Evaluate the financial performance of pharmaceutical companies using fuzzy DEA

Babak Mazandaranian¹, Maryam Mosleh²*

(¹) Department of Management, Firoozkooh Branch, Islamic Azad University, Firoozkooh, Iran
(²) Department of Mathematics, Firoozkooh Branch, Islamic Azad University, Firoozkooh, Iran

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Abstract
In the past, company’s financial inputs was the only factor for evaluating the performance of the company. Now a day, by complicating the organizational structures, for assessing the performance, we have to choose the high ability techniques for evaluating the organizational structures and processes, which the results of them have defense capability.
The purpose of this article is answering to this question:
“How can we evaluate the efficiency of the pharmaceutical Company by using FDEA Method?”
This practical research includes an evaluating method based on Mathematic Modeling.
The Pharmaceutical Companies which are the Member of the Stock Exchange, have been selected as a Statistical Society.
All selected companies evaluated based on Census Report. The experts of Pharmaceutical Companies advised us for choosing the inputs and outputs in environmental fuzzy. The factor of development and increase the efficiency in pharmaceutical company is:
Pay attention to the research and development in Pharmaceutical Industry and trying to have a joint production with international pharmaceutical companies.

Keywords: Data Envelopment Analysis, Pharmaceutical Companies, Fuzzy, Efficiency.

1 Introduction

Data envelopment analysis (DEA) initially proposed by charnes et al. [3] is a non-parametric technique for measuring and evaluating the relative efficiencies of a set of entities, called decision-making units (DMUs), with the common inputs and outputs. Examples include school, hospital, library and, more recently, whole economic and society systems, in which outputs and inputs are always multiple in character. Most of DEA papers make an assumption that input and output data are crisp ones without any variation. In fact, inputs and outputs of DMU, are ever-changeful. For example, for evaluating operations efficiencies of airline, seat-kilometers available, cargo-kilometers available, fuel and labor are regard as the inputs and
passenger kilometers performed as the output [5]. It is common sense that these inputs and outputs are easy to change because of weather, season, operating state, and so on. Because DEA is a ‘boundary’ method sensitive to outliers, it is very difficult to evaluate the efficiency of DMU with varying inputs and outputs by conventional DEA models. Some researchers have proposed several models to challenge how to deal with the variation of data in efficiency evaluation problems by stochastic frontier models [1, 6, 9]. On the other hand, in more general cases, the data for evaluation are often collected from investigation by polling where the natural language such as good, medium, and bad are used to reflect a kind of general situation of the investigated entities rather than a specific case [7, 8]. In the above example, the expert can make a general conclusion that airline A’s passenger-kilometer is about 200 passenger-kilometers and fuel cost is high based on his rich experience. These fuzzy concepts are used to summarize the general situations of inputs and outputs and reflect the ambiguity of expert’s judgment. The center of fuzzy number represents the most general case and the spread reflect some possibilities.

In this paper CCR model is extended to be fuzzy DEA model for evaluating the fuzzy efficiency of DMU with the given fuzzy input and output data.

2 Models and methodology

2.1. Data Envelopment Analysis (DEA)

The DEA began in the 1950s, with a measurement model for technical efficiency for the case where a unique resource generate a unique product, named input and output, respectively. Charnes, cooper and Rhodes extended the original model by using many inputs and outputs and by casting them into a unique virtual input and output [4].

The developed model has a nonparametric approach, based on linear programing, which allows comparisons between homogeneous entities, the so-called Decision Making Unities (DMU). These DMUs should use the inputs and outputs in the same way. Each of these factors may be in different measurement unities, that is, it is not necessary to convert these factors into a standard measurement unit, as a monetary unit, for it may be infeasible.

The DEA requires the data on the input and output levels solely for evaluating the efficiency of each DMU. The model has been named CCR on account of its authors. In this model, the DMU efficiencies are obtained by considering a linear programming for each DM, as follows:

\[
\begin{align*}
\text{Max} & \quad h_0 = v^t y_0 \\
\text{S.t.} & \quad u^t x_0 = 1 \\
& \quad v^t Y - u^t X \leq 0, k = 1, \ldots, n \\
& \quad u, v \geq 0 
\end{align*}
\]  

(2.1)

where:

- \(h_0\): Efficiency of DMU_0;
- \(u^t, v^t\): Weights associated with inputs and outputs, respectively;
- \(x_0, y_0\): Input and output levels for DMU_0 under analysis, respectively;
- \(X, Y\): Matrices with the input and output levels of the other DMUs, respectively;

The solution to this problem return the values of the variables \(u\) and \(v\), which maximize the efficiency of the DMU under analysis. This procedure is repeated for each DMU, which result in different values for the weights for each DMU. In this way, the relative efficiencies of the entities under study are obtained. This approach generates a DEA efficiency boundary. As to ranking, the DMUs on the boundary are considered to be 100% efficient. The efficiency of the remaining ones is measured in terms of their distance to the boundary.
Farrel (1957) established an empirical measure of relative efficiency based on the distance function (figure 1). In figure 1, $x_1$ and $x_2$ are the necessary inputs for producing $y$, $L(y)$ is the combination of feasible inputs for producing $y$, and $\text{Isoq } L(y)$ stands for the necessary minimum of $x_1$ and $x_2$ for producing $y$. $B$ represents an observed input vector to be evaluated in relation to $\text{Isoq } L(y)$. The distance function $D(y, x)$ is the inverse of the ratio $OA/OB$: the proportion by which the inputs may be reduced in $B$ in order to reach the efficient point of the Isoquanta in $A$. $D(x, y)^{-1}$ is the measure of the relative efficiency in relation to input combination in $B$ for producing $y$.

In order to measure the efficiency, it is necessary to estimate the Isoquanta from the observed inputs and outputs and then evaluate the relative efficiency as to the best practice observed in the boundary. The measurement of the Farrel technical efficiency is similar to the resource Utilization Coefficient, Debreu (1951), Farrel (1957), see figure 2. This measure indicate the rate at which the production would be multiplied for a given input level. It is obtained by radially projecting the output vector over the boundary.

In [2] presented the application of a DEA model for ranking failure modes. The operational context was considered as the input factor and the output factors were the FMEA indices. In order to make the model even more discriminating, a partial aggregation with fixed severity model (APGF) has been developed, where the profiles were evaluated by combining occurrence and severity indices and later on severity and detection indices and finally the arithmetic mean of both risk indices established for each profile was evaluated. In this sense, the developed model could take into account the fact that the effects associated to the occurrence of given failure model be more important than the occurrence probability and the detection potential, if considered each at a turn. The combination of the occurrence and detection indices may turn to be as much important as the severity index. This approach aims at considering the risk perception when determining the failure mode criticality.
The aim here is to improve the model developed by Gracia et al. (2001a), so as to consider linguistic variables coming from expert opinion and keep a good discriminating power the failure modes and also uncertainties related to the variables.

2.2. Fuzzy DEA model

The inputs and outputs of the traditional DEA models are crisp number. Now fuzzy number are introduced so that part or all of the inputs and outputs are fuzzy numbers. The BCC model with fuzzy numbers and its dual are given below [10].

\[
\begin{align*}
\text{Max } & \mu^T \tilde{y}_0 - \mu_0 \\
\text{s.t. } & w^T \tilde{x} - \mu^T + \mu_0 \geq 0, \\
& w^T \tilde{x}_0 = 1 \\
& w, \mu \geq 0, \mu_0 \text{ unrestricted}
\end{align*}
\]

The dual program is formulated as

\[
\begin{align*}
\text{Min } \theta \\
\text{s.t. } & \tilde{x} \lambda \leq \theta \tilde{x}_0, \\
& \tilde{y} \lambda \geq \tilde{y}_0, \\
& \tilde{1} \lambda = 1 \\
& \lambda \geq 0, \theta \text{ free in sign}
\end{align*}
\]

Where

\[
\begin{align*}
\tilde{Y} &= \begin{pmatrix}
\tilde{y}_{11} & \tilde{y}_{12} & \cdots & \tilde{y}_{1s} \\
\tilde{y}_{21} & \tilde{y}_{22} & \cdots & \tilde{y}_{2s} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{y}_{n1} & \tilde{y}_{n2} & \cdots & \tilde{y}_{ns}
\end{pmatrix}, \\
\tilde{X} &= \begin{pmatrix}
\tilde{x}_{11} & \tilde{x}_{12} & \cdots & \tilde{x}_{1m} \\
\tilde{x}_{21} & \tilde{x}_{22} & \cdots & \tilde{x}_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{x}_{n1} & \tilde{x}_{n2} & \cdots & \tilde{x}_{nm}
\end{pmatrix}
\end{align*}
\]

\(\tilde{x}_i(i = 1, 2, \ldots, m)\) and \(\tilde{y}_j(r = 1, \ldots, s)\) are fuzzy input and output of DMU \(j = 1, 2, \ldots, n\), respectively, \(w\) and \(\mu\) refer to the \(m\) –dimension and \(s\) –dimension input and output column weight vector associated with the fuzzy inputs and outputs, respectively. \(\lambda\) in model (2.1) represent \(n\) –dimension column vector.

The fuzzy DEA model cannot be solved like a crisp model. In [11]. Adopted a possibility approach and \(\alpha\) –cut technique to convert the fuzzy CCR model to the standard linear programming (LP) problem.

\[
\begin{align*}
\text{Max } & \left(\mu^T \tilde{y}_0\right)_U^\alpha - \mu_0 \\
\text{s.t. } & \left(-w^T \tilde{x} + \mu^T\right)_L^\alpha \leq \mu_0 \leq 0, \\
& \left(w^T \tilde{x}_0\right)_U^\alpha \geq 1, \\
& \left(w^T \tilde{x}_0\right)_L^\alpha \leq 1, \\
& w, \mu \geq 0, \mu_0 \text{ unrestricted },
\end{align*}
\]

\[
\begin{align*}
\text{Min } \theta \\
\text{s.t. } & \left(\theta \tilde{x}_0 - \tilde{x} \lambda\right)_U^\alpha \geq 0, \\
& \left(\tilde{y} \lambda - \tilde{y}_0\right)_U^\alpha \geq 0, \\
& \tilde{1} \lambda = 1 \\
& \lambda \geq 0, \theta \text{ unrestricted}
\end{align*}
\]
Where $\alpha, \alpha_0, \tilde{a}_1$ and $\tilde{a}_2$ are all $\alpha-$cut levels used to convert associated fuzzy number to a crisp one. In model (2.2) and (2.3), fuzzy number are initially converted to an interval number with upper and lower bounds by use of $\alpha-$cut level technique [24]. The notation “$T$” is a transpose in (950). Based upon the interval number, linear programming (LP) models with crisp numbers are yielded in (2.2) and (2.3), where upper and lower bounds are represented by notation “U” and “L”. For example, $(\mu^T \tilde{y}_0)_{\beta}^U$ refers to upper bound of fuzzy number $\mu^T \tilde{y}_0$ at the $\beta -$cut level. Similar notations are viewed in models (2.2) and (2.3).

Following the same logic, the fuzzy BCC model with crispy number can be formulated as

Max $\left( \mu^T \tilde{y}_0 \right)^U_{\beta} + \mu^T y_0 - \mu_0$

$s.t. \left( -w^T \tilde{x} + \mu^T \tilde{y} \right)^L_{\alpha} - w^T x + \mu^T y - \mu_0 \leq 0,$

$\left( w^T \tilde{x}_0 \right)^U_{\alpha_0} + w^T x_0 \geq 1,$

$\left( w^T \tilde{x}_0 \right)^L_{\alpha_0} + w^T x_0 \leq 1,$

$w^T, w^T, \mu^T, \mu^T \geq 0, \mu_0 \text{ unrestricted}$

Min $\theta$

$\lambda, \theta$

$s.t. \left( \theta \tilde{x}_0 - \tilde{x} \lambda \right)^U_{\tilde{a}_1} + \theta x_0 - x \lambda \geq 0,$

$\left( \tilde{y} \lambda - \tilde{y}_0 \right)^U_{\tilde{a}_2} + Y \lambda - y_0 \geq 0,$

$\tilde{l} \lambda = 1$

$\lambda \geq 0, \theta \text{ unrestricted}.$

In models (2.4) and (2.5), inputs and outputs are clearly divided into two types, i.e. fuzzy type and crisp one, with $\tilde{x}, \tilde{y}, X$ and $Y$ referring to fuzzy input matrix, fuzzy output matrix, crisp input matrix and crisp output matrix, respectively. $w^T, \mu^T, w^T, \mu^T$ are weight vectors attached to them. All $\alpha -$cut levels $\alpha, \alpha_0, \tilde{a}_1$ and $\tilde{a}_2$ are those defined in models (2.2) and (2.3), the notation “$T$” is also a transpose in (2.4).

If the convexity constraint ($\tilde{l} \lambda = 1$) in (2.4) and the variable $\mu_0$ in (2.5) are removed, the feasible is enlarge, then we yield the following reduced fuzzy CCR model (2.6) and (2.7).

Max $\left( \mu^T \tilde{y}_0 \right)^U_{\beta} + \mu^T y_0$

$s.t. \left( -w^T \tilde{x} + \mu^T \tilde{y} \right)^L_{\alpha} - w^T x + \mu^T y \leq 0,$

$\left( w^T \tilde{x}_0 \right)^U_{\alpha_0} + w^T x_0 \geq 1,$

$\left( w^T \tilde{x}_0 \right)^L_{\alpha_0} + w^T x_0 \leq 1,$

$w^T, w^T, \mu^T, \mu^T \geq 0, \mu_0 \text{ unrestricted}$
Min $\theta$

$\lambda, \theta$

s.t.

$\left(\theta \bar{x}_0 - \bar{x}\lambda\right)_{\bar{a}_1}^U + \theta x_0 - X\lambda \geq 0,$

$\left(\bar{Y}\lambda - \bar{y}_0\right)_{\bar{a}_2}^U + Y\lambda - y_0 \geq 0,$

$\lambda \geq 0, \theta \text{ unrestricted}.$

The traditional DEA analysis requires a consistent operating environment in which the DMUs operate in bank branch evaluation studies. But the consistent culture does not exist when attempting a cross-region branch comparison. Branches in different regions may face completely different external environments, which exert significant influence to branches performance. This research overcome the above limitation by deeming all environmental variables as fuzzy variables. The proposed approach considers the influences by environmental variable and ensure that the branch’s ability to produce financial products at the customer level is comparable for all the regions. Environmental variables such as the income level and population density can be well represented by fuzzy data and thus the developed fuzzy DEA models can evaluate the branch’s performance from different provinces. Meanwhile, in this setting, branch performances from different subsystem, i.e., different provinces, are evaluated from a systematic point of view. Note that our models (2.4) and (2.5) differ from fuzzy models in [10] where only fuzzy numbers are taken into account.

3 Inputs and Outputs data:

In this study we are used 22 Pharmaceutical Companies which has accepted in Stock Exchange as a Statistical Society.

Sampling is not necessary because we are using from whole Pharmaceutical Companies.

By introducing the following inputs & outputs, we want to evaluate the efficiency of Pharmaceutical Companies:

Inputs data:
- The average cost of raw materials at intervals,
- The average discount for the whole order at once,
- The cost of repair and maintenance at the time,
- The costs arising from the return of the goods by the player and the entire outbound goods,

Outputs data:
- The annual sales rate (ratio)% net profit,
- Annual sales rate (ratio)% Gross profit,
- Percent based on sales growth in the period of time,
- Cash flow in the period of time.
4 FDEA model evaluate for ranking the pharmaceutical companies

By using CCR method (The fuzzy input) and TORA software, we evaluate the efficiency of Pharmaceutical companies. The results are as follows:

<table>
<thead>
<tr>
<th>No.</th>
<th>Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(0.48,0.56,0.68)</td>
</tr>
<tr>
<td>2</td>
<td>(0.24,0.39,0.53)</td>
</tr>
<tr>
<td>3</td>
<td>(0.27,0.58,0.71)</td>
</tr>
<tr>
<td>4</td>
<td>(0.42,0.71,1)</td>
</tr>
<tr>
<td>5</td>
<td>(0.37,0.56,0.79)</td>
</tr>
<tr>
<td>6</td>
<td>(0.11,0.24,0.51)</td>
</tr>
<tr>
<td>7</td>
<td>(0.46,0.81,1)</td>
</tr>
<tr>
<td>8</td>
<td>(0.21,0.36,0.47)</td>
</tr>
<tr>
<td>9</td>
<td>(0.43,0.50,1)</td>
</tr>
<tr>
<td>10</td>
<td>(0.29,0.49,0.59)</td>
</tr>
<tr>
<td>11</td>
<td>(0.14,0.26,0.41)</td>
</tr>
<tr>
<td>12</td>
<td>(0.22,0.46,0.69)</td>
</tr>
<tr>
<td>13</td>
<td>(0.31,0.61,1)</td>
</tr>
<tr>
<td>14</td>
<td>(0.27,0.41,0.52)</td>
</tr>
<tr>
<td>15</td>
<td>(0.11,0.23,0.61)</td>
</tr>
<tr>
<td>16</td>
<td>(0.42,0.61,0.73)</td>
</tr>
<tr>
<td>17</td>
<td>(0.24,0.56,1)</td>
</tr>
<tr>
<td>18</td>
<td>(0.17,0.56,0.66)</td>
</tr>
<tr>
<td>19</td>
<td>(0.16,0.21,0.39)</td>
</tr>
<tr>
<td>20</td>
<td>(0.51,0.76,1)</td>
</tr>
<tr>
<td>21</td>
<td>(0.44,0.70,0.92)</td>
</tr>
<tr>
<td>22</td>
<td>(0.24,0.46,0.82)</td>
</tr>
</tbody>
</table>

The result of Table 1 shows that, the companies with numbers 4,7,9,13,17, 20 are more efficient than the other companies.

For ranking the efficient units, we are using the following formula:

If $A_1$ and $A_2$ be two fuzzy numbers as $A_1 = (l_1, m_1, u_1)$, $A_2 = (l_2, m_2, u_2)$ then:

$A_1 < A_2$ and if $\frac{l_1 + 2m_1 + u_1}{4} < \frac{l_2 + 2m_2 + u_2}{4}$. Therefore, we have:

<table>
<thead>
<tr>
<th>Rank</th>
<th>Performance</th>
<th>Company No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.71</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>0.77</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>0.61</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>0.63</td>
<td>13</td>
</tr>
<tr>
<td>6</td>
<td>0.59</td>
<td>17</td>
</tr>
<tr>
<td>2</td>
<td>0.75</td>
<td>20</td>
</tr>
</tbody>
</table>

The result of Table 2 shows that, the company with number 7 has the first grade between the other Pharmaceutical Companies.
5 Conclusion

In this paper, we have presented FDEA method for ranking of pharmaceutical companies. Then, we compare twenty two pharmaceutical companies. In this work six pharmaceutical companies be efficient and another branches be not efficient. In this work we used FDEA model for evaluation these branches and in the end we obtain rank for these efficient pharmaceutical companies where the company with number seven has the first grade between the other Pharmaceutical Companies.

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