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Research Article

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Determination Bounds for Intermediate Products in a Two-Stage Network DEA

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Abstract

The internal structure of decision making unit (DMU) is the key element at extension of network DEA. In general considering internal performance evaluation of system is a better criterion than the conventional DEA-models, essentially based on the initial inputs and final outputs of the system. The internal performance of a system is dependent on the relation between sub-DMUs and intermediate products. Since the intermediate measures are consumed by some sub-DMUs produced by the others, it is possible to produce systems; the role of intermediate production is twice output and input. Thats why they can be analyzed based on conventional mathematical modeling. In this paper we introduce a new method for determining bounds for intermediate product in a two stage network DEA structure.

Keywords: Data envelopment analysis, Network DEA, Intermediate products, Efficiency.

1 Introduction

Data envelopment analysis (DEA) is a well-known non-parametric mathematical approach for evaluating a set of homogeneous decision-making units (DMUs) [1] formulated the first DEA model under constant returns-to-scale(CRS)assumption. By far, many researchers a have expanded this approach [2] - [4]. In last two decades, a large number of DEA studies have been focused on DMUs with internal network structures that is in this paper we called it NDEA of the sake of simplification [5]. In such models, the internal performance evaluation of the system is included to present a better criterion in than the traditional structure, which where primarily based on the inputs and outputs of the final system [6]-[13].We propose a new model for measuring overall efficiency in a two stage network. The rest of the paper is organized as follows: Section 2 briefly introduces the Production possibility set of a two stage structure, Section3 calculates the bound of intermediate measure; In Section 4 we explain the overall efficiency in two stages, while, system Section 5 paper is concluded.

2 Production Possibility Set of a Two-Stage Model

In the first stage, DMU_o consumes initial inputs X_o to produce intermediate production Z_o for the consumption of the second stage; which generates final outputs Y_o. The PPS for evaluating performance based on concept of DEA for the first stage shown by P_{stg1} is considered as the set of all (X,Z) that X can produce Z. P_{stg2} is considered as the set of all (Z,Y) in which Z can produce Y.

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$$P_{stg1} = \{(X,Z)|X \text{ can produce output } Z\}$$
$$P_{stg2} = \{(Z,Y)|Z \text{ can produce output } Y\}$$

P is considered as the set of whole system can be defined as follow:

$$P = \{(X,Y)|\exists Z((X,Z) \in P_{stg1},(Z,Y) \in P_{stg2})\}$$

Based on the concept of PPS at two stages, our goal is to find the bound of intermediate corresponding to the unit under evaluation DMU_o in a way that this unit can still produce Y_o by consuming Z_o. At first, given the free disposability principle for input and outputs not possible to produce more than Z_o as Z_o is input to the second stage; however without lost of the generality PPS in different stages it may be unlike free disposability with the chance of decreasing or increasing PPS. It provides product possibility of Y_o by Z_o. If DMU is inefficient at the output, there is possibility that it can produce large amount of intermediate products than Z_o in the second stage. On the other hand, if , DMU_o is inefficient for its own inputs at the second stage, it is a possibility to decrease consumption of intermediate products for generating Y_o. This idea is shown in Fig 1.

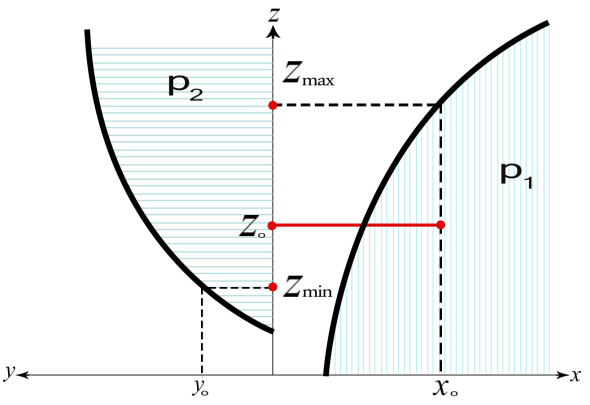

Figure 1: Production possibility of two stage structure: The minimum and maximum production possibility of Z_o is shown by Z_{min} and Z_{max} in figure.

As shown in Fig 1, DMU_o yields Z_o by consuming $X_o$while based on the production possibility set shown in Fig1, it is able to produce as much as Z_o^{max}. In practice the first stage is inefficiency in its output. In the second stage, this unit produces the final output of $Y_o$with the use of Z_o. However, the production possibility set for the production target output Y_o lower outputZ_{min}^o can be used as well. So far for the production possibility of (X_o,Y_o) to enable the production of each intermediate product belongs to the interval $[Z_o^{min},Z_o^{max}]$. Note that this interval is the result of inefficiency in the unit under evaluation and efficiency at each stages, it will further constraint it. In the next section the proposed bounds of Z_o^{min} and Z_o^{max} will be calculated.

3 Determination Bounds For Intermediate Products

As discussed in the previous section, let the vector R be shortage of production in the first stage and T the surplus vector of the intermediate production Z consumer of the second stage. Regardless of the technology used by production possibility set, calculation of R and T can be calculated by the following two models:

$$\begin{aligned} \max \quad & R \\ s.t. \quad & (X_o, Z_o + R) \in P_{stg1} \end{aligned} \tag{3.1}$$

$$\begin{aligned} \max \quad & T \\ s.t. \quad & (Z_o - T, Y_o) \in P_{stg2} \end{aligned} \tag{3.2}$$

Obviously, the optimum value of R for the first model, is the deviation from upper bound of intermediate products and the optimal value of T in the second model is its deviation from lower bound corresponding to the unit under evaluation.
It is clear that $Z_o^{min} \leq Z_o \leq Z_o^{max}$, if $Z_o^{min} = Z_o$ and $Z_o^{max} = Z_o$ meaning that the unit under evaluation is input and output oriented efficient at the second and first stages, respectively. If $Z_o^{min} = Z_o^{max}$ there will not any inefficiency in the intermediate products oriented. We propose this concept for the first time in the NDEA literature. Thus we propose a DMU with two-stage structure with the introduction of the new oriented is efficient when there is no inefficiency in inputs, outputs and intermediate products. In the next section we are going to explain this concept.

4 Two-Stage Network Efficiency

In accordance with the efficiency concept in data envelopment analysis issues, a DMU is efficient if it bereft of any surplus and shortfall in input and output. Excessive unit under evaluation in the data envelopment analysis are indicated by positive slacks corresponding to inputs, Similarly the production shortage will be shown by positive corresponding to outputs. Ina two-stage network, in addition to above inefficiencies, inefficiency in intermediate products should also be considered. This inefficiency is shown by $Z_o^{min} \neq Z_o^{max}$. Using the charactristics and pattern presented in SBM models, the next section proposes a model that checks three above inefficiencies in unit under evaluation and ultimately, presents a model for measuring the overall efficiency.

5 Conclusion

As explained earlier, the internal structure of a system depends on the relations of intermediate production but the handling of intermediate in NDEA literatures is similar to conventional inputs and outputs. In this paper, the intermediate products are introduced from a different perspective. By determining these bounds, a new concept will be introduce in the NDEA literature for evaluating performance and measuring the efficiency in a two- stage structure for intermediate products. The extension of our proposed model to dynamic structure and its specific applications can also be considered as implications for highly efficient future systems. Moreover, the multi stage of this paper offers a framework for future studies.

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