units is one of the most important administrative issues; therefore, selecting a proper model is of high importance. Some organizations such as factories, hospitals etc … in production and activity process may produce undesirable outputs such as Aerosols and waste in addition to desirable required outputs. In this study, a model based on MBSM with negative data is proposed to reduce undesirable outputs despite fixed inputs.

2 MSBM model

Sharp et al (2006) [4] have introduced a modified slack based measure model known as MSBM for negative data that has the ability to handle negative inputs and outputs.
Sharp et al (2006) rewrote SBM model for calculating the efficiency measure using SBM model in the presence of negative variables as well as applying Portela method [4] and placing the improving directions (R_{io},R_{ro}) and called it MSBM:

$$\text{Min} \quad e_o = \frac{1 - \sum_{i=1}^{m} \frac{w_i s_i^-}{R_{io}}}{1 + \sum_{r=1}^{s} \frac{v_r s_r^+}{R_{ro}}}$$

$$\text{S.t} \quad \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io}, \qquad i = 1.\dots.m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro}, \qquad r = 1.\dots.s \tag{2.1}$$

$$\sum_{j=1}^{n} \lambda_j = 1$$

$$\lambda_j \geq 0\,.\; s_i^- \geq 0.\; s_r^+ \geq 0\,. \qquad j = 1.\dots.n\,.\; r = 1.\dots.s\,.\; i = 1.\dots.m$$

Where:
s_i^- : the value of i[th] input slack
s_r^+ : the value of i[th] output slack
w_i‘ v_r: the weights predetermined by decision maker (DM).
In addition, vectors in the model are as below:

$$R_{ro} = \max_j\{y_{rj}\} - y_{ro} \qquad . \qquad R_{io} = x_{io} - \min_j\{x_{ij}\}$$

When R_{io} and R_{ro} are equal to zero, it is assumed that $\frac{w_i s_i^-}{R_{io}}$ and $\frac{v_r s_r^+}{R_{ro}}$ terms are eliminated from nominator and denominator.

3 Undesirable Outputs in Data Envelopment Analysis

Generally, there are wastes in the production process that expose additional cost on institutions. Therefore, waste can be considered as undesirable output. In the primary models of DEA as well as the above-mentioned model, undesirable outputs are ignored. Most undesirable outputs are produced jointly with the desirable output (desirable value of output cannot be produced without producing undesirable outputs).

In the DEA model, inputs and outputs are presented by X symbol and (Y^{D} ,Y^{UD}) symbol respectively, so that Y^{UD} and Y^{D} are the desired (good) and undesirable (bad) outputs. Obviously, desirable output should be increased and undesirable output should be reduced to improve the performance. In order to increase desirable output and reduce undesirable outputs, model (4.2) is proposed.

4 The Proposed Model to Evaluate the Efficiency with Fixed inputs and Undesirable Outputs

To evaluate the performance of decision making units, each unit is evaluated based on the inputs consumed to product output. UMD achieves better efficiency in the evaluation, when the input is lower and output is more. In some cases, there are some fixed inputs and the decision maker is not allowed to disturb in selecting the inputs. Such inputs are fixed inputs. Also, there are some undesirable outputs and it is better to have lower rate of such outputs. Such outputs are undesirable outputs. In this section, a model is proposed to estimate the efficiency of the units under the evaluation in non-radial form, when the outputs mentioned above are undesirable. So, a new model is proposed assuming fixed inputs and undesirable outputs in which the inputs are divided in two fixed and non-fixed inputs. Also, outputs are divided in two desirable and undesirable outputs. It is noteworthy, undesirable outputs are considered as desirable inputs in the new model.

Therefore, new model is introduced as below:

$$
\begin{aligned}
\text{Min} \quad & p = \frac{1 - \sum_{i \in I_1} \frac{w_i s_i^-}{R_{io}} - \frac{v_r s_r^+}{R_{ro}}}{1 + \sum_{r \in D} \frac{v_r s_r^+}{R_{ro}}} \\
\text{S.t} \quad & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io}, \qquad i \in I_1 \\
& \sum_{j=1}^{n} \lambda_j x_{ij} = x_{io}, \qquad i \in I_2 \\
& \sum_{j=1}^{n} \lambda_j y_{rj}^{D} - s_r^+ = y_{ro}^{D}, \qquad r \in D \qquad\qquad (4.2) \\
& \sum_{j=1}^{n} \lambda_j y_{rj}^{UD} + s_r^+ = y_{ro}^{UD}, \qquad r \in UD \\
& \sum_{j=1}^{n} \lambda_j = 1
\end{aligned}
$$

$\lambda_j \geq 0 \,.\; s_i^- \geq 0.\; s_r^+ \geq 0 \,.\qquad j = 1.\ldots.n \;.\; r = 1.\ldots.s \;.\; i = 1.\ldots.m$

$\underset{r \in D}{R_{ro}} = \underset{j}{Max}\{y_{rj}\} - y_{ro} \;.\quad \underset{r \in UD}{R_{ro}} = y_{ro} - \underset{j}{Min}\{y_{rj}\} \;.\quad \underset{i \in I_1}{R_{io}} = x_{io} - \underset{j}{Min}\{x_{ij}\}$

$y_r^{min} = \underset{j}{Min}\{y_{rj}\} \;.\quad x_i^{min} = \underset{j}{Min}\{x_{ij}\}$

v_r, w_i is the weight selected already by the decision maker and provided that $v_r \geq 0. w_i \geq 0$

D: Set the desired output UD: Set the Undesirable output

I_1: Set the Fixed Inputs I_2: Set the Non-fixed inputs

Theorem 4.1. *Model (4.2) is always possible.*

Proof. It is obvious that, $\lambda_o = 1.\ \lambda_j = 0\ .(j = 1. \dots . n$ و $j \neq 0)$ and $s_i^- = 0\ .(i = 1. \dots . m)$ and $s_r^+ = 0\ .(r = 1. \dots . s)$ is a feasible solution for the model mentioned above.

Theorem 4.2. *Optimized value of objective function in model (4.2) is always between zero and one. DMU$_O$ is efficient when, value of objective function is equal to one.*

Proof. According to model (4.2):

$$s_i^- = x_{io} - \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io} - \sum_{j=1}^{n} \lambda_j x_i^{min} = x_{io} - x_i^{min}\left(\sum_{j=1}^{n} \lambda_j\right)$$

Because of $\sum_{j=1}^{n} \lambda_j = 1$: $s_i^- \leq x_{io} - x_i^{min}$

Since $R_{io} = x_{io} - \underset{j}{Min}\{x_{ij}\}$ thus:

$s_i^- \leq R_{io}$

Now, for undesirasble output variables we have:

$$\xrightarrow{r\in UD} s_r^+ = y_{ro} - \sum_{j=1}^{n} \lambda_j y_{rj} \leq y_{ro} - \sum_{j=1}^{n} \lambda_j y_r^{min} = y_{ro} - y_r^{min}\left(\sum_{j=1}^{n} \lambda_j\right)$$

Because of $\sum_{j=1}^{n} \lambda_j = 1$:

$s_r^+ \leq y_{ro} - y_r^{min}$

Since $\underset{r\in UD}{R_{ro}} = y_{ro} - y_r^{min}$ thus:

$s_r^+ \leq R_{ro}$

Since, $s_r^+ \leq R_{ro}$, $s_i^- \leq R_{io}$, therefore we have:

$$s_i^- \leq R_{io} \rightarrow \frac{s_i^-}{R_{io}} \leq 1 \tag{4.3}$$

$$s_r^+ \leq R_{ro} \rightarrow \frac{s_r^+}{R_{ro}} \leq 1 \tag{4.4}$$

$$(4.3) \text{ and } (4.4) \rightarrow \sum_{j=1}^{n} w_i\left(\frac{s_i^-}{R_{io}}\right) + \sum_{r=1}^{s} v_r\left(\frac{s_r^+}{R_{ro}}\right) \leq 1 \quad \rightarrow \quad 0 \leq \sum_{j=1}^{n} w_i\left(\frac{s_i^-}{R_{io}}\right) + \sum_{r=1}^{s} v_r\left(\frac{s_r^+}{R_{ro}}\right) \leq 1$$

Then:

$0 \leq 1 - \sum_{j=1}^{n} w_i\left(\frac{s_i^-}{R_{io}}\right) - \sum_{r=1}^{s} v_r\left(\frac{s_r^+}{R_{ro}}\right) \leq 1$

And also we can be concluded:

$\rightarrow \sum_{r=1}^{s} \frac{v_r s_r^+}{R_{ro}} \geq 0 \quad \rightarrow \quad 1 + \sum_{r=1}^{s} \frac{v_r s_r^+}{R_{ro}} \geq 1$

Therefore we have: $0 \leq \frac{1 - \sum_{j=1}^{n} w_i\left(\frac{s_i^-}{R_{io}}\right) - \sum_{r=1}^{s} v_r\left(\frac{s_r^+}{R_{ro}}\right)}{1 + \sum_{r=1}^{s} v_r\left(\frac{s_r^+}{R_{ro}}\right)} \leq 1$

When DMU is efficient, $s_i^- . s_r^+$ are equal to zero and as a result, the value of objective function is equal to one. $s_i^- . s_r^+$ is zero, when the value of objective function is equal to one; i.e. DMU$_O$ is efficient**.**

5 A numerical example

Assume that there are ten DMUs with one Fixed Input and Unfixed Input and one Desirable Output and one Undesirable Output according to the table below.

Table 1: Ten DMU with two inputs and two outputs

DMU	I_F	I_{UF}	Y_D	Y_{UD}
1	11	10	13	7
2	35	6	19	2
3	24	12	25	4
4	22	20	27	1
5	39	24	6	1
6	49	27	22	-6
7	34	5	19	20
8	40	22	8	-12
9	25	19	11	8
10	15	8	6	49

For more investigation, the Proposed model with the data in table 1 is run by GAMS software, and The efficiency of each unite is presented in table 2.
Furthermore, We solve the proposed model by assigning weight of 0.50 for each input slack variables in the model and weight of 0.5 for each input slack variable in the objective function.
The obtained efficiencies have been brought in Table 2 for each of DMUs.

Table 2: Efficiency results for model 2

DMU_j	p
1	0.78
2	1
3	0.63
4	0.80
5	0.94
6	1
7	0.77
8	1
9	0.89
10	0.71

Given that in this example, the first input is fixed and the second input is intended variable. As well as a first output desirable and the second output is considered undesirable, we see that $\{\mathrm{DMU}_2.\mathrm{DMU}_6.\mathrm{DMU}_8\}$

efficient were evaluated and the DMU's, respectively$\{\mathrm{DMU}_1.\mathrm{DMU}_3.\mathrm{DMU}_4.\mathrm{DMU}_5.\mathrm{DMU}_7.\mathrm{DMU}_9.\mathrm{DMU}_{10}\}$ were inefficient.

6 Conclusion

In The article basic model data envelopment analysis has been studied. And given that there are conditions that some inputs are fixed and do not allow us to change them, as well as some outputs are undesirable, evaluating the performance of DMUs have different conditions, In this paper we present models that were able to calculate the performance in these conditions. Also, given that some of the negative input and output models presented in this article are able to evaluate the performance of the unit in such conditions.

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