Journal of Data Envelopment Analysis and Decision Science 2014 (2014) 1-24

ISPACS
International Scientific Publications and Consulting Services

Available online at www.ispacs.com/dea
Volume: 2014, Year 2014 Article ID: dea-00063, 24 Pages
doi:10.5899/2014/dea-00063
Research Article

DEA&DS
Data Envelopment Analysis and Decision Science

A location-routing problem model with multiple periods and fuzzy demands

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Abstract
This paper puts forward a dynamic capacitated location-routing problem with fuzzy demands (DCLRP-FD). It is given on input a set of identical vehicles (each having a capacity, a fixed cost and availability level), a set of depots with restricted capacities and opening costs, a set of customers with fuzzy demands, and a planning horizon with multiple periods. The problem consists of determining the depots to be opened only in the first period of the planning horizon, the customers and the vehicles to be assigned to each opened depot, and performing the routes that may be changed in each time period due to fuzzy demands. A fuzzy chance-constrained programming (FCCP) model has been designed using credibility theory and a hybrid heuristic algorithm with four phases is presented in order to solve the problem. To obtain the best value of the fuzzy parameters of the model and show the influence of the availability level of vehicles on final solution, some computational experiments are carried out. The validity of the model is then evaluated in contrast with CLRP-FD's models in the literature. The results indicate that the model and the proposed algorithm are robust and could be used in real world problems.

Keywords: Capacitated location-routing problem, Fuzzy demand, Dynamic, Credibility theory, Stochastic Simulation.

1 Introduction

Distribution systems in supply chain management comprise all operations related to the transportation of final products from plants to customers, considering all intermediate steps, such as the ones relating to depots and distribution centers. The location of depots and the distribution of products from the depots to customers are two key components of each distribution system [1]. Many researchers indicated that if the routes are ignored while locating the depots, the costs of distribution systems might be immoderate [2-4].
At first, Salhi and Rand firstly showed that the solving of the location problem without route consideration may lead to a sub-optimal solution [5]. In operations research context, the location-routing problem (LRP) overcomes this drawback by simultaneously considering the location and routing decisions [6,7]. The LRP

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combines the facility location problem (FLP) and the vehicle routing problem (VRP) [8,9]. Since both FLP and VRP belong to the class of NP-hard problems [10], the LRP is also an NP-hard problem [11,12]. In most general form, the LRP seeks to minimize the total cost by simultaneously selecting a subset of potential depots and making routes by allocating customers to different depots while satisfying the following constraints: (i) each customer's demand is satisfied, (ii) each customer is visited by exactly one route, and is assigned to exactly one depot without exceeding vehicle or depot capacities, (iii) each route begins and ends at the same depot [13].

The LRP is applicable to a wide variety of fields such as food and soft drink distribution, newspapers delivery, waste collection, bill delivery, military applications, parcel delivery, drug distribution and various consumer goods distribution [14-16]. Laporte [17] was the first researcher who discussed and classified the LRP models. Min et al. [18] reviewed the LRP literature using a hierarchical classification based on the problem characteristics such as the number of depots, the capacity of depots and vehicles, the form of the objective function, etc. Nagy and Salhi [19] also performed a comprehensive literature review on the LRP models, solution approaches, application areas and some future works.

In this paper, the dynamic capacitated location-routing problem with fuzzy demands (DCLRP-FD) is considered. In the DCLRP-FD, a planning horizon with multiple periods is included to LRP. In this problem, depots can only be opened at the beginning of the planning horizon and remain unchanged throughout the planning horizon. On the other hand, the routing of vehicles can be changed at each period due to fluctuations of demands. Moreover, vehicles are heterogeneous and have an availability level and a maximum travel distance. The vehicles and depots have a limited capacity to serve the customers that their demands change in each time period. Furthermore, it is assumed that demands of customers are fuzzy variables. The objective is to minimize the total cost of opening depots and routing the vehicles (i.e., transportation and fixed costs). A fuzzy chance-constrained programming is designed based upon the fuzzy credibility theory to model the DCLRP-FD. The high complexity of this problem makes it impossible to be solved in practice with commercial software. For this reason, a hybrid heuristic algorithm (HHA) with four phases including the stochastic simulation is proposed to solve the problem. To the best of our knowledge, this paper is the first work in the field of LRP that consider both the planning horizon and uncertainty for the customers' demand.

This paper is organized as follows, in Section 2 a discussion about previous related works is presented and some basic concepts of fuzzy theory are given in Section 3. The problem description and its mathematical formulation are comprised in Section 4. The solution approach description is given in Section 5, while the generation of data instances and computational results are presented in Section 6. In Section 7 some concluding remarks are offered.

2 Literature review

The dynamic location-routing problem is an important area of the LRP which has not been addressed much in the literature. The static (single-period) LRP is very much prone to the criticism that the planning horizons of the location and routing do not match. It is important to note that, the LRP integrates the strategic (facility location) and tactical (vehicle routing) levels and balancing strategic with tactical objectives is the challenge [20]. Location decisions are usually quite stable in time, because of implementation costs and set-up times. On the other hand, routing decisions (even master tour decisions at a tactical level) are more often changed than location decisions, especially when they refer to the transportation of goods to customers with varying demands [21]. Therefore, by considering a planning horizon for facility location that contain shorter planning intervals for route planning, dynamic LRPs are a much better model of real-life location problems with routing aspects and provide an important means of refuting the above criticism [19].

The first effort on dynamic LRP dates back to the research of Laporte and Dejax [22]. They considered multiple planning periods for the LRP, so that in each period both the locations and the routes may be changed. They proposed an ingenuous network representation of the problem. The resulting network optimization problem was then solved by exact and heuristic approaches. Salhi and Nagy [23] assumed that the depots were fixed throughout the planning horizon but the vehicle routes changed following changes in customers' demand. It was also assumed that the customer set did not change. In their work, a number of solution approaches were investigated. Ambrosino and Scutella [24] considered a multi-level LRP with static and dynamic planning horizons and applied commercial software to solve the integer linear programming (ILP) formulation of the problem.

Prodhon [4] considered the periodic location-routing problem. The objective of the problem was to determine the set of depots to be opened, the combination of service days to be assigned to customers and the routes originating from each depot for each period of the horizon, in order to minimize the total cost. To solve large size instances of the periodic location-routing problem, a hybrid evolutionary algorithm was proposed. The algorithm was hybridized with a heuristic based on the randomized extended Clarke and Wright algorithm to create feasible solutions. Finally, the proposed method was evaluated over three sets of instances and the results showed that it outperforms the previous methods. Albareda-Sambolaet al. [21] presented the multiperiod location-routing problem with decoupled time scales. Their problem was defined over a finite time horizon, in which location and routing decisions were made at different time scales. They also assumed that locations could be opened or modified only in some selected time periods of the planning horizon and then they remain unchanged during the time periods between them. Due to the complexity of the model, they proposed an approximation based on replacing vehicle routes by spanning trees, and its capability for providing good quality solutions was assessed in a series of computational experiments.

Dynamic problems divide the planning horizon into multiple periods. Normally within the planning horizon there is some uncertainty about some of the parameters (typically the customers' demand). Very few works in literature have been done on stochastic or fuzzy LRP. At first Laporte et al. [25] described a family of stochastic location-routing problems which consist of a set of customers having random supplies. They assumed two stages for the problem: In the first stage, decisions regarding depot location, fleet size and planned routes were made without knowing the actual supplies. Since it was possible that the total supply of a route exceed the vehicle capacity in this stage, a corrective recourse action was taken at the second stage: (i.e., the vehicle returned to the depot and empties its load before resuming its journey). The problems was modeled as (ILP) and solved to optimality. Albareda-Sambola et al. [26] also considered a version of stochastic LRP in which a set of potential customers was given but only a subset of them would require service after a priori decision was made. The uncertainty of the problem was modeled by using a vector of independent Bernoulli random variables as the demand vector. In their work, a two-phase heuristic was developed. An initial feasible solution was built by solving a sequence of sub-problems, and an improvement phase was then applied. They also developed a lower bound based on bounding separately different parts of the cost of any feasible solution.

In this paper a fuzzy version of dynamic LRP is considered. In operations research context, recently fuzzy logic has been used to some problems. The need to use fuzzy logic arises whenever there are some vague or uncertain parameters in the problem. Moreover, credibility theory has been used in with fuzzy parameters so far, in parallel with some meta-heuristics to solve the problem (see [27]). There is some works on the LRP with fuzzy variables in the literature. The work of Zarandi et al. [28] is the first attempt to model the LRP using fuzzy variables and credibility theory along with a meta-heuristic technique as a tool for solving the problem. They presented a LRP in which travel time between two nodes was a fuzzy variable and the problem was solved using simulated annealing (SA) approach. Their proposed method was tested using a standard test problem of LRP and the results showed that the method was robust and could be used in real world problems. In the second work, Fazel Zarandi et al. [29] considered the location-routing problem with time windows under uncertainty. They assumed that demands of customers and travel times were fuzzy variables. In their work, a fuzzy chance-constrained programming model was designed using credibility theory and a simulation-embedded SA algorithm was presented in order to solve the problem. To initialize solutions of SA, a heuristic method based on fuzzy c-means clustering with Mahalanobis distance and sweep method was employed. They attested the proposed solution approach with some numerical experiments. In next work, Zare Mehrjerdi and Nadizadeh [27] considered the LRP with fuzzy demands. They modeled the problem with a fuzzy chance-constrained programming based upon the fuzzy credibility theory. To solve this problem, a greedy clustering method (GCM) including the stochastic simulation was proposed. In the proposed GCM, iterative and clustering approaches were used to solve the problem. To obtain the best value of the dispatcher preference index of the model and to analyze its influence on the final solution, numerical experiments were carried out. Consequently, to show the performance of their proposed method, associated results were compared with the lower bound of the solutions. In work of Ghaffari-Nasab et al. [30], the location-routing problem with fuzzy demands was considered, and a fuzzy chance-constrained program was designed to model it, based on the fuzzy credibility theory. A hybrid SA based heuristic incorporated with stochastic simulation was developed and proposed to solve the problem. The efficiency of the solution procedure was then demonstrated via comparing its performance with those of some other existing solution procedures from literature using a standard benchmark set of test problems.

3 Fuzzy credibility theory

The concept of fuzzy set was initiated by Zadeh [31] via the membership function and applied to the wide varieties of real problems thereafter. To measure a fuzzy event, the term fuzzy variable was proposed by Kaufmann [32] and later Zadeh [33] proposed the possibility measure theory of fuzzy variable. Although, possibility measure has been widely used, it has no self-duality property. However, a self-dual measure is absolutely necessary in both theory and practice. In order to define a self-dual measure, a modified form of the possibility theory called credibility theory was founded by Liu [34] and studied very recently by many scholars all around the world. Since a fuzzy version of dynamic LRP with credibility theory will be modeled in this paper, a brief introduction on the basic concepts and definitions used are presented as follows:
Let Θ be a nonempty set, and P the power set of Θ. Each element in P is called an event, and ϕ is an empty set. In order to present an axiomatic definition of possibility, it is necessary to assign a number $\text{Pos}\{A\}$ to each event A, which indicates the possibility that A will occur. To ensure that the number $\text{Pos}\{A\}$ has certain mathematical properties, the following four axioms are approved [34]:

Axiom 3.1 $\text{Pos}\{\Theta\} = 1$;

Axiom 3.2 $\text{Pos}\{\phi\} = 0$;

Axiom 3.3 For each $A_i \in p(\Theta)$, $\text{Pos}\left\{\bigcup_{i=1}^{n} A_i\right\} = \sup_i \text{Pos}\{A_i\}$;

Axiom 3.4 If Θ_i is a non-empty set, and the set function $\text{Pos}_i\{\}$; i = 1, 2, ..., n, satisfies above three axioms, and $\Theta = \Theta_1 \times \Theta_2 \times \cdots \times \Theta_n$, then for each $A \in p(\Theta)$, $\text{Pos}\{A\} = \sup_{(\theta_1,\theta_2,\ldots,\theta_n)\in A} \text{Pos}_1\{\theta_1\} \wedge \text{Pos}_2\{\theta_2\} \wedge \cdots \text{Pos}_n\{\theta_n\}$.

The above four axioms form the basis of credibility measure theory, all concepts of credibility theory can be obtained from them [34].

Definition 3.5. *[34]. Let (Θ, P(Θ), Pos) be a possibility space, and A be a set in p(Θ), then the necessity measure of A is defined by Nec{A} = 1 –Pos{*A^c*}.*

Definition 3.6 *[34]. Let (Θ, P(Θ), Pos) be a possibility space, and A be a set in p(Θ), then the credibility measure of A is defined by Cr{A}* $= \frac{1}{2}$ *(Pos{A} + Nes{A}).*

Considering definition 3.6, the credibility of a fuzzy event is defined as the average of its possibility and necessity. A fuzzy event may fail even though its possibility achieves 1, and hold even though its necessity is 0. However, the fuzzy event must hold if its credibility is 1, and fail if its credibility is 0 [34]. The credibility measure is self-dual, and in the theory of fuzzy subsets, the law of credibility plays a role similar to that played by the law of probability in measurement theory for ordinary sets [35].

Now let consider a triangular fuzzy variable $\tilde{d} = (d_1, d_2, d_3)$ for demand of a customer that $\tilde{d}$ is described by its left boundary d_1, its right boundary d_3, and the value of d_2 corresponding to a grade of membership of 1. Thus, the dispatcher or analyst studying that problem can subjectively estimate, based upon his experience and intuition and/or available data, the demand of the customer will not be less than d_1 or greater than d_3. If the actual demand of a customer is considered by r, the possibility, necessity, and credibility are easily obtained as follows [36]:

$$\text{Pos}\,\{\tilde{d} \geq r\} = \begin{cases} 1, & \text{if } r \leq d_2 \\ \dfrac{d_3 - r}{d_3 - d_2}, & \text{if } d_2 \leq r \leq d_3 \\ 0, & \text{if } r \geq d_3 \end{cases} \tag{3.1}$$

$$\text{Nec}\,\{\tilde{d} \geq r\} = \begin{cases} 1, & \text{if } r \leq d_1 \\ \dfrac{d_2 - r}{d_2 - d_1}, & \text{if } d_1 \leq r \leq d_2 \\ 0, & \text{if } r \geq d_2 \end{cases} \tag{3.2}$$

$$\text{Cr}\,\{\tilde{d} \geq r\} = \begin{cases} 1, & \text{if } r \leq d_1 \\ \dfrac{2d_2 - d_1 - r}{2(d_2 - d_1)}, & \text{if } d_1 \leq r \leq d_2 \\ \dfrac{d_3 - r}{2(d_3 - d_2)}, & \text{if } d_2 \leq r \leq d_3 \\ 0, & \text{if } r \geq d_3 \end{cases} \tag{3.3}$$

4 Fuzzy chance-constrained programming model for the DCLRP-FD

In this section definition and assumptions of the DCLRP-FD is detailed and the mathematical formulation is developed. In the DCLRP-FD, there is a time horizon with multiple periods that in each time period the demand of every customer should be supplied by a single vehicle and the total load of each route must not exceed the definite capacity of the vehicle. The routes start and end to the same depot and the total load of all allocated customers to a depot must be less than or equal to the limit capacity of that depot. Vehicles are heterogeneous and have the maximum travel distance. Furthermore, each vehicle in each time period has an availability level that varies within [0,1]. This means that a vehicle can serve some customers in a part time of a period. The main objective of the problem is to minimize the total cost of the system, taking the costs of depot, routing costs and lost opportunity costs due to the lack of vehicle capacity into consideration.

In the DCLRP-FD, in addition to the above assumptions, the customers' demand at each period is a triangular fuzzy number such as $\tilde{d} = (d_1, d_2, d_3)$. To model the problem with fuzzy credibility theory, the fuzzy number representing demand of the j^{th} customer at the t^{th} period is denoted by $\tilde{d}_j^t = (d_{1j}^t, d_{2j}^t, d_{3j}^t)$ Let each vehicle has a limited capacity that is denoted by Q_k. After serving the first c customers at period t, the available capacity of the vehicle will equal $Q_{kk} = Q_k - \sum_{j=1}^{c} \tilde{d}_j^t$ in which Q_{kk} is also a triangular fuzzy number by using the rules of fuzzy arithmetic, and

$$Q_{kk} = \left(Q_k - \sum_{j=1}^{c} d_{3j}^t,\ Q_k - \sum_{j=1}^{c} d_{2j}^t,\ Q_k - \sum_{j=1}^{c} d_{1j}^t \right) = (q_{1,kk}^t,\ q_{2,kk}^t,\ q_{3,kk}^t).$$

The credibility that the next customer demand does not exceed the remaining capacity of the vehicle can be obtained as follows:

$$Cr\left\{\tilde{d}_{c+1}^t \le Q_{kk}\right\} = Cr\left\{\left(d_{1,c+1}^t - q_{3,kk}^t,\ d_{2,c+1}^t - q_{2,kk}^t,\ d_{3,c+1}^t - q_{1,kk}^t\right) \le 0\right\} \tag{4.4}$$

$$Cr\left\{\tilde{d}_{c+1}^t \le Q_{kk}\right\} = \begin{cases} 0, & \text{if } d_{1,c+1}^t \ge q_{3,kk}^t \\ \dfrac{q_{3,kk}^t - d_{1,c+1}^t}{2\times(q_{3,kk}^t - d_{1,c+1}^t + d_{2,c+1}^t - q_{2,kk}^t)}, & \text{if } d_{1,c+1}^t \le q_{3,kk}^t,\ d_{2,c+1}^t \ge q_{2,kk}^t \\ \dfrac{d_{3,c+1}^t - q_{1,kk}^t - 2\times(d_{2,c+1}^t - q_{2,kk}^t)}{2\times(q_{2,kk}^t - d_{2,c+1}^t + d_{3,c+1}^t - q_{1,kk}^t)}, & \text{if } d_{2,c+1}^t \le q_{2,kk}^t,\ d_{3,c+1}^t \ge q_{1,kk}^t \\ 1, & \text{if } d_{3,c+1}^t \le q_{1,kk}^t \end{cases} \tag{4.5}$$

Similarly, let the capacity of i^{th} candidate depot is given by P_i. After allocating c customers to the i^{th} depot, the available capacity of the depot at period t will equal $P_{ii} = P_i - \sum_{j=1}^{c} \tilde{d}_j^t$ in which P_{ii} is also a triangular fuzzy number by using the rules of fuzzy arithmetic, and

$$P_{ii} = \left(P_i - \sum_{j=1}^{c} d_{3j}^t,\ P_i - \sum_{j=1}^{c} d_{2j}^t,\ P_i - \sum_{j=1}^{c} d_{1j}^t \right) = (p_{1,ii}^t,\ p_{2,ii}^t,\ p_{3,ii}^t).$$

The credibility that the next allocated customer demand does not exceed the remaining capacity of the depot can be shown as follows:

$$Cr\left\{\tilde{d}_{c+1}^t \le P_{ii}\right\} = Cr\left\{\left(d_{1,c+1}^t - p_{3,ii}^t,\ d_{2,c+1}^t - p_{2,ii}^t,\ d_{3,c+1}^t - p_{1,ii}^t\right) \le 0\right\} \tag{4.6}$$

$$Cr\left\{\tilde{d}^{t}_{c+1} \le P_{ii}\right\} = \begin{cases} 0, & \text{if } d^{t}_{1,c+1} \ge p^{t}_{3,ii} \\ \dfrac{p^{t}_{3,ii} - d^{t}_{1,c+1}}{2\times(p^{t}_{3,ii} - d^{t}_{1,c+1} + d^{t}_{2,c+1} - p^{t}_{2,ii})}, & \text{if } d^{t}_{1,c+1} \le p^{t}_{3,ii},\ d^{t}_{2,c+1} \ge p^{t}_{2,ii} \\ \dfrac{d^{t}_{3,c+1} - p^{t}_{1,ii} - 2\times(d^{t}_{2,c+1} - p^{t}_{2,ii})}{2\times(p^{t}_{2,ii} - d^{t}_{2,c+1} + d^{t}_{3,c+1} - p^{t}_{1,ii})}, & \text{if } d^{t}_{2,c+1} \le p^{t}_{2,ii},\ d^{t}_{3,c+1} \ge p^{t}_{1,ii} \\ 1, & \text{if } d^{t}_{3,c+1} \le p^{t}_{1,ii} \end{cases} \tag{4.7}$$

There is no doubt that if the remaining goods in the vehicle is high and the demand at the next customer is low, then the vehicle's chance of being able to finish the next customer's service become greater. This means that the greater the difference between available goods and demand at the next customer, the greater preference to send the vehicle to serve the next customer. According to formulation (4.5), the preference index is designated by *Cr* which denotes the magnitude of the preference for sending the vehicle to the next customer after it served current customer. It is obvious that $Cr \in [0, 1]$. When $Cr = 0$ driver is completely sure that he should return to the depot. When $Cr = 1$, the driver is absolutely certain that he can serve the next customer by the remaining goods having in his vehicle. Let the dispatcher preference index designated by *DPI*, where $DPI \in [0, 1]$. So, according to the *DPI* value and the credibility that the next customer demand does not exceed the remaining capacity of the vehicle, a decision must be made as to whether to send the vehicle to the next customer or return that to the depot. Thus, if relation $Cr \ge DPI$ is fulfilled, then the vehicle should be sent to the next customer; otherwise, the vehicle should be returned to the depot, and send it back again to the next customer after loading sufficient goods.

Similarly, in formulation (4.7) if the depot's remaining capacity for serving customers is high and the demand at the next customer being low, then the depot's chance of being able to serve the next customer become greater. This means that the greater the difference between the available capacity of the depot and the demand at the next customer, the greater the preference to allocate the next customer to the depot for receiving the service. The preference index is designated by *Cr* with the value of $Cr \in [0,1]$. When $Cr = 0$, then the depot manager is completely sure that he should not accept the next customer for giving the service. On the other hand, when $Cr = 1$, the depot manager is absolutely certain that he can serve the next customer. Let the assignment preference index for allocating customers to a depot is designated by *API*, $API \in [0, 1]$. So, according to the *API* value and the credibility that the next customer demand does not exceed the remaining capacity of the depot, a decision must be made as to whether to allocate it to the current depot or the next opened depot should accept it. Thus, if the relation $Cr \ge API$ is fulfilled, then the depot should serve the next customer; otherwise, the customer should receive service from another opened depot.

Moreover, the planned routes are designed in advance by applying the proposed hybrid heuristic algorithm. But, the actual value of a customer demand is only known when the vehicle reaches the customer. Due to demand uncertainty at the customers, a vehicle might not be able to serve a customer once it arrives there due to insufficient capacity when the vehicle implements the planned route. It is assumed that in such situations the vehicle returns to its depot to load itself and then returns to the customer where it had a "failure" and continue its service along the rest of the planned route. This arises an additional distance due to route failure. Hence, an additional distance should be considered for the vehicle due to the "failure" arises at some customers' locations along the route when evaluating the planned route [27].

Both parameters *DPI* and *API* which are empirically determined have an extremely great impact on both the total length of the planned routes and on the additional distance. For example, lower values of parameter *DPI* express the dispatcher's desire to use vehicle capacity the best he can. These values result in shorter planned distances. Moreover, lower values of parameter *DPI* increase the number of circumstances where a vehicle meet a customer but is unable to serve that, thereby increasing the total distance it covers due to the "failure". In this work, stochastic simulation is used to evaluate the additional distance due to route failure. On the other hand, higher values of parameter *DPI* are characterized by less utilization of vehicle capacity along the planned routes and less additional distance to cover due to failures. Furthermore, higher values of *DPI* increase the number of vehicles that are used. In other hand, it may be due to the unavailability of vehicles, the cost of lost opportunity increases due to the not serving of some customers. As a result, the sensitive parameters *DPI* and *API* significantly influence the sum of planned route lengths, additional distances and lost opportunity costs that should be determined properly by modeling the problem.

The notations used to represent the mathematical programming formulation for the DCLRP-FD are as follows:

Sets and parameters:

I: Set of candidate depots indexed by i and $I = \{1, 2, ..., M\}$, that M is the number of candidate depots.

J: Set of customers indexed by j and $J = \{1, 2, ..., N\}$, that N is the number of customers.

V: Set of all points: $V = I \cup J$ and $V = \{1, 2, ..., M, M+1, M+2, ..., M+N\}$.

E: Set of arcs (i,j) connecting every pair of nodes $i, j \in V$.

K: Set of vehicles indexed by k and $K = \{1, 2, ..., \bar{K}\}$, that $\bar{K}$ is the number of exiting vehicles.

T: Set of time periods indexed by t and $T = \{1, 2, ..., \bar{T}\}$, that $\bar{T}$ is the number of periods.

$\tilde{d}_j^t$: Demand of customer j in period t.

P_i: Capacity of i^{th} depot.

Q_k: Capacity of k^{th} vehicle.

O_i: Fixed cost of opening a depot at the candidate site i.

F_k: Fixed cost of employing the vehicle k.

c_{ij}: Traveling cost associated with arc $(i, j) \in E$.

B_j^t: Cost of the lost opportunity for the lack of serving to customer j in period t.

s_j: Time of serving to customer j.

f_k: Additional distances traveled by vehicle k.

D: Maximum travel distance of vehicles.

A_k^t: Availability level of vehicle k in period t and $A_k^t \in [0,1]$.

Decision variables:

$$x_{ijk}^t = \begin{cases} 1 & \text{if vehicle } k \text{ in period } t \text{ goes directly from customer } i \text{ to customer } j \\ 0 & \text{other wise} \end{cases}$$

$$Y_{ij}^t = \begin{cases} 1 & \text{if in period } t \text{ the demand of customer } j \text{ is served by the depot } i \\ 0 & \text{other wise} \end{cases}$$

$$Z_i = \begin{cases} 1 & \text{if a depot at candidat site } i \text{ is opened} \\ 0 & \text{other wise} \end{cases}$$

U_{jk}^t = Auxiliary variables for sub-tour elimination constraints in route k.

The corresponding fuzzy chance-constrained programming (FCCP) formulation of the DCLRP-FD based upon the credibility theory is shown as follows:

$$Minimize \quad \sum_{i\in I} O_i Z_i + \sum_{i\in I}\sum_{j\in J}\sum_{k\in K}\sum_{t\in T} F_k x_{ijk}^t + \sum_{i\in V}\sum_{j\in V}\sum_{k\in K}\sum_{t\in T} c_{ij} x_{ijk}^t + \dots$$

$$\dots + \sum_{j\in J}\sum_{t\in T}\Big(1-\sum_{i\in V}\sum_{k\in K} x_{ijk}^t\Big)B_j^t \tag{4.8}$$

$$Minimize \quad \sum_{k\in K} f_k \tag{4.9}$$

Subject to

$$Cr\left(\sum_{i\in V}\sum_{j\in J}\tilde{d}_j^t x_{ijk}^t \le Q_k\right) \ge DPI \qquad \forall\ k\in K;\ \forall\ t\in T \tag{4.10}$$

$$Cr\left(\sum_{j\in J}\tilde{d}_j^t Y_{ij}^t \le P_i Z_i\right) \ge API \qquad \forall\ i\in I;\ \forall\ t\in T \tag{4.11}$$

$$\sum_{i\in V}\sum_{k\in K} x_{ijk}^t \le 1 \qquad \forall\ j\in J;\ \forall\ t\in T \tag{4.12}$$

$$U_{lk}^t - U_{jk}^t + N x_{ljk}^t \le N-1 \qquad \forall\ l,j\in J;\ \forall\ k\in K;\ \forall\ t\in T \tag{4.13}$$

$$\sum_{j\in V} x_{ijk}^t - \sum_{j\in V} x_{jik}^t = 0 \qquad \forall\ i\in V;\ \forall\ k\in K;\ \forall\ t\in T \tag{4.14}$$

$$\sum_{i\in I}\sum_{j\in J} x_{ijk}^t = 1 \qquad \forall\ k\in K;\ \forall\ t\in T \tag{4.15}$$

$$\sum_{u\in J} x_{iuk}^t + \sum_{u\in V\setminus\{j\}} x_{ujk}^t \le 1 + Y_{ij}^t \qquad \forall\ i\in I;\ \forall\ j\in J;\ \forall\ k\in K;\ \forall\ t\in T \tag{4.16}$$

$$\sum_{i\in V}\sum_{j\in V}(c_{ij}+s_j)x_{ijk}^t + f_k \le A_k^t D \qquad \forall\ k\in K;\ \forall\ t\in T \tag{4.17}$$

$$x_{ijk}^t \in \{0,1\} \qquad \forall\ i\in V;\ \forall\ j\in V;\ \forall\ k\in K;\ \forall\ t\in T \tag{4.18}$$

$$Y_{ij}^t \in \{0,1\} \qquad \forall\ i\in I;\ \forall\ j\in J;\ \forall\ t\in T \tag{4.19}$$

$$Z_i \in \{0,1\} \qquad \forall\ i\in I \tag{4.20}$$

$$U_{jk}^t \in \{N\cup 0\} \qquad \forall\ j\in J;\ \forall\ k\in K;\ \forall\ t\in T \tag{4.21}$$

The objective function (4.8) represents the sum of the fixed depot location costs, the fixed costs of employing vehicles, the travel costs, total cost of the lost opportunities, respectively. The objective function (4.9) shows the total additional travel distances. Note that, the total additional travel distances of vehicle k, denoted by f_k, can be obtained by stochastic simulation algorithm in section 5.4.2. Fuzzy chance-constraints (4.10) and (4.11) assure that all customers are visited within vehicle capacity and are allocated within depot capacity with a confidence level, respectively. Constraints (4.12) stated that a customer should be served within one route only and should have only one predecessor, if it received the service. The sub-tour elimination constraints are assured in (4.13). The continuity of the routes and return to the original depot are guaranteed through constraints (4.14). Constraints (4.15) ensure every vehicle k must be used in each time period t once. Constraints (4.16) ensure that a customer must be assigned to a depot if there is a route connecting it. Constraints (4.17) express the limitation of travel distance of vehicles. Note

that, the service times should be transformed to the distance scale in constrains (4.17). Constraints (4.18), (4.19), and (4.20) specify the binary variables used in the formulation and finally, auxiliary variables taking positive values are declared in (4.21).
Figure 1 depicts the relationship between the DCLRP-FD and its special cases. For example, if the number of period is considered 1 (i.e., T=1), the DCLRP-FD is reduced to the CLRP-FD. In the other hand, after preliminary setting such as T=1, M=1 and deterministic demand for the customers, the master problem changed to the capacitated vehicle routing problem (CVRP). Other problems related to the DCLRP-FD are shown in Figure 1.

5 Proposed heuristic algorithm for the DCLRP-FD

A hybrid heuristic algorithm (HHA) is presented in this section to solve the DCLRP-FD. In general, HHA consists of four phases in each time period, which is illustrated in Figure 2. In the first phase, the depots are opened between the candidate depots sites based on the coordination of customers (Figure 2(a)). In the second phase, customers are clustered using a greedy search algorithm (Figure 2(b)). The clusters are allocated to the opened depot(s) in the third phase, considering the distance between the depots and the gravity center of the clusters as well as the capacity of the depots (Figure 2(c)). Finally, in the fourth phase an admissible tour between each cluster and depot is formed by ant colony system (ACS) (Figure 2(d)). In this phase, the stochastic simulation is also used to determine the actual demands of customers. The problem is initialized by defining a plane comprising the set of depots, M, customers, N, and their coordinate points. The hybrid heuristic algorithm is repeated for a predefined number of iterations. When algorithm obtains a better solution, it replaced with the last best known solution. Moreover, since in the second phase of HHA, the first customer in each cluster is selected randomly, the constituted clusters in each iteration of the proposed algorithm are different together. Thus, the proposed algorithm can search some feasible solutions among all the solution space. This can ensure that HHA avoid confining suboptimal solutions. Details of HHA are described in the following sections.

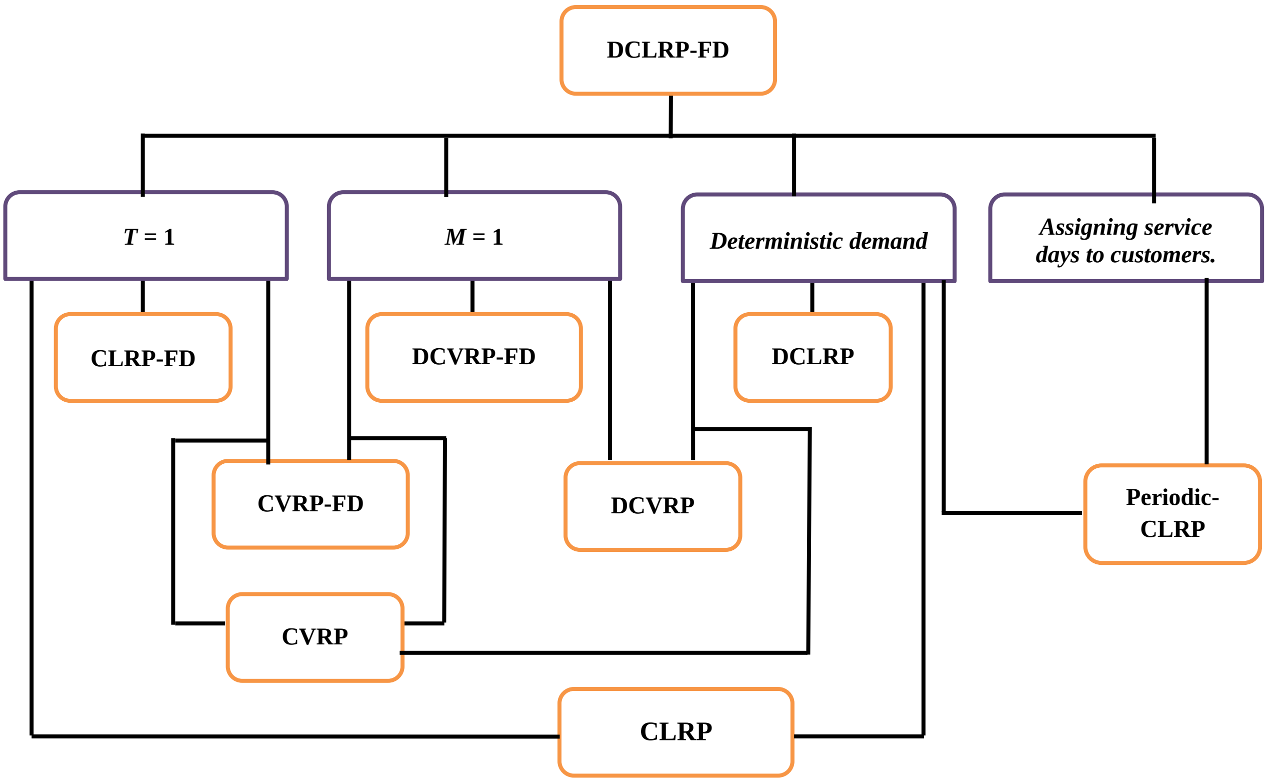

Figure 1: Relationship between the DCLRP-FD and its variants.

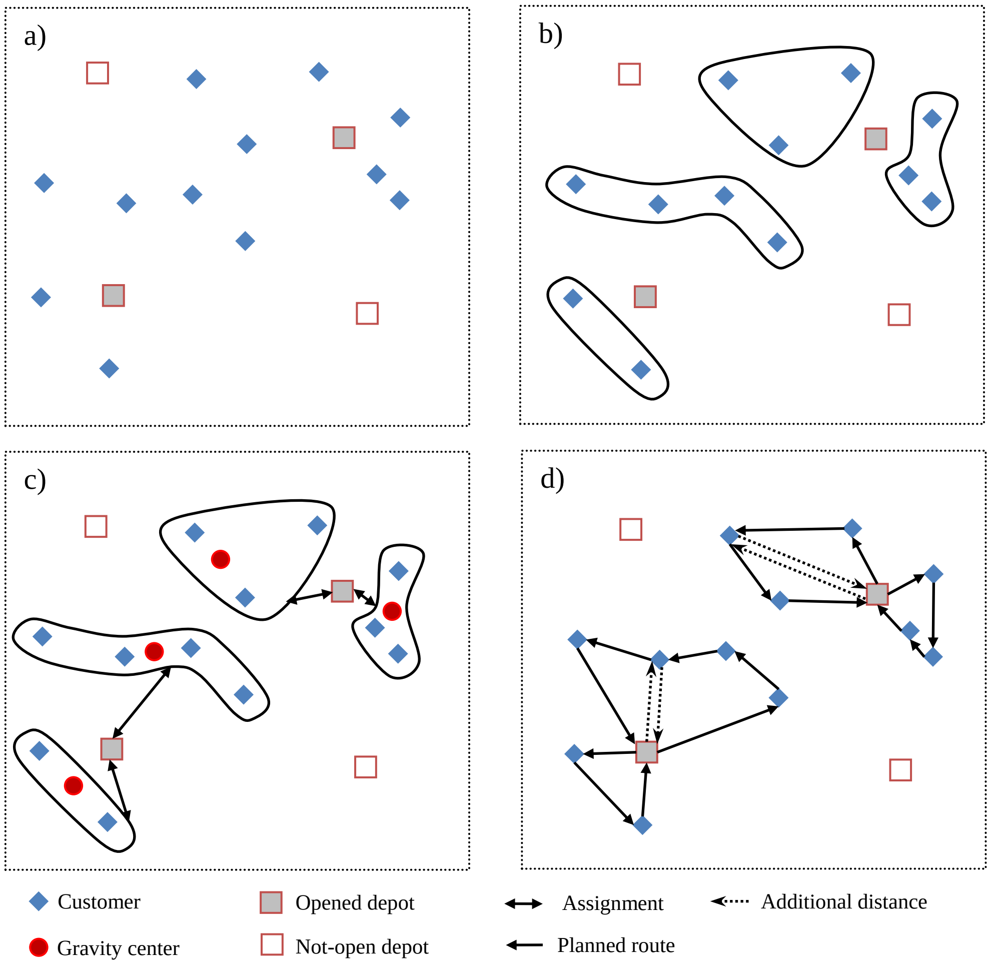

Figure 2: Illustrative procedure of the HHA in each period.

5.1. Establishing the depot(s)

As mentioned before, in the beginning of the planning horizon of the DCLRP-FD, the depot(s) should be opened and should be fixed during the planning horizon. Thus, the first phase of the HHA searches among the potential sites to establish the depot(s). Firstly, the sum of distances between the locations of customers and each potential site is calculated. Secondly, the potential sites are sorted according to their Euclidean distance to customers and their capacity. The Euclidean distance is calculated by equation (5.22). In this equation, w_i is the total Euclidean distance between potential site i and the locations of customers, (x_i,y_i) is the coordinates of potential site i, (a_j,b_j) is the coordinates of customer j, N is the number of customers, and M is the number of potential sites.

$$w_i = \sum_{j=1}^{N} \left[(x_i - a_j)^2 + (y_i - b_j)^2 \right]^{1/2} \qquad \forall\ i = 1, \ldots, M \tag{5.22}$$

For each potential depot, the amount of factor $\dfrac{P_i}{O_i \times w_i}$ is calculated. In this factor, P_i and O_i are the capacity and fixed cost of i^{th} depot, respectively and w_i is the amount of equation (5.22). After that, the depots are sorted in a descending order and ranked from 1 to M. Finally, the top-ranked potential site, that has the maximum amount of $\dfrac{P_i}{O_i \times w_i}$, is selected to be established. The number of depots that should be opened in this phase depends on the total upper bound of fuzzy demands. To do this, for each period of t

the value of $d_t = \sum_{j=1}^{N} d_{3j}^t$ is calculated and the maximum value of d_t, denoted by d^*, is then selected. Consequently, the depots in sorted list should be opened one by one until the total capacity of opened depots can support the value of d^*.

5.2. Clustering the customers

The second phase of the HHA is the clustering of the customers. The customers are grouped considering their intra distance, their fuzzy demands and the capacity of the vehicles. A greedy search algorithm is used to form a cluster of customers. At first, to form a cluster, a customer is selected randomly from the set of non-clustered customers belongs to *N*. The algorithm searches for the nearest customer to the last selected customer of the current cluster. The nearest customer is not assigned to the cluster if its demand exceeds the remaining capacity of the vehicle, considering the *DPI* value and the credibility of the customer. When a new customer is selected to be assigned to a cluster, total fuzzy demand of current members of the cluster is calculated and compared to the capacity of the vehicle. If the relation $Cr \geq DPI$ is fulfilled (according to the formulation (4.10)), the new customer is allowed to assign to the current cluster. Otherwise, last customer is withdrawn from the cluster. The greedy search algorithm searches for a new customer close to the last added member of the cluster among the ungrouped customers of *N*. This procedure helps to use the maximum capacity of a vehicle. The algorithm forms a new cluster if there is no customer to be assigned to current cluster considering the capacity of vehicle and fuzzy demand of customers. When there is no unassigned customer, the process of clustering stops.
It is important to note that, since the capacity of vehicles is not equal (i.e., vehicles are heterogeneous), the clustering of customers may be different together in terms of the number of customers and their total demands in each cluster.

5.3. Allocating clusters to depot(s)

In the third phase of HHA, the clusters are respectively allocated to the opened depots. Each depot serves as many clusters as possible, based on the *API* value and the credibility that the next cluster demand does not exceed the remaining capacity of the depot. To allocate the clusters, the Euclidian distance of gravity center of each cluster to the first opened depot is calculated. The gravity center of each cluster is calculated according to equation (5.23), in which $(a_{(g)},b_{(g)})$ is the coordinates of the gravity center of cluster g, (a_j,b_j) is the coordinates of customer j, and n_g is the number of customers assigned to cluster g.

$$\left(a_{(g)},b_{(g)}\right)=\left(\frac{\sum_{j\in g}a_j}{n_g},\frac{\sum_{j\in g}b_j}{n_g}\right) \tag{5.23}$$

Afterwards, the unassigned clusters are ranked in an ascending order based upon the distance of their gravity centers to the depot. The top-ranked cluster is allocated to the first opened depot, if the relation $Cr \geq API$ is fulfilled (according to the formulation (4.11)). If there is an empty capacity for the current opened depot, the second-ranked cluster is allocated to the depot considering the above relation. The allocation process to the depot will be finished when there is not enough capacity to allocate new cluster. In this situation, the allocating procedure is repeated for the next-opened depot until all clusters are allocated.

5.4. Routing

In the fourth and last phase of the HHA the routing problem is solved for each cluster with the relevant depot. The routing problem of the DCLRP-FD is the same as traveling salesman problem (TSP), which is solved by using ACS.

5.4.1 Ant colony system

ACS is referred to ants' treatment to find food [37]. The ants spread a material called pheromone and put it on their way so that other ants can pass the same route. The pheromone of shorter route increases due to lower evaporation and therefore more ants move from that way. Artificial ants construct a solution by selecting a customer to visit sequentially until all the customers in a route are visited. Ants select the next customer to visit using a combination of heuristic and pheromone information. A local updating rule is applied to modify the pheromone on the selected route, during the construction of a route. When all ants construct their tours, the amount of pheromone of the best selected route and the global best solution, are updated according to the global updating rule. More details on ACS can be found in [38,39].
As mentioned before, the demand of each customer is a triangular fuzzy number, so it cannot be directly considered as a deterministic number like other algorithms that solve the deterministic CLRP. Since the real value of demand is identified as the vehicle reaches the customer, the simulation experiment is used to determine the deterministic value of each customer demand. The actual demands help the decision maker to evaluate the planned routes designed by fuzzy demands. Moreover, for each feasible planned route that the solution of the HHA stands for, additional distances due to route failures (f_k) are obtained by a stochastic simulation algorithm.

5.4.2 Stochastic Simulation

To reveal the "actual" demand of each customer and to determine the additional distances due to route failures, the following stochastic simulation with four steps is proposed:

Step 1: For each customer in each time period, estimate the additional distances by simulating "actual" demands. The "actual" demands were generated by following processes: (1) randomly generate a real number d in the interval between the left and right bounds of the triangular fuzzy number representing demand of the customer, and compute its membership m; (2) generate a random number r; $r \in [0,1]$; (3) compare r and m, if $r \leq m$, then "actual" demand of the customer is adopted as d; otherwise, it is not accepted. In this case, random numbers d and r are generated again and again until random numbers d and r are found such that relation $r \leq m$ is satisfied; (4) check and repeat (1) till (3), and terminate the process when each customer in each time period has a simulation "actual" demand quantity.

Step 2: For each time period, move along the route designed by ACS and calculate the additional distance due to route failures in terms of the "actual" demands.

Step 3: Repeat Steps 1 and 2 R times. In this work, the proper value of R is considered 400 after some computational experiments.

Step 4: For each time period, compute the average additional distances that comes out of simulation, and return it as the additional distance.

Note that, the routing cost of the DCLRP-FD consists of two amounts: additional distances and planned routes distances. In the DCLRP-FD, each planned routes distances between the depots and allocated clusters are obtained by ACS and additional distances are calculated by stochastic simulation algorithm.

6 Computational results

6.1. Sensitivity analysis on both parameters DPI and API

In this section, some numerical experiments are given to show the performance of the DCLRP-FD's model and the efficiency of the HHA. In the first experiment, to evaluate the sensitivity of parameters *DPI* and *API* on solving the model, different size of instances is considered to conduct computational experiments. It is assumed that there are 30 customers and 5 candidate depots for a small size instance and 100 customers and 7 candidate depots for a large size instance. In two examples, the coordinates of all customers and depots are generated randomly in [100×100]. Furthermore, six time periods are considered in two test instances and the fuzzy demands of customers in each time period, which are triangular fuzzy numbers such as $\tilde{d}_j^t = (d_{1j}^t, d_{2j}^t, d_{3j}^t)$, are selected randomly. In such a way, d_{1j}^t, d_{2j}^t and d_{3j}^t are generated within [10,35], [36,60] and [61,110], respectively. The number of vehicles available is 5 and 10 for small and large-size test instances, respectively. Moreover, in two instances, the availability level of each vehicle is considered 1, the capacity of vehicles and depots are selected equally and maximum travel distances of each vehicle are given 240 in each time period. The relative data for two test instances are listed in Table 1. Note that, in Table 1, the serving and loading times of vehicles are expressed based on the distance scale. Consequently, the name of each instance can be summarized as the number of potential depots, $|I|$, the number of customers, $|J|$, the number of time periods, $|\bar{T}|$, and the number of vehicles, $|\bar{K}|$, (i.e., $|I|\times|J|\times|\bar{T}|\times|\bar{K}|$).

Table 1**:** The relative values of the test instances.

ID of Instances	Vehicle capacity	Depot capacity	Fixed cost of depots	Fixed cost of vehicles	Serving time to customers	Loading time of a vehicle from a depot
5 × 30 × 6 × 5	300	900	100	20	2	4
7 × 100 × 6 × 10	800	10000	100	20	2	4

The HHA is encoded in MATLAB 7.10.0 on a computer, holding Intel® Core™ Duo CPU T2450 2.00 GHz and RAM of 1.00 GB. The average computational results of 10 times are given in Tables 2 and 3 for small and large size instances, respectively. In first and second columns of Tables 2 and 3, the values of *DPI* and *API* varied with the interval of 0.2 to 1 with a step of 0.2, respectively. Next columns of the tables respectively labeled: the planned routes, the additional distances, the routing costs that include the planned routes and additional distances, the lost opportunity costs, the depot costs, the vehicle costs, the total costs that consist of routing costs as well as lost opportunity, depots and vehicles costs. The last column also shows the CPU time of solutions. As shown in last column of Tables 2 and 3 by bold number, when the value of the dispatcher preference index equals 0.6 and the value of assignment preference index equals 1, the total cost has a minimum value.

It is noted that, when the value of *API* grows, it ensures that the depot is more able to respond customers' demand. In the other hand, establishment of more depots or depots with high capacity can provide high confidence for supporting more customers. As seen in Table 2, for each constant value of *DPI*, when the *API* value has increased, the cost (or the number) of deploying the depots is also grown. According to Tables 2 and 3 for the different *DPI* values and each constant *API* value, planned route and additional distances are changed as follows: the lower values of parameter *DPI* denote a tendency to use total vehicle capacity. These values are associated with the routes with the shorter planned distances. Furthermore, lower values of parameter *DPI* increase the number of cases in which vehicles visit customers but are unable to serve them, thereby increase the total additional distance due to the "route failure". In the other hand, higher values of parameter *DPI* are characterized by less utilization of vehicle capacity along less additional distance to cover due to failures. But, these values are associated with the routes with the longer planned distances. At high *DPI* value, it also requires a greater number of vehicles. In this case, it may be due to the unavailability of vehicles, the cost of lost opportunity increases due to the lack of serving to customers. For example, the results of planned routes, additional distances and total costs of Tables 2 and 3, for the different *DPI* values and the *API* value of 1, are depicted by Figures 3 and 4, respectively. As is shown in Figures 3 and 4, when the value of *DPI* equals 0.6, the total cost has a minimum value.

Table 2: Computational results for 5 × 30 × 6 × 5 instance with different *DPI* and *API*.

DPI	*API*	Planned routes	Additional distances	Routing cost	Lost opportunity	Depot cost	Vehicle cost	Total cost	CPU Time (second)
	0.2	4888.3	1146.1	6034.4	13.2	200	580	6827.6	2245
	0.4	4850.1	1307.6	6157.7	10.4	200	580	6948.1	3821
0.2	0.6	4788.5	1226.3	6014.9	2.2	300	560	6877.1	3783
	0.8	4568.5	1107.4	5675.9	6.0	400	600	6681.9	3730
	1.0	4553.9	1217.5	5771.4	0.0	400	560	6731.4	3584
	0.2	4895.9	1151.4	6047.3	25.2	200	600	6872.5	1861
	0.4	5104.6	1219.6	6324.2	17.4	200	600	7141.6	4386
0.4	0.6	5032.1	1101.2	6133.3	16.4	300	600	7049.7	4570
	0.8	4862.1	774.5	5636.6	35.0	300	600	6571.6	4765
	1.0	4651.4	1146.0	5797.4	0.0	400	600	6797.4	4185
	0.2	5499.9	509.4	6009.2	107.8	200	600	6917.0	2229
	0.4	5541.4	708.8	6250.2	101.2	200	600	7151.4	2700
0.6	0.6	5682.9	743.9	6426.8	93.8	200	600	7320.6	2780
	0.8	5077.4	580.5	5657.9	106.2	300	600	6664.1	2590
	1.0	4876.8	572.5	5449.3	96.2	400	600	**6545.5***	2180
	0.2	5991.5	73.3	6064.8	233.6	100	600	6998.4	1830
	0.4	6262.6	78.8	6341.4	280.6	100	600	7322.0	1631
0.8	0.6	6309.4	99.7	6409.2	279.2	200	600	7488.4	1655
	0.8	5722.3	71.4	5793.7	294.8	200	600	6888.5	1518
	1.0	5571.5	78.6	5650.1	281.2	400	600	6931.3	1408
	0.2	6928.4	0.0	6928.4	378.2	100	600	8006.6	768
	0.4	7120.4	0.0	7120.4	366.6	100	600	8187.0	866
1.0	0.6	7222.1	0.0	7222.1	371.0	100	600	8293.1	789
	0.8	6460.2	0.0	6460.2	369.8	200	600	7630.0	808
	1.0	6177.9	0.0	6177.9	365.5	400	600	7543.4	817

*Bold number indicates the minimum total cost between all *DPI* and *API* values.

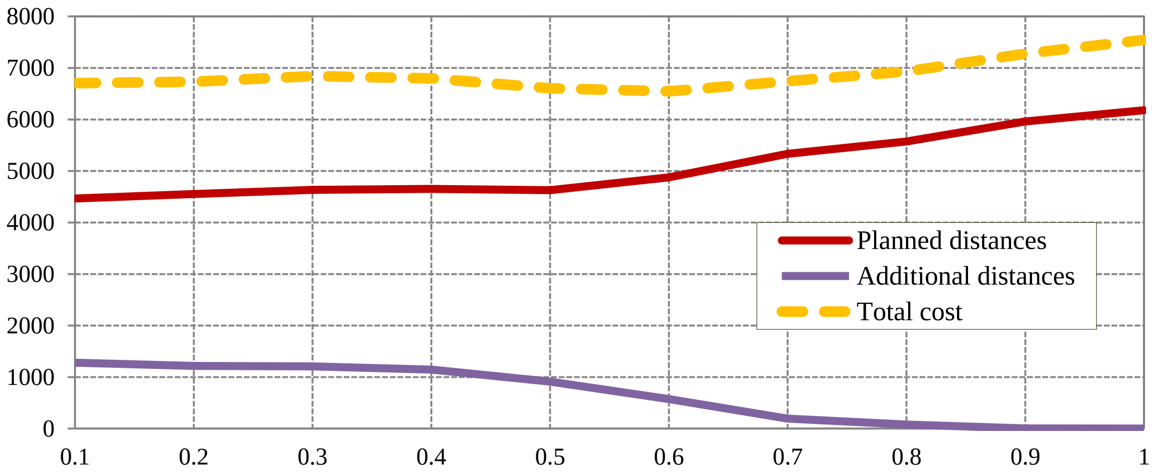

Figure 3: The cost changes with different *DPI* and *API* of 1 for 5 × 30 × 6 × 5 instance.

Table 3: Computational results for 7 × 100 × 6 × 10 instance with different *DPI* and *API*.

DPI	*API*	Planned routes	Additional distances	Routing cost	Lost opportunity	Depot cost	Vehicle cost	Total cost	CPU Time (second)
0.2	0.2	6350.6	1430.9	7781.5	0.0	100	800	8681.5	16547
	0.4	6477.2	1312.4	7789.7	0.0	100	820	8709.7	17201
	0.6	6321.9	1377.0	7698.9	0.0	100	780	8578.9	16531
	0.8	6201.1	1553.4	7754.5	0.0	100	740	8594.5	15766
	1.0	6227.6	1595.6	7823.1	0.0	100	600	8523.1	16388
0.4	0.2	6754.1	1216.5	7970.6	0.0	100	840	8910.6	14485
	0.4	6687.7	1232.7	7920.4	0.0	100	840	8860.4	14340
	0.6	6778.9	1144.2	7923.1	0.0	100	860	8883.1	14479
	0.8	6690.4	1161.4	7851.8	0.0	100	840	8791.8	14563
	1.0	6447.1	1159.4	7606.5	0.0	100	700	8406.5	14157
0.6	0.2	6991.5	506.3	7497.9	0.0	100	960	8557.9	9758
	0.4	7045.0	469.1	7514.2	0.0	100	960	8574.2	12765
	0.6	7033.2	506.2	7539.3	0.0	100	960	8599.3	9919
	0.8	7093.6	414.3	7507.9	0.0	100	960	8567.9	9765
	1.0	6893.7	497.8	7391.5	0.0	100	800	**8291.5***	9877
0.8	0.2	7719.0	2.8	7721.9	0.0	100	1160	8981.9	6532
	0.4	7738.7	3.9	7742.6	0.0	100	1140	8982.6	6533
	0.6	7862.1	2.5	7864.6	0.0	100	1180	9144.6	6618
	0.8	7652.7	4.2	7656.9	0.0	100	1160	8916.9	6722
	1.0	7780.2	1.9	7782.1	0.0	100	1120	9002.1	6857
1.0	0.2	8694.0	0.0	8694.0	301.2	100	1200	10295.2	4759
	0.4	8743.2	0.0	8743.2	254.4	100	1200	10297.6	5374
	0.6	8813.9	0.0	8813.9	236.4	100	1200	10350.3	5713
	0.8	8555.8	0.0	8555.8	256.0	100	1200	10111.8	4660
	1.0	8580.5	0.0	8580.5	245.2	100	1200	10125.7	4576

*Bold number indicates the minimum total cost between all *DPI* and *API* values.

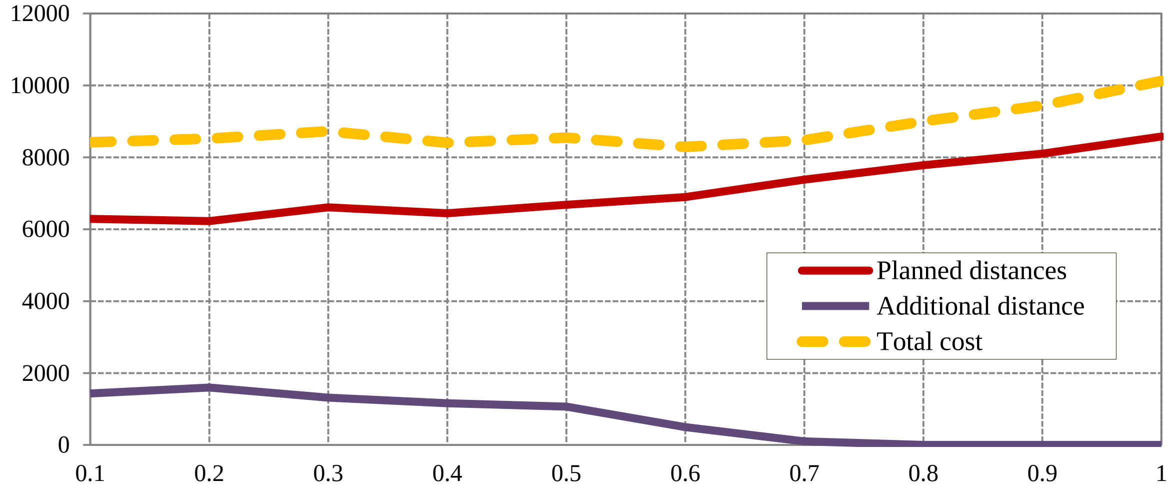

Figure 4: The cost changes with different *DPI* and *API* of 1 for 7 × 100 × 6 × 10 instance.

6.2. Sensitivity analysis on availability level of vehicles

As mentioned before, vehicles due to some reasons such as technical flaws, going off by driver, etc. have availability level that can be different for each vehicle ($0 \le A_k^t \le 1$). In this section numerical experiment on availability level of vehicles is performed to show its role and influence on solving the model. Constraints (4.17) in the model of DCLRP-FD express the limitation of travel distance of vehicles that it depends on the availability level. It is interesting that A_k^t is considered as a random variable that it may be more adapted to real cases. If the A_k^t be a random variable, the constraints (4.17) can be transferred to chance-constraints (6.24) that is realized with a minimum probability of $1-\alpha_k^t$, $0 \le \alpha_k^t \le 1$ for each vehicle in each period.

$$p\left(\sum_{i \in V} \sum_{j \in V} [(c_{ij} + s_j) x_{ijk}^t + f_k] \le A_k^t D \right) \ge 1 - \alpha_k^t \qquad \forall\ k \in K;\ \forall\ t \in T \tag{6.24}$$

In this section, it is assumed that availability level is normally distributed with known mean of μ_k^t and standard variance (SD) of σ_k^t (i.e., $A_k^t \sim N(\mu_k^t, \sigma_k^t)$). In constraints (6.24) $A_k^t D$ is also normal with mean $D\mu_k^t$ and SD of $D\sigma_k^t$ and the constraints can be changed as follows:

$$p\left(\frac{\sum_{i \in v} \sum_{j \in V} [(c_{ij} + s_j) x_{ijk}^t + f_k] - D\mu_k^t}{D\sigma_k^t} \le \frac{A_k^t D - D\mu_k^t}{D\sigma_k^t} \right) \ge 1 - \alpha_k^t \qquad \forall\ k \in K;\ \forall\ t \in T \tag{6.25}$$

In constraints (6.25), $Z = \dfrac{A_k^t D - D\mu_k^t}{D\sigma_k^t}$ is standard normal variable with mean zero and SD 1. This means that

$$p\left(\frac{\sum_{i \in v} \sum_{j \in V} [(c_{ij} + s_j) x_{ijk}^t + f_k] - D\mu_k^t}{D\sigma_k^t} \le Z \right) \ge 1 - \alpha_k^t \qquad \forall\ k \in K;\ \forall\ t \in T$$

and the result is

$$1 - F\left(\frac{\sum_{i \in v} \sum_{j \in V} [(c_{ij} + s_j) x_{ijk}^t + f_k] - D\mu_k^t}{D\sigma_k^t} \right) \ge 1 - \alpha_k^t \qquad \forall\ k \in K;\ \forall\ t \in T$$

or

$$F\left(\frac{\sum_{i \in v} \sum_{j \in V} [(c_{ij} + s_j) x_{ijk}^t + f_k] - D\mu_k^t}{D\sigma_k^t} \right) \le \alpha_k^t \qquad \forall\ k \in K;\ \forall\ t \in T \tag{6.26}$$

where *F* represents the cumulative distribution function (CDF) of the normal standard distribution. Let $z_{\alpha_k^t}$ be the standard normal value such that $F(z_{\alpha_k^t}) = \alpha_k^t$, then the constraints (6.26) is realized if and only if

$$\frac{\sum_{i\in v}\sum_{j\in V}[(c_{ij}+s_j)x_{ijk}^t+f_k]-D\mu_k^t}{D\sigma_k^t}\le z_{\alpha_k^t} \qquad \forall\ k\in K;\ \forall\ t\in T$$

This yields the following deterministic constraints (6.27):

$$\sum_{i\in v}\sum_{j\in V}[(c_{ij}+s_j)x_{ijk}^t+f_k]\le D\mu_k^t+z_{\alpha_k^t}D\sigma_k^t \qquad \forall\ k\in K;\ \forall\ t\in T \tag{6.27}$$

In other word, instead of chances-constraints (4.17) with $A_k^t \sim N(\mu_k^t,\sigma_k^t)$, the deterministic constraints of (6.27) can be used in the model. Given the constraints (6.27), Tables 4 and 5 shows the sensitive of the model's solution on parameter A_k^t for two test instances described in pervious section. It is assumed that the value of A_k^t for each vehicle in each period be normal distribution with mean 0.6 and SD 0.15. In Tables 4 and 5, *DPI* of 0.6 and *API* of 1 are selected because these values have minimum total costs among other *DPI* and *API* values in Tables 2 and 3. The first column of Tables 4 and 5 indicates different value of α_k^t and other columns are similar to Tables 2 and 3. As shown in Tables 4 and 5, when the value of α_k^t is lower, all costs will be lower. Low value of α_k^t (i.e., high value of $1-\alpha_k^t$) implies that the vehicle should be more available and it can serve the planned route with high probability. Moreover, lower values of α_k^t increase the cases that each vehicle can use its maximum travel distances as possible as and leads to the number of trips and the routing costs will be reduced.

Table 4: Summarized results of 5 × 30 × 6 × 5 instance compare to different value of α_k^t.

α_k^t	Planned routes	Additional distances	Routing cost	lost opportunity	Depot cost	Vehicle cost	Total cost	CPU Time (second)
0.99	15877.3	0.0	15877.3	602.4	300	600	17379.7	715
0.9	10673.7	68.1	10741.8	334.0	300	600	11975.8	878
0.8	9576.5	158.3	9734.8	296.6	300	600	10931.4	966
0.7	7661.1	164.4	7825.5	277.8	300	600	9003.3	1482
0.6	6812.9	195.4	7008.3	256.2	300	600	8164.5	1392
0.5	6543.5	249.4	6792.8	229.4	300	600	7922.2	1377
0.4	5802.4	319.6	6122.0	178.6	400	600	7300.6	1437
0.3	5506.3	333.8	5840.1	191.4	400	600	7031.5	1600
0.2	5423.4	343.8	5767.2	155.0	400	600	6922.2	1928
0.1	5078.0	346.6	5424.6	142.6	400	600	6567.2	2158
0.01	5017.2	390.1	5407.3	111.2	400	600	6518.5	2357

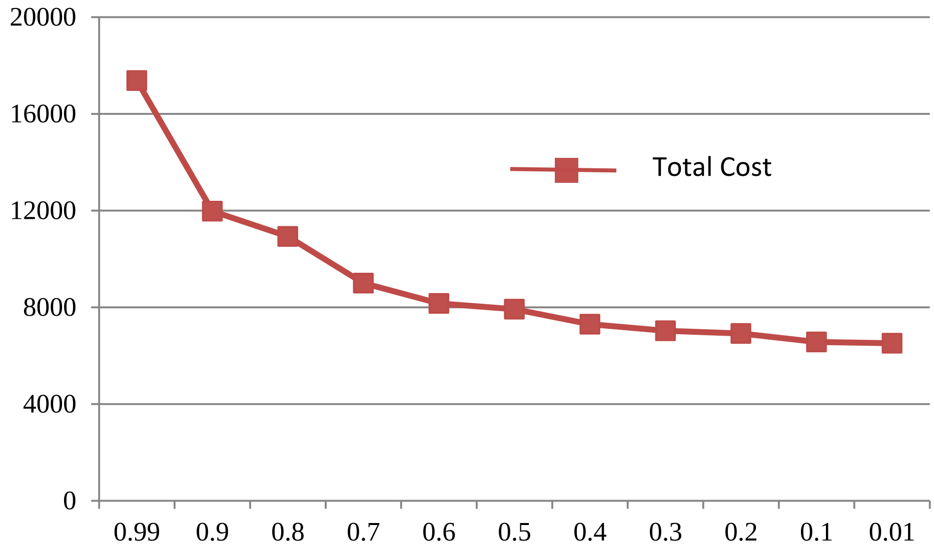

Figure 5: Total cost of 5 × 30 × 6 × 5 instance compared to different value of α_k^t.

In the other hand, high value of α_k^t causes the number of trips and routing costs will be increased. At higher values of α_k^t, a vehicle cannot use its maximum travel distances as possible as and leads to the number of trips and finally, the routing costs will be increased. For convenience, the total cost of Tables 4 and 5 compared to the α_k^t values are depicted by Figures 5 and 6, respectively. Consequently, as it is clear in Figures 5 and 6, for the lower values of α_k^t, the total cost has a lower amount.

Table 5: Summarized results of 7 × 100 × 6 × 10 instance compare to different value of α_k^t.

α_k^t	Planned routes	Additional distances	Routing cost	lost opportunity	Depot cost	Vehicle cost	Total cost	CPU Time (second)
0.99	42460.7	0.0	42460.7	10075	100	1200	53835.7	683
0.9	27492.9	0.0	27492.9	6304	100	1200	35096.9	999
0.8	19647.4	6.6	19654.0	4924	100	1200	25878.0	1733
0.7	13762.1	63.4	13825.5	3131	100	1200	18256.5	2461
0.6	11938.2	85.2	12023.5	2570	100	1200	15893.5	3907
0.5	10311.0	95.3	10366.3	1825	100	1200	13491.3	4980
0.4	9602.3	125.4	9727.7	1308	100	1200	12335.7	5188
0.3	8839.5	148.8	8948.3	829	100	1200	11077.3	5861
0.2	8180.4	182.1	8362.5	161	100	1200	9823.5	6662
0.1	7609.6	291.9	7901.5	0	100	1100	9101.5	7750

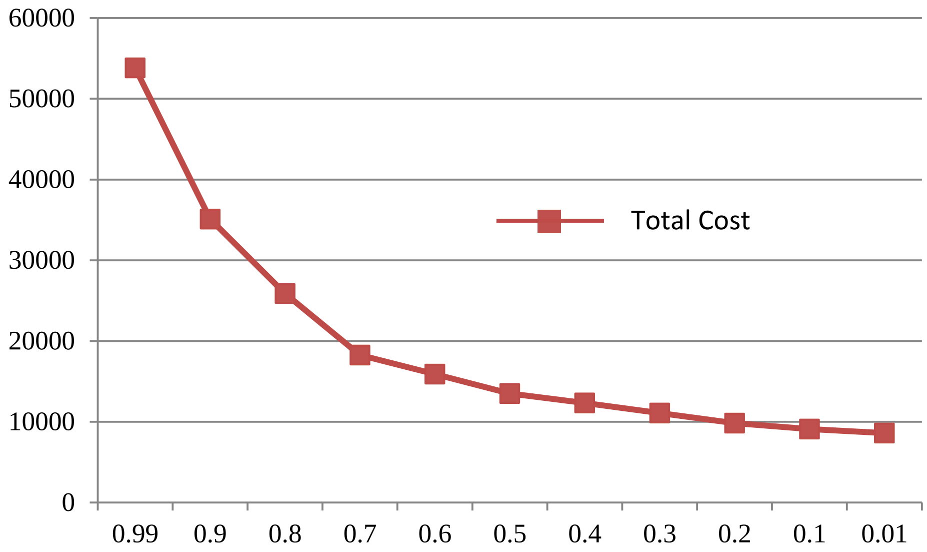

Figure 6: Total cost of 7 × 100 × 6 × 10 instance compared to different value of α_k^t .

6.3. Validity of the DCLRP-FD's model

To evaluate the validation of the DCLRP-FD's model, numerical experiment is carried out in this section. As mentioned before, a time horizon with some planning periods exists in the DCLRP-FD. It may be interest to evaluate the validity of DCLRP-FD compared to CLRP-FD that considers single-period for the problem. In fact, it will be shown that solving an instance with DCLRP-FD produces better solution than solving several CLRP-FD problems. Tables 6 and 7 demonstrate the validation of the proposed model for two test instances described before. Each of the Table of 6 and 7 is divided into two sections of top and bottom. The first column of the tables shows static and dynamic models that are related to results of CLRP-FD and DCLRP-FD, respectively. Next columns of Tables 6 and 7 are also similar to four previous tables. DCLRP-FD's solution with *DPI* of 0.6 and *API* of 1 are detailed for each period at bottom of the tables. As seen in the Tables 6 and 7, in dynamic model, the planned routes and additional distances can be changed during the planning periods due to fluctuations of demands. In static model, the planned routes are constant or fixed for all periods but, the additional distances and lost opportunity costs may be changed due to fluctuations in demands. Regarding to the last column of Tables 6 and 7, it is clear that dynamic model is superior to static model in terms of quality of solution. Consequently, the results state the validation of the DCLRP-FD's model for instances that can be solved by CLRP-FD's model.

7 Conclusion

In this work a DCLRP-FD that is related to logistics system of supply chain was addressed. A mathematical model using credibility theory consists of fuzzy constraints was proposed. Since the model was NP-hard, a HHA with four phases including stochastic simulation for estimating the additional distances due to route failures was presented. To evaluate the HHA and to obtain the best sensitive fuzzy and stochastic parameters of the model, two test instances with different size which are compatible with real data were generated. The first computational experiment showed that fuzzy parameters of *DPI* and *API* greatly influence on cost of transportation system. Precisely, first experiment indicates that *DPI* value influences more on the planned routes' length, additional distance, lost opportunity and fixed cost of vehicles. But, another parameter had more impact on fixed cost of depots. Second experiment expressed that if availability level of vehicles was considered as random variable such as normal distribution, it can effect on total cost. In this case, constraints (6.27) were replaced with constraints (4.17). Furthermore, to show the validity of the proposed model, comparison results between DCLRP-FD and CLRP-FD' models are carried out. This paper suggests some future researches as follows: (a) improving each phase of the proposed HHA such as using improvement process of 2-Opt or 3-Opt for second phase (b) considering the model with pickup and delivery demands.

Table 6: Summarized results of 5 × 30 × 6 × 5 instance for validation of DCLRP-FD

	DPI	*API*	Period	Planned cost	Additional distances	Routing cost	Lost opportunity	Depot cost	Vehicle cost	Total cost
Static model	0.6	1.0	1	782.3	27.2	809.5	3	400	100	6812.5
			2	782.3	165.3	947.6	23		100	
			3	782.3	160.6	942.9	30		100	
			4	782.3	207.3	989.6	37		100	
			5	782.3	231.0	1013.3	31		100	
			6	782.3	164.3	946.6	39		100	
	DPI	*API*	Period	Planned cost	Additional distances	Routing cost	Lost opportunity	Depot cost	Vehicle cost	Total cost
Dynamic model	0.6	1.0	1	782.3	27.2	809.5	3	400	100	6545.5
			2	845.6	80.6	926.2	21		100	
			3	778.3	87.5	865.8	19		100	
			4	876.0	145.7	1021.7	16		100	
			5	765.3	173.8	939.1	19		100	
			6	829.2	57.7	886.9	19		100	

Table 7: Summarized results of 7 × 100 × 6 × 10 instance for validation of DCLRP-FD

	DPI	*API*	Period	Planned cost	Additional distances	Routing cost	Lost opportunity	Depot cost	Vehicle cost	Total cost
Static Problem	0.6	1.0	1	1196.2	61.1	1257.3	0	100	140	8770.4
			2	1196.2	154.2	1350.4	0		140	
			3	1196.2	156.1	1352.3	0		140	
			4	1196.2	40.7	1236.9	0		140	
			5	1196.2	155.4	1351.6	0		120	
			6	1196.2	125.7	1321.9	0		120	
	DPI	*API*	Period	Planned cost	Additional distances	Routing cost	Lost opportunity	Depot cost	Vehicle cost	Total cost
Dynamic Problem	0.6	1.0	1	1196.2	61.1	1257.3	0	100	140	8291.5
			2	1138.5	95.5	1234.0	0		140	
			3	1149.6	66.9	1216.5	0		140	
			4	1121.4	87.8	1209.2	0		140	
			5	1158.2	93.8	1251.9	0		120	
			6	1129.8	92.8	1222.6	0		120	

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