Credit Rating via Dynamic Slack-Based Measure And It’s Optimal Investment Strategy

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Abstract

In this paper we check the credit rating of firms applied for a loan. In this regard we introduce a model, named Dynamic Slack-Based Measure (DSBM) for measuring credit rating of applicant companies. Selection of financial ratios that represent the financial state of a company -in the best possible way- is one of the most challenging parts of any credit rating analysis. At first, ranking needs to identify the appropriate variables. Therefore we introduce five financial variables to provide a ranking. As noted above, we assess the performance of these firms. Then we introduce the dynamic model of SBM and theorems, also we discuss the overall structure of DSBM. Then we will present the implementation and the simulation model. After that, we propose a stochastic controlled dynamic system model to express the optimal strategy. Banks expect companies selected with DSBM model, act in accordance with this strategy. This stochastic dynamic system is originated from the balance sheets of firms applying for a loan. Finally we evaluate the performance of the system and strategy problem.

Keywords: Credit rating, Dynamic Slack-Based measure, Stochastic optimal control, Hamilton–Jacobi–Bellman (HJB) equation.

1 Introduction

Today, competition in the field of international trade and economic boundaries due to fading and other aspects of efforts to improve productivity based on economic rationality, Should be emphasized. Efficiency shows the extent to which an organization uses inputs optimally to produce outputs. One of the convenient and efficient tools in this context is DEA, a nonparametric method used to calculate efficiency. In Farrell (1957), we see the first approach of Data Envelopment Analysis (DEA). He combined productivity with the concept of efficiency. Credit rating for the first time was performed in 1909 on bonds [1]. In the 1980s and 1990s, mathematical programming model was introduced in most studies. These studies present an extension of the comprehensive (overall) concordance index of ELECTRE methods, which takes the interaction between criteria into account [8].
Data Envelopment Analysis includes various models, but SBM is the best among them. SBM model was introduced by Tone in 2001. He improved SBM to the dynamic model in 2010. In 2013, he could improve the study to the state of network structure [10], [11].

DEA is a simple and explicit instrument for optimization and some of its models such as SBM are dynamic and time dependent. Dependence on time in these models makes efficiency more accurate. But in many cases the accuracy is not sufficient and we need another instrument to make sure the model is useful. This instrument is stochastic optimal control and we can use it to have more accuracy.

Initially optimal control was applied to sciences such as economics and financial, but today it is used in most of them. Performance index and differential equation are control problems parts, to solve these problems we can use Hamilton–Jacobi–Bellman equation by using them [3], [6], [7].

Using optimal control in economics and financial is more attractive, for example finding a model for analyzing dynamic pairs trading strategies. This model is about a portfolio which consists of bank account and stocks. The main idea of this model is to maximize its wealth process [12].

Another attractive example is presenting production system consisting of two parallel machines with production-dependent failure rates. The main idea of the system is to find a productivity policy for both machines that will minimize the inventory and shortage costs over an infinite horizon [4].

As you have seen, optimal control and data envelopment analysis are suitable tools for evaluating. Therefore, finding a consolidated model of them is the main objective of this paper.

2 Preliminaries and notations

2.1. DSBM Model

Let there are n decision making units (DMU j; j=1,…,n) such that their production activities in the t courses (t=1,2,…,T) are tested and we assume that m, s, n good and n bad are the number of inputs, outputs, desirable (good) carry-over and undesirable (bad) carry-over [10].

![DSBM Model Diagram]

Figure 1: DSBM Model

Items are as follows:

a) $x_{ijt}$, represents m (i=1,…,m) optional inputs.

b) $x_{ijt}^{fix}$, represents p (i=1,…,p) non-optional inputs.

c) $y_{ijt}$, represents s (j=1,…,s) optional outputs.

d) $y_{ijt}^{fix}$, represents r (j=1,…,r) non-optional outputs.

e) $z_{ijt}^{good}$, represents desirable (good) carry-over. (i=1,…,ngood; j=1,…,n; t=1,…,T)

f) $z_{ijt}^{bad}$, represents undesirable (bad) carry-over. (i=1,…,nbad; t=1,…,T)

g) $z_{ijt}^{free}$, represents Discretionary (free) carry-over. (i=1,…,nfree; t=1,…,T)

h) $z_{ijt}^{fix}$, represents Non-discretionary (fixed) carry-over. (i=1,…,nfree; t=1,…,T)
Mathematical Model

In this section we describe a mathematical model including objective functions, data analysis and related optimal control problems for finding optimal investment strategy.
3.1. Objective functions and efficiency

We evaluate the overall efficiency of $\text{DMU}_1 (o=1, \ldots, n)$ taking $\{(\lambda^*)\}, \{s^-\}, \{s^+\}, \{s^{\text{good}}\}, \{s^{\text{bad}}\}, \{s^{\text{free}}\}$ as variables, in three orientations, input-oriented case, output-oriented case, non-oriented case.

a) Input-oriented case

Input-oriented case acts with minimum inputs and compliance minimal output. Overall performance $\theta^*$ is defined as,

$$\theta^* = \text{Min} \frac{1}{T} \sum_{t=1}^{T} w^t \left[ 1 - \frac{1}{m+\text{nbad}} \left( \sum_{i=1}^{m} w^t_i s^-_{i,ot} + \sum_{i=1}^{\text{nbad}} s^{\text{bad}}_{i,ot} \right) \right]. \quad (3.4)$$

Let (2.2) and (2.3) be established. $w^t$ and $w^t_i$ are weights term $t$, and input $i$ will be considered according to their importance and satisfy in the following conditions,

$$\sum_{t=1}^{T} w^t = T, \quad \sum_{i=1}^{m} w^t_i = m \quad (3.5)$$

Let an optimal solution of (4-2) be $\{\lambda^*, \{s^-\}, \{s^+\}, \{s^{\text{good}}\}, \{s^{\text{bad}}\}, \{s^{\text{free}}\}\}$ the term efficiency $\theta_{ot}^*$ define as follow,

$$\theta_{ot}^* = 1 - \frac{1}{m+\text{nbad}} \left( \sum_{i=1}^{m} w^t_i s^-_{i,ot} + \sum_{i=1}^{\text{nbad}} s^{\text{bad}}_{i,ot} \right), \quad (t=1,\ldots,T) \quad (3.6)$$

This formula expresses period efficiency. Overall efficiency $\theta^*$ during the period is weighted average of the efficiency period $\theta_{ot}^*$, which is shown as below,

$$\theta^* = \frac{1}{T} \sum_{t=1}^{T} w^t \theta_{ot}^* \quad (3.7)$$

b) Output-oriented case

The output-oriented overall efficiency $\tau^*$ is defined by

$$\frac{1}{\tau^*} = \text{Max} \frac{1}{T} \sum_{t=1}^{T} w^t \left[ 1 + \frac{1}{s+\text{ngood}} \left( \sum_{i=1}^{s} w^t_i s^+_{i,ot} + \sum_{i=1}^{\text{ngood}} s^{\text{good}}_{i,ot} \right) \right] \quad (3.8)$$

It should be noted, $w^t_i$ satisfies the following condition.

$$\sum_{i=1}^{s} w^t_i = s \quad (3.9)$$

Using an optimal solution $\{(\lambda^*), \{s^-\}, \{s^+\}, \{s^{\text{good}}\}, \{s^{\text{bad}}\}, \{s^{\text{free}}\}\}$, we define the term efficiencies $\theta_{ot}$ by

$$\tau_{ot} = \frac{1}{s+\text{ngood}} \left( \sum_{i=1}^{s} w^t_i s^+_{i,ot} + \sum_{i=1}^{\text{ngood}} s^{\text{good}}_{i,ot} \right) \quad (3.10)$$

Overall performance $\tau^*$ of the output oriented is weighted. Thus, the efficiency of the Period Performance $\tau^*$ is shown as below,

$$\frac{1}{\tau^*} = \frac{1}{T} \sum_{t=1}^{T} \frac{w^t}{\tau_{ot}} \quad (3.11)$$

c) Non-oriented case

As the combination of input- and output-oriented cases, we define the non-oriented efficiency measure by solving below program,
\[ \rho_0^* = \min \left\{ 1 + \frac{1}{\sum^{T}_{t=1} w^t} \left[ 1 - \frac{1}{m+nbad} \left( \sum^n_{i=1} w^i \left( s^t_{it} + s^\theta_{it} \right) + \sum^m_{i=1} \frac{s^t_{it} + s^\theta_{it}}{x^t_{it}} \right) \right] \right\} \]  

(3.12)

Using an optimal solution \( (\{\lambda^t\}, \{s^{-}\}, \{s^{+}\}, \{s^{good}\}, \{s^{bad}\}, \{s^{free}\}, \{s^{fix}\}) \) to (3.12), we define the non-oriented term efficiency by

\[ \rho_{ot} = \frac{1 + \frac{1}{s^{good} \sum^{T}_{t=1} w^t} \left[ 1 - \frac{1}{s^{bad} \sum^{T}_{t=1} w^t} \left( \sum^{T}_{t=1} \frac{s^t_{it} + s^\theta_{it}}{x^t_{it}} \right) \right] } {1 + \frac{1}{s^{good} \sum^{T}_{t=1} w^t} \left[ 1 - \frac{1}{s^{bad} \sum^{T}_{t=1} w^t} \left( \sum^{T}_{t=1} \frac{s^t_{it} + s^\theta_{it}}{x^t_{it}} \right) \right] } \]

(3.13)

**Theorem 3.1.** DMU\(_o\) is overall efficient, if and only if it is term efficient for all terms[9].

**Theorem 3.2.** The projected DMU\(_o\) is overall efficient [9].

**Proof.** Let \((\{\lambda^t\}, \{s^{-}\}, \{s^{+}\}, \{s^{good}\}, \{s^{bad}\}, \{s^{free}\}, \{s^{fix}\})\) be an optimal solution. We have:

\[ x^t_{it} = \sum^n_{j=1} x^t_{ijt} \lambda^j + s^t_{it} \quad (i=1, ..., m; t=1, ..., T) \]

\[ z^t_{it} = \sum^n_{j=1} z^t_{ij} \lambda^j + s^t_{it} \quad (i=1, ..., nbad; t=1, ..., T) \]

With replacement \( x^t_{it} \) (i = 1, ..., m) and \( z^t_{it} \) (i = 1, ..., nbad) in (3.1.14) we obtain

\[ x^t_{it} = \sum^n_{j=1} x^t_{ijt} \lambda^j + s^t_{it} \quad (i=1, ..., m; t=1, ..., T) \]

\[ z^t_{it} = \sum^n_{j=1} z^t_{ij} \lambda^j + s^t_{it} \quad (i=1, ..., nbad; t=1, ..., T) \]

Overall efficiency is as follows,

\[ \overline{\theta}_0^* = \frac{1}{T} \sum_{t=1}^{T} w^t \left[ 1 - \frac{1}{m+nbad} \left( \sum^n_{i=1} \frac{w^i \left( s^t_{it} + s^\theta_{it} \right)}{x^t_{it}} + \sum^m_{i=1} \frac{s^t_{it} + s^\theta_{it}}{x^t_{it}} \right) \right] \]

If any member of \((\{\overline{s^{-}_{it}}\}, \{\overline{s^{+}_{it}}\})\) is positive, the result is \( \overline{\theta}_0^* < \theta_0^* \). This contradicts the \( \theta_0^* \) optimality, so \( \overline{s^{-}_{it}} \)\( \overline{s^{+}_{it}} \) are zero, hence DMU\(_o\) is overall-efficient [9].

**3.2. Data Analysis**

To select the variables, we use the Goss perspective, (2009). Choosing variables must represent various aspects of the company such as firm liquidity, profitability, leverage, and asset structure [1].

**Table 1:** Output, input and link variable selecting

<table>
<thead>
<tr>
<th>Data</th>
<th>Name</th>
<th>Aspect</th>
<th>Formulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input 1</td>
<td>Total Liabilities to total Assets</td>
<td>Leverage</td>
<td>(Total Liabilities) / (Total Assets)</td>
</tr>
<tr>
<td>Input 2</td>
<td>Asset Turnover</td>
<td>Asset structure</td>
<td>Sales / (Total Assets)</td>
</tr>
<tr>
<td>Output 1</td>
<td>Working Capital to Total Assets</td>
<td>Liquidity</td>
<td>(Current Assets-Current Liabilities) / (Total Assets)</td>
</tr>
<tr>
<td>Output 2</td>
<td>Earnings before Taxes to Total Assets</td>
<td>Profitability</td>
<td>(Net Income + Income Tax + Interest Expense) / (Total Assets)</td>
</tr>
<tr>
<td>Badlink</td>
<td>Current Liabilities</td>
<td></td>
<td>...</td>
</tr>
</tbody>
</table>
3.2. Challenge
Companies will be selected using the above method. This selection is based on past information and we do not give a lot of confidence in the future of the company.
But how can we guarantee that the company will not be affected by the financial crisis?
For this reason, bank chooses a strategy for risky liquid asset portfolio of each company. When companies follow this strategy, they will be able to cover their debts in the future. Therefore bank expects companies to act in accordance with the proposed strategy.
We can find the proposed strategy using optimal control. Some numerical methods for solving optimal control have been investigated in [5], [6].

3.3. Optimal Control Problem
In the DSBM model we evaluate the efficiency of companies by using ratios. These assessments measure the company’s ability to pay its obligations such as loan and liabilities. Therefore we shall discuss the company’s ability to cover liquidity. In this regard, the following assumptions are made.

In this paper we consider the assumption used in [5]. Cash is the first type of stock of high-quality (liquid asset) that is made up of coins and bills. Other kinds of high-quality stocks are marketable securities which are held by the companies and the next one is each item of their commodities.
Net cash outflows are composed of all the debts that the company forced to pay.
Cash inflows are composed of all earnings from company’s investments and sales.

We are working with a filtered probability space \((\Omega, \mathcal{F}, \mathbb{P})\) with filtration \(\mathcal{F}_{t\in[0,T]}\) on a time index set \([0, T]\). Under \(\mathbb{P}\), \(\{W_t; 0 \leq t \leq T, W_0 = 0\}\) is an \(\mathcal{F}_t\)-Brownian motion.

Stochastic process of stock of high-quality liquid assets, \(x^1 = \Omega \times [0, T] \to \mathbb{R}^+\), at time \(t\), denoted by \(x^1_t\) and stochastic process of net cash outflows, \(x^2 = \Omega \times [0, T] \to \mathbb{R}^+\), at time \(t\), denoted by \(x^2_t\). Furthermore, for stochastic process \(x = \Omega \times [0, T] \to \mathbb{R}^2\) we use the notation,

\[
x_t = \begin{bmatrix} x^1_t \\ x^2_t \end{bmatrix},
\]

liquidity coverage ratio (LCR), \(l = \Omega \times [0, T] \to \mathbb{R}^+\), represents by

\[
l_t = \frac{x^1_t}{x^2_t}.
\]

It is important for the bank that \(l_t\) is adequately high to cover liquidity risk.
Consider

\[
dh_t = r^h_t dt + \sigma^h_t dW^h_t, \quad h(t_0) = h_0.
\]

Where the stochastic processes \(h : \Omega \times [0, T] \to \mathbb{R}^+\) are the return per unit of liquid assets, \(r^h \to \mathbb{R}^+\) is the rate of return per liquid asset unit, the scalar \(\sigma^h : \Omega \times [0, T] \to \mathbb{R}\) is the volatility in the rate of returns, and \(W^h : \Omega \times [0, T] \to \mathbb{R}\) is standard Brownian motion.

Suppose that liquid assets are held with \(n+1\) asset classes in financial market. One of these assets is riskless (cash) while the assets 1, 2, ..., \(n\) are risky.
The risky liquid assets modeled by using an \(n\)-dimensional Brownian motion. The asset returns in the \(k\)th liquid asset class per unit of the \(k\)th class is denoted by \(y^{k}_t\), \(k \in \mathbb{N}_n = \{0, 1, 2, ..., n\}\) where \(y : \Omega \times [0, T] \to \mathbb{R}^{n+1}\). Thus, the return per liquid asset unit is
Because one of the liquid assets being

\[ y = (C(t), y_1^t, ..., y_n^t), \] (3.17)

\[ \text{C}(t) \text{ is the return on cash and } y_1^t, ..., y_n^t \text{ are the risky return. The dynamics of } y \text{ is as} \]

\[ dy_t = r^y_t dt + \sum_{i=1}^{n} dW^y_i, \quad y(t_0) = y_0. \] (3.18)

Where \( r^y_T : T \rightarrow \mathbb{R}^{n+1} \) denotes the rate of liquid asset returns \( \sum_{i=1}^{n} y_i(t) \in \mathbb{R}^{(n+1)\times n} \) is a matrix of liquid asset returns, \( W^b : \Omega \times [0, T] \rightarrow \mathbb{R}^n \) is standard Brownian motion. Because one of the liquid assets being riskless; there are only n scalar Brownian motions. We assume that the investment strategy \( \pi : T \rightarrow \mathbb{R}^{n+1} \) is

\[ S = \{ \pi \in \mathbb{R}^{n+1} : \pi = (\pi^0, ..., \pi^n)^T, \pi^0 + \cdots + \pi^n = 1, \pi^0 \geq 0, ..., \pi^n \geq 0 \}. \] (3.19)

The liquid asset return is then \( h^y : \Omega \times R \rightarrow \mathbb{R}^+ \) where the dynamics of \( h \) can be written as

\[ dh_t = \pi^T r^y dt + \pi^T \sum_{i=1}^{n} dW^y_i, \] (3.20)

And after simplification we have

\[ dh_t = [r^C(t) + \pi^T r^y] dt + \pi^T \sum_{i=1}^{n} dW^y_i, \quad h(t_0) = h_0. \] (3.21)

The stochastic process \( u^1 : \Omega \times [0, T] \rightarrow \mathbb{R}^+ \) is the normal cash inflow rate per net cash inflow unit that \( u^1 \), is its value at time \( t \). A notion related to the cash inflow rate per unit of the net cash inflow rate for a higher or lower LCR, \( u^2 : \Omega \times [0, T] \rightarrow \mathbb{R}^+ \) that \( u^2 \), is its value at time \( t \). We denote the cash inflow rate \( u^3 : \Omega \times [0, T] \rightarrow \mathbb{R}^+ \), by

\[ u^3 = u^1 + u^2, \] (3.22)

For all \( t \in [0, T] \). The cash inflow \( u^3 \) is predictable with respect to \( \{ \mathcal{F} \}_{t \geq 0} \). The cash inflow provides us with a means of controlling LCR dynamics. The dynamics of the cash outflow per unit of the net cash outflow, \( e : \Omega \times [0, T] \rightarrow \mathbb{R} \), is given by

\[ de_t = r^e_t dt + \sigma^e dW^e_t, \quad e(t_0) = e_0. \] (3.23)

\( e^i \) is the cash outflow per unit of the net cash outflow, \( r^e : T \rightarrow \mathbb{R} \) is the rate of outflow per unit of the net cash outflow, the scalar \( e^i : T \rightarrow \mathbb{R} \) is the volatility in the outflow per net cash outflow unit, and \( W^e : \Omega \times [0, T] \rightarrow \mathbb{R}^+ \) is standard Brownian motion. Next, we take \( i : \Omega \times [0, T] \rightarrow \mathbb{R}^+ \) as the net cash outflow increase before cash outflow per monetary unit of the net cash outflow, \( r^1 : T \rightarrow \mathbb{R}^+ \) is the rate of increase of net cash outflows before cash outflow per net cash outflow unit, the scalar \( i \) is the volatility in the increase of net cash outflows before outflow, and \( W^i : \Omega \times [0, T] \rightarrow \mathbb{R} \) represents standard Brownian motion. Then, we set

\[ di_t = r^i_t dt + \sigma^i dW^i_t, \quad i(t_0) = i_0. \] (3.24)

We introduce the dynamics of liquid assets \( x_1^t \) and net cash outflow \( x^2_t \), as follows

\[ dx_1^t = x_1^t dh_t + x_2^t du^1^t dt - x_2^t^2 dt, \]
\[ = [r^C(t)x_1^t + x_1^t \pi^T r^y + x_1^t u^1 + x_2^t u^2 - x_2^t r^e] dt + [x_1^t \pi^T \sum_{i=1}^{n} dW^y_i - x_2^t \sigma^e dW^e_t]. \] (3.25)

\[ dx_2^t = x_2^t^2 dt - x_2^t^2 de_t = x_2^t [r^i_t - r^e_t] dt + x_2^t [\sigma^i dW^i_t - \sigma^e dW^e_t]. \]

We want to build dynamic of liquidity coverage ratio. First, we explain the basics.
Theorem 3.3. (Ito Formula)
Suppose that $X(\cdot)$ has a stochastic differential
$$dX = F \, dt + G \, dW,$$
then
$$Y(t) := u(X(t), t).$$

For $F \in L^1(0, T), G \in L^2(0, T)$. Assume $u: \mathbb{R} \times [0, T] \to \mathbb{R}$ is continuous and that $\frac{\partial u}{\partial t}, \frac{\partial u}{\partial x}, \frac{\partial^2 u}{\partial x^2}$ exist and are continuous.

Then $Y$ has the stochastic differential
$$dY = \frac{\partial u}{\partial t} \, dt + \frac{\partial u}{\partial x} \, dx + \frac{1}{2} \frac{\partial^2 u}{\partial x^2} \, G^2 \, dt = \left( \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} F + \frac{1}{2} \frac{\partial^2 u}{\partial x^2} G^2 \right) \, dt + \frac{\partial u}{\partial x} G \, dW.$$

We call the above formula Ito’s formula or Ito’s chain rule [9].

Theorem 3.4. (Ito Product Rule)
Suppose
$$\begin{align*}
    dX_1 &= F_1 \, dt + G_1 \, dW \\
    dX_2 &= F_2 \, dt + G_2 \, dW (0 \leq t \leq T),
\end{align*}$$
For $F_i \in L^1(0, T), G_i \in L^2(0, T) \ (i = 1, 2)$. Then
$$d(X_1 X_2) = X_2 \, dX_1 + X_1 \, dX_2 + G_1 G_2 \, dt, \ [8].$$

We use two above theorems and create a dynamic for $l_t = \frac{x_t^1}{x_t^2}$.
$$d \left( \frac{x_t^1}{x_t^2} \right) = \left( -\frac{x_t^1}{x_t^2} F_2 + \frac{(G_2)^2}{(x_t^2)^2} x_t^1 + \frac{F_1}{x_t^2} \right) dt + \left( \frac{G_1}{x_t^2} \frac{x_t^1}{(x_t^2)^2} G_2 - \frac{G_1 G_2}{(x_t^2)^2} \right) dW_t.$$

Now, with replacement (3.25) in the above equations and use the following notation
$$W_t = \begin{bmatrix}
W_t^y \\
W_t^e \\
W_t^l
\end{bmatrix}, \ \tilde{c}_t = \begin{bmatrix}
\tilde{y}_t \\
\tilde{e}_t \\
\tilde{l}_t
\end{bmatrix},$$
we get
$$dl_t = l_t \left[ c^e(t) + r_t - r^e_t + (\sigma^y)^2 + (\sigma^l)^2 + \frac{1}{2} \bar{\pi}_t^T \bar{\pi}_t \right] dt + [u_1^l + u_2^l + (\sigma^e)^2 - r_t^e] \, d\tilde{W}_t.$$ 

$d l_t$ is liquidity coverage ratio dynamics. Now we need to get performance index to cover the company’s liquidity.

Definition 3.1. A nondecreasing, concave utility function is defined on the set of real numbers. $Ln(x)$ is a utility function, which is defined for $x > 0$. A whole class of utility functions by choosing a number $p < 1$, $p \neq 0$, and another number $c \in \mathbb{R}$, is defined as follows
$$U_p(x) = \begin{cases}
\frac{1}{p} (x - c)^p, & \text{if } x > c, \\
0, & \text{if } 0 < p < 1 \text{ and } x = c, \\
-\infty, & \text{if } p < 0 \text{ and } x = 0, \\
-\infty, & \text{if } x < c.
\end{cases}$$
According to [2] for these functions, the index of absolute risk aversion \( \frac{u''(x)}{u'(x)} \) is the hyperbolic function \( \frac{1-p}{x-c} \) for \( x>c \).

This class of functions is called the HARA. The HARA function corresponding to \( p=0 \) is

\[
U_0(x) = \begin{cases} 
\ln(x-c), & \text{if } x > c, \\
0 & \text{if } 0 < p < 1 \text{ and } x = c.
\end{cases}
\]

Now according to the utility function (Hara) create a performance index in this form that have liquidity coverage ratio, \( l_t \), and security allocation strategy, \( \hat{m}_t \), as follows

\[ EU_p(l_t) \]

We need to maximize the utility function above for each company. In this way we can achieve to the best liquidity strategy that indicates the most optimal weights are invested in liquid assets for each of that selected companies [3].

4 Numerical examples

In this paper, we use the dynamic SBM model to evaluate seven companies that are assumed, apply for a loan. These companies have been examined in four consecutive years.

In this example we use Excel and Dea-solver software. This example is composed of two inputs, two outputs and one badLink. Due to the presence of badLink, input oriented approach is used to calculate the overall and period efficiency. Using 3.1.1 and Conditions 2.1.2 and 2.1.3, the objective function and constraints in the first year of the first company are as follows,

\[
\text{Min } 1 - \left( \frac{1}{3} \left( \frac{s^-_1}{0.8067} + \frac{s^-_2}{0.4443} + \frac{s_{\text{bad}}}{0.0473} \right) \right)
\]

\[
0.8067\lambda_1 + 0.6912\lambda_2 + 0.5911\lambda_3 + 0.6952\lambda_4 + 0.609\lambda_5 + 0.5696\lambda_6 + 0.9497\lambda_7 + s^-_1 = 0.8067
\]

\[
0.4443\lambda_1 + 0.8163\lambda_2 + 0.11754\lambda_3 + 0.10022\lambda_4 + 0.58029\lambda_5 + 0.36642\lambda_6 + 0.39049\lambda_7 + s^-_2 = 0.4443
\]

\[
-0.2563\lambda_1 + 0.2764\lambda_2 + 0.12833\lambda_3 + 0.1464\lambda_4 + 0.3033\lambda_5 + 0.038\lambda_6 + 0.1209\lambda_7 + s_{\text{bad}}^+ = -0.2563
\]

\[
0.0552\lambda_1 + 0.2239\lambda_2 + 0.1131\lambda_3 + 0.207\lambda_4 + 0.0152\lambda_5 + 0.1366\lambda_6 + 0.0037\lambda_7 + s_{\text{bad}}^- = 0.0552
\]

\[
0.0048\lambda_1 + 0.0245\lambda_2 + 0.0202\lambda_3 + 0.0198\lambda_4 + 0.0223\lambda_5 + 0.0480\lambda_7 = 0
\]

\[
0.0473\lambda_1 + 0.0437\lambda_2 + 0.0418\lambda_3 + 0.0499\lambda_4 + 0.0989\lambda_5 + 0.06 + 0.47\lambda_7 + s_{\text{bad}} = 0.0473
\]

\[
\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7 \geq 0, s^-_1 \geq 0, s^-_2 \geq 0, s_{\text{bad}}^+ \geq 0, s_{\text{bad}}^- \geq 0
\]

Above device provides the performance in the first year of Company one. By changes in the objective function and right hand side values we can count efficiency of other companies in the first year.

Variables \( \lambda_1 \ldots \lambda_7 \) indicate the Dea weights of seven Iranian companies, Tehran shims, Daroosazi Farabi, Sanati Bootan, Daroobakhsh, Palayesh Naft Shiraz, Palayesh Naft Esfahan and Palayesh Naft Bandarabbas. These companies applied for a loan. In this example we evaluate the efficient companies. The real data used for this example are as follows [13].
<table>
<thead>
<tr>
<th>Company</th>
<th>Input1</th>
<th>Input2</th>
<th>Link</th>
<th>Output1</th>
<th>Output2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-Tehran Shimi</td>
<td>0.8067</td>
<td>0.4443</td>
<td>842486</td>
<td>-0.2563</td>
<td>0.0552</td>
</tr>
<tr>
<td>2-Daroosazi Farabi</td>
<td>0.6912</td>
<td>0.8163</td>
<td>777742</td>
<td>0.2764</td>
<td>0.2239</td>
</tr>
<tr>
<td>3-Sanati Bootan</td>
<td>0.5911</td>
<td>1.1754</td>
<td>744277</td>
<td>0.1283</td>
<td>0.1131</td>
</tr>
<tr>
<td>4-Daroobakhsh</td>
<td>0.6952</td>
<td>1.0022</td>
<td>888981</td>
<td>0.146</td>
<td>0.207</td>
</tr>
<tr>
<td>5-Palayesh Naft Shiraz</td>
<td>0.6090</td>
<td>5.8029</td>
<td>1760847</td>
<td>0.3033</td>
<td>0.0152</td>
</tr>
<tr>
<td>6-Palayesh Naft Esfahan</td>
<td>0.5696</td>
<td>3.6642</td>
<td>17805300</td>
<td>0.038</td>
<td>0.1366</td>
</tr>
<tr>
<td>7-Palayesh Naft Bandarabbas</td>
<td>3.9497</td>
<td>3.9049</td>
<td>8368623</td>
<td>0.1209</td>
<td>0.0037</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Company</th>
<th>Input1</th>
<th>Input2</th>
<th>Link</th>
<th>Output1</th>
<th>Output2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-Tehran Shimi</td>
<td>0.7874</td>
<td>0.3644</td>
<td>893373</td>
<td>-0.2062</td>
<td>0.0749</td>
</tr>
<tr>
<td>2-Daroosazi Farabi</td>
<td>0.3965</td>
<td>0.3456</td>
<td>1038234</td>
<td>0.1569</td>
<td>0.1005</td>
</tr>
<tr>
<td>3-Sanati Bootan</td>
<td>0.6390</td>
<td>1.2026</td>
<td>959112</td>
<td>0.105</td>
<td>0.0538</td>
</tr>
<tr>
<td>4-Daroobakhsh</td>
<td>0.7174</td>
<td>0.7184</td>
<td>1099762</td>
<td>0.1325</td>
<td>0.179</td>
</tr>
<tr>
<td>5-Palayesh Naft Shiraz</td>
<td>0.6090</td>
<td>5.8029</td>
<td>1998085</td>
<td>0.3033</td>
<td>0.0152</td>
</tr>
<tr>
<td>6-Palayesh Naft Esfahan</td>
<td>0.5851</td>
<td>3.6852</td>
<td>28442037</td>
<td>0.1307</td>
<td>0.2082</td>
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<tr>
<td>7-Palayesh Naft Bandarabbas</td>
<td>0.3859</td>
<td>4.1697</td>
<td>8879665</td>
<td>0.1494</td>
<td>0.0268</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Company</th>
<th>Input1</th>
<th>Input2</th>
<th>Link</th>
<th>Output1</th>
<th>Output2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-Tehran Shimi</td>
<td>0.7584</td>
<td>0.3913</td>
<td>1016804</td>
<td>-0.0986</td>
<td>0.0812</td>
</tr>
<tr>
<td>2-Daroosazi Farabi</td>
<td>0.6208</td>
<td>0.6127</td>
<td>1035502</td>
<td>0.2829</td>
<td>0.1662</td>
</tr>
<tr>
<td>3-Sanati Bootan</td>
<td>0.7124</td>
<td>1.3039</td>
<td>1450330</td>
<td>0.1317</td>
<td>0.0518</td>
</tr>
<tr>
<td>4-Daroobakhsh</td>
<td>0.762</td>
<td>0.6355</td>
<td>1501338</td>
<td>0.1017</td>
<td>0.1561</td>
</tr>
<tr>
<td>5-Palayesh Naft Shiraz</td>
<td>0.7207</td>
<td>4.7232</td>
<td>3992010</td>
<td>0.1817</td>
<td>0.1275</td>
</tr>
<tr>
<td>6-Palayesh Naft Esfahan</td>
<td>0.5704</td>
<td>2.9136</td>
<td>37463395</td>
<td>0.2048</td>
<td>0.0679</td>
</tr>
<tr>
<td>7-Palayesh Naft Bandarabbas</td>
<td>0.4982</td>
<td>4.5891</td>
<td>14380362</td>
<td>0.1228</td>
<td>0.0186</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Company</th>
<th>Input1</th>
<th>Input2</th>
<th>Link</th>
<th>Output1</th>
<th>Output2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-Tehran Shimi</td>
<td>0.7368</td>
<td>0.3791</td>
<td>1173307</td>
<td>-0.0986</td>
<td>0.0812</td>
</tr>
<tr>
<td>2-Daroosazi Farabi</td>
<td>0.6456</td>
<td>0.593</td>
<td>1394636</td>
<td>0.2829</td>
<td>0.1774</td>
</tr>
<tr>
<td>3-Sanati Bootan</td>
<td>0.7337</td>
<td>1.1005</td>
<td>3179286</td>
<td>0.1051</td>
<td>0.0560</td>
</tr>
<tr>
<td>4-Daroobakhsh</td>
<td>0.8053</td>
<td>0.6225</td>
<td>2210289</td>
<td>-0.0152</td>
<td>0.1255</td>
</tr>
<tr>
<td>5-Palayesh Naft Shiraz</td>
<td>0.7589</td>
<td>3.4112</td>
<td>6840500</td>
<td>0.1940</td>
<td>0.1018</td>
</tr>
<tr>
<td>6-Palayesh Naft Esfahan</td>
<td>0.5778</td>
<td>1.1434</td>
<td>44614277</td>
<td>0.2315</td>
<td>0.0679</td>
</tr>
<tr>
<td>7-Palayesh Naft Bandarabbas</td>
<td>0.5480</td>
<td>2.9924</td>
<td>30231451</td>
<td>0.4599</td>
<td>0.2443</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Company</th>
<th>Overall Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-Tehran Shimi</td>
<td>0.78827386</td>
</tr>
<tr>
<td>2-Daroosazi Farabi</td>
<td>0.86116832</td>
</tr>
<tr>
<td>3-Sanati Bootan</td>
<td>0.70657486</td>
</tr>
<tr>
<td>4-Daroobakhsh</td>
<td>0.91211037</td>
</tr>
<tr>
<td>5-Palayesh Naft Shiraz</td>
<td>0.82063261</td>
</tr>
<tr>
<td>6-Palayesh Naft Esfahan</td>
<td>0.86116832</td>
</tr>
<tr>
<td>7-Palayesh Naft Bandarabbas</td>
<td>0.78827386</td>
</tr>
</tbody>
</table>

Table 2: 1388 (solar year)

Table 3: 1389 (solar year)

Table 4: 1390 (solar year)

Table 5: 1391 (solar year)

Table 6: Overall and period input-efficiency
Now selected companies (Tehran Simi and Daroosazi Farabi) have been known through the DSBM model. These companies ranking, provided based on the past company’s information. But how can we guarantee that the company will not be affected by the financial crisis? For this reason, the bank chooses a strategy for risky liquid asset portfolio of each selected company. Following this strategy, companies are able to cover their debts in the future. Therefore bank expects companies to act in accordance with the proposed strategy. In this regard, for each selected Companies we will solve the following control problem,

$$\text{Max } \mathbb{E} U_p(l_t), (p < l, p \neq 0)$$

$$dl_t = l_t \left[ r^C(t) + r^e_t - r^l_t + (\sigma^e)^2 + (\sigma^l)^2 + \bar{n}^T_t r^e_t \right] dt + \left[ u^1_t + u^2_t + (\sigma^e)^2 - r^e_t \right] dt$$

$$+ \left[ (\sigma^e)^2 (1 - l_t)^2 + (\sigma^e)^2 x^2_t + (l_t)^2 \bar{n}^T_t \bar{C}_t \bar{n}_t \right]^{1/2} dW_t.$$ 

$l(t_0) = l_0$. 

$x^1_0$ is the amount of cash that are held in the liquid assets portfolio, and $x^2_0$ is the first payment of debt at the beginning of the year.

This control problem can be solved by Hamilton–Jacobi–Bellman (HJB) equation, in which $\bar{n}^T_t$ is the control factor [3]. It should be mentioned, selected companies (Tehran Simi and Daroosazi Farabi) data for LCR dynamic model are closed and confidential, but these companies present them to the bank in a financial crisis for the loan.

5 Conclusion

In this paper, we have presented DSBM model and its theorems. Then we explained how to select data. A numerical example has been solved and we have the rankings of seven companies. After that a question arised as follows, How can we guarantee that selected companies will not be affected by the financial crisis? Hence banks provide an optimal liquid asset portfolio strategy for selected companies by stochastic optimal control. It should be mentioned; companies must accept and perform this strategy.

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