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Research Article

Allocating the Fixed Resources and Setting Targets in Integer Data Envelopment Analysis

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Abstract

Data envelopment analysis (DEA) is a non-parametric approach to evaluate a set of decision making units (DMUs) consuming multiple inputs to produce multiple outputs. Formally, DEA use to estimate the efficiency score into the empirical efficient frontier. Also, DEA can be used to allocate resources and set targets for future forecast. The data are continuous in the standard DEA model whereas there are many problems in the real life that data must be integer such as number of employee, machinery, expert and so on. Thus in this paper we propose an approach to allocate fixed resources and set fixed targets with selective integer assumption that is based on an integer data envelopment analysis (IDEA) approach for the first time. The major aim in this approach is preserving the efficiency score of DMUs. We use the concept of benchmarking to reach this aim. The numerical example gets to illustrate the applicability of the proposed method.

Keywords: Data Envelopment Analysis, Integer Data, Resource Allocation, Target Setting.

1 Introduction

Data envelopment analysis (DEA) pioneered by Farrell (1957) [1] and later developed by Charnes et al. (1978) [2]. It is a nonparametric method to frontier analysis for measuring efficiency of a set of decision making units (DMUs). Mathematically, DEA utilizes a linear programming (LP) model which characterizes the relationship among multiple inputs and multiple outputs by envelopment of the observed data to determine a best practice frontier for production. The conventional DEA models construct a non-parametric piecewise surface or frontier over the data. DMUs placed on the frontier take unity score and they are called the frontier (efficient) DMUs otherwise called inefficient DMUs.

In many organizations the resource allocation (e.g., labor and asset) to individual DMUs can be limited and in most of the time the fixed cost exists for assignment among all DMUs. In addition the organization

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is willing to set targets for the individual DMUs with respect to future performance (e.g., output targets for the next evaluating time) and it is clear that these targets setting should be related to the allocated resources. Thus, the resource/cost allocation and target setting recently is one of the most interesting realistic topics in DEA. According to Beasley (2003, p. 208) [3], we first clarify the difference between the terms “resource allocation” and “target setting” in the DEA terminology. The resource allocation may happen when the organization has restricted input resources or restricted output possibilities. In such circumstances, the organization must allocate equitably the fixed input/output levels among the DMUs. For example, add a new line in Gas Company by fixed capacity as fixed input and reach fixed profit corresponding it as fixed output. The target setting for input and output can be defined as a certain input/output value for each DMU without citation to organizational limitations. In the pioneering work, Cook and Kress (1999) [4] proposed an equitable resource allocation method in DEA where the allocated fixed cost considered as an extra input measure. They obtained the suitable allocation by solving several linear programming problems with regard to the efficiency invariance and parato minimality. The method of Cook and Kress (1999) [4] cannot be utilized in a straightforward manner for determining a cost allocation between the DMUs. Therefore, Cook and Zhu (2005) [5] extended Cook and Kress's method (1999) [4] as a practical cost allocation method. With the similar assumption proposed by Cook and Kress (1999) [4] on treating the costs allocation as an extra input, Beasley (2003) [3] developed a DEA method for allocating fixed cost and setting output target so that a unique cost allocation was determined by maximizing the average efficiency of all DMUs. Amirteimoori and Kordrostami (2005) [6] proposed an alternative DEA-based allocation approach to obtain a unique allocation in terms of combining the efficiency invariance proposed by Cook and Kress (1999) [4] as well as taking account of the additional constraints proposed by Beasley (2003) [3]. Li et al. (2009) [7] developed a DEA-based approach to allocating fixed cost among various DMUs based on the combination of the allocated cost with other cost measures to determine a unique allocation. Li et al. (2009) [7] introduced the relationship between the allocated cost and the efficiency score. They show that the fixed cost was a dependent input while most researches often have regarded it as an independent factor. Amirteimoori and Mohaghegh Tabar (2010) [8] proposed a DEA approach in the presence of the fixed resource allocation and the fixed output target by defining an additional input and output for all DMUs, respectively. Hosseinzadeh Lotfi et al. (2013) [9] introduced a new approach in the presence of resource allocation and target setting by common-weight DEA and goal programming methods. Their method allocated fixed resources and set fixed targets in three steps such that all DMUs are efficient.

In the traditional DEA, the input and output variables are always considered to be continuous. In some real applications, the input and output variables cannot be satisfied in real conditions and may be taken integer-valued. Therefore, the integer-valued DEA models have been introduced for dealing with the integral constraints (Lozano and Villa, 2007 [10]). Lozano and Villa (2006) [11] were the first to deal with this problem by introducing a mixed integer linear programming problem based on DEA model. Kuosmanen and Kazemi Matin (2009) [12] reviewed the shortcoming of the Lozano and Villa’s (2006) [11] model and resolve them. Indeed, they proposed a new axiomatic foundation to deal integer-valued DEA model. They also showed an improved kind of the classic Farrell input efficiency measure, and introduced a mixed integer linear programming problem for estimating efficiency scores. Kazemi Matin and Kuosmanen (2009) [13] addressed the axiomatic foundation under variable, non-decreasing and non-increasing returns to scale where the integer DEA employed. Their model in question was made by two new axioms as natural convexity and natural augment ability. Wu and Zhou (2011) [14] suggested an integer-valued DEA model to deal with input excesses and output shortfalls simultaneously, in a way that maximizes both. Foroughi (2011) [15] presented a linear mixed integer model that is feasible and is employed for producing a single efficient DMU. Also, his proposed model is used for ranking all extreme efficient DMUs.

In this paper, we propose a new IDEA method to allocate fixed resources and set fixed targets by selective integer assumption. The major aim of the proposed method is preserving efficiency scores. For this aim, first, DMUs are evaluated by the IDEA model which has been presented by Kazemi Matin and Kuosmanen (2009) [13]. Second, the fixed resources are allocated and fixed targets will be set by selective integer assumption based on our new benchmarking method. By incorporating the proposed model, the related efficiency scores of all DMUs remain unchanged.

Consequently, we will introduce a new method to allocate fixed resources and set fixed target that preserve the efficiency score of each DMU as three steps: at first, evaluating DMUs by solving IDEA model, then classifying the DMUs in the two classes; efficient and inefficient DMUs, finally determining multiplier all inefficient units. So, corresponding to the said three steps, we will be allocated the fixed integer and non-integer resources and will be set fixed integer and non-integer targets among DMUs equitably.

The remainder sections of the paper are organized as follows. In Section 2, we get a general study the IDEA model and in Section 3, we represent the details of the proposed method based on allocating the fixed resource and setting target for integer data. Numerical example is used to illustrate the applicability of the proposed model in Section 4. The paper ends with conclusion in Section 5.

2 IDEA Preliminaries

Conventional DEA models are able to evaluate DMUs with real valued data. In the real applications, there are many problems with integer valued data. So, some authors worked on problems with integer data based on DEA approach. Suppose that there are n DMUs to be evaluated, $j=1,...,n$, such that each DMU is produce s outputs by consuming m inputs. DMUs have inputs and outputs include integer and non-integer data. Kazemi Matin and Kuosmanen (2009) [13] produced a new IDEA model that partitioned the set of inputs as $I = I^I \cup I^{NI}$ and the set of outputs as $O = O^I \cup O^{NI}$, where subsets I^I and O^I are integer-valued and subsets I^{NI} and O^{NI} are real-valued. They formed the following production possibility set (PPS) with selective integer technology for the variable returns to scale case (VRS):

$$T = \{(x^I, x^{NI}, y^I, y^{NI}) | (x^I, y^I) \in Z^+,\ x^I \geq \sum_{j=1}^{n} \lambda_j x_j^I,\ x^{NI} \geq \sum_{j=1}^{n} \lambda_j x_j^{NI}, y^I \leq \sum_{j=1}^{n} \lambda_j y_j^I, \\ y^{NI} \leq \sum_{j=1}^{n} \lambda_j y_j^{NI}, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, j=1,...,n\}. \tag{1}$$

Applying the technology proposed in (1), they introduced the following input-oriented model for measuring efficiency with integer valued data:

$$
\begin{array}{llll}
\min & \theta & & \\
s.t.\ (Ia) & \sum_{j=1}^{n}\lambda_j x_{ij}^{I} + s_i^{I} = \tilde{x}_i & i \in I^{I}, & \\
(IIa) & \theta x_{io}^{I} - s_i'^{I} = \tilde{x}_i & i \in I^{I}, & \\
(IIIa) & \sum_{j=1}^{n}\lambda_j x_{ij}^{NI} + s_i^{NI} = \theta x_{io}^{NI} & i \in I^{NI}, & \\
(Ib) & \sum_{j=1}^{n}\lambda_j y_{rj}^{I} - s_r^{I} = \tilde{y}_r & r \in O^{I}, & \\
(IIb) & \tilde{y}_r - s_r'^{I} = y_{ro}^{I} & r \in O^{I}, & (2)\\
(IIIb) & \sum_{j=1}^{n}\lambda_j y_{rj}^{NI} - s_r^{NI} = y_{ro}^{NI} & r \in O^{NI}, & \\
(Ic) & s_i'^{I}, \tilde{x}_i \in Z^{+} & i \in I^{I}, & \\
(IIc) & s_r'^{I}, \tilde{y}_r \in Z^{+} & r \in O^{I}, & \\
(d) & \sum_{j=1}^{n}\lambda_j = 1,\ \ \lambda_j \geq 0, & j = 1,\ldots,n, &
\end{array}
$$

where λ_j are the convex multipliers and defining unit *o* is the evaluating DMU. Constraints (Ia), (IIa) and (IIIa) state that the efficiency corrected use of input ($\theta x_{io}^{I}, \theta x_{io}^{NI}$) must at least equal the amounts used by the reference unit *o* because the correction number is the same for all types of inputs, there is a radial reduction of observed inputs. Constraints (Ib), (IIb) and (IIIb) represent that the reference unit must produce at least as much output as unit *o.* constraints (Ic) and (IIc) show that the integer variables and constraints (d) represents the non-negative multipliers and convex condition for a VRS model.

Definition 2.1.
DMU_o is input-efficient if and only if $\theta^* = 1$, otherwise, DMU_o is input-inefficient in which θ^* is the optimal solution of model (2).

3 Fixed resource allocation and target setting with selective integer data

Suppose that a manager decides to allocate fixed resources to the inputs and set fixed targets to the outputs across a set of DMUs in which resource/target indexes can be integer valued. Namely, a manager of a chain manufactory decides to add a new production line where for this line he/she needs to allocate fixed expert workers across the manufactories, and wants to produce certain machines related to this line. Obviously, in this problem, fixed expert workers and certain machines are integer indexes. So, the number of expert workers can be considered as an extra integer input (resource) and the number of machines can be treated as an extra output (target) to new production line. In this case, the aim of the manager is preserving efficiency scores of manufactories. In the other word, the manager wants to allocate new integer resource and set new integer target with fixed capacity that are preserved the efficiency scores of manufactories by add a new production line.
Similar to this real problem, there are many cases in which managers decide to allocate resources and set targets with selective integer assumption. Therefore in this section, we present an IDEA-based method for allocating fixed resources among a set of DMUs. In addition, we show how fixed targets can be set at the

same time as decisions are made about allocating input resources by selective integer assumption. For the present, consider a system inclusive n DMUs under the evaluation such that each DMU consumes m inputs, $x_{ij} \in R^+,\ i=1,...,m;\ j=1,...,n$ to produce s outputs, $y_{rj} \in R^+,\ r=1,...,s;\ \ j=1,...,n$. We consider two types of fixed resources as $F_1 \in Z^+$ and $F_2 \in R^+$ where F_1, F_2 are indexes with integer and non-integer values, respectively. The manager decides to allocate resources among DMUs in an equitable and fair way. Accordingly, the manager wants to achieve two types of fixed targets as $G_1 \in Z^+$ and $G_2 \in R^+$ such that G_1, G_2 are indexes with integer and non-integer values, respectively and set among DMUs equitably. We introduce the non-negative integer variables f_j^I , $j=1,...,n$ and non-negative real variables f_j^{NI} , $j=1,...,n$ to represent the fixed integer and non-integer resources, respectively that allocate to DMU$_j$. Where, the relations $\sum_{j=1}^n f_j^I = F_1, \sum_{j=1}^n f_j^{NI} = F_2,$ must be held. Moreover, We set the non-negative integer variables g_j^I, $j=1,...,n$ and non-negative real variables g_j^{NI} , $j=1,...,n$ to represent the fixed integer and non-integer targets to each DMU$_j$, respectively that the relations $\sum_{j=1}^n g_j^I = G_1, \sum_{j=1}^n g_j^{NI} = G_2,$ must be held. Therefore, we create the following model based on the above concepts and model (2):

$$
\begin{aligned}
\min \quad & \theta \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij}^{I} \le \tilde{x}_i & i \in I^{I}, \\
& \theta x_{io}^{I} - s_i^{+I} = \tilde{x}_i & i \in I^{I}, \\
& \sum_{j=1}^{n} \lambda_j x_{ij}^{NI} \le \theta x_{io}^{NI} & i \in I^{NI}, \\
& \sum_{j=1}^{n} \lambda_j f_j^{I} \le \tilde{f} & \\
& \theta f_o^{I} - s' = \tilde{f} & \\
& \sum_{j=1}^{n} \lambda_j f_j^{NI} \le \theta f_o^{NI} & \\
& \sum_{j=1}^{n} \lambda_j y_{rj}^{I} \ge \tilde{y}_r & r \in O^{I}, \\
& \tilde{y}_r - s_r^{-I} = y_{ro}^{I} & r \in O^{I}, \\
& \sum_{j=1}^{n} \lambda_j y_{rj}^{NI} \ge y_{ro}^{NI} & r \in O^{NI}, \\
& \sum_{j=1}^{n} \lambda_j g_j^{I} \ge \tilde{g} & \\
& \tilde{g} - s'' = g_o^{I} & \\
& \sum_{j=1}^{n} \lambda_j g_j^{NI} \ge g_o^{NI} & \\
& \sum_{j=1}^{n} f_j^{I} = F_1, \ \sum_{j=1}^{n} f_j^{NI} = F_2, & \\
& \sum_{j=1}^{n} g_j^{I} = G_1, \ \sum_{j=1}^{n} g_j^{NI} = G_2, & \\
& f_j^{I}, g_j^{I} \in Z^{+}, & j = 1, \ldots, n, \\
& \tilde{f}, s', \tilde{g}, s'' \in Z^{+}, & \\
& s_i^{+I}, \tilde{x}_i \in Z^{+}, & i \in I^{I}, \\
& s_r^{-I}, \tilde{y}_r \in Z^{+}, & r \in O^{I}, \\
& \sum_{j=1}^{n} \lambda_j = 1, \ \lambda_j \ge 0, & j = 1, \ldots, n.
\end{aligned}
\tag{3}
$$

where (F_1, F_2) and (G_1, G_2) are total amount of extra inputs and outputs, respectively and distribute among DMUs equitably. We must be solve a mixed integer non-linear programming (MINLP) model (3) to achieve the optimal values of resource allocation and target setting related to each DMU. Whereas, there are n DMUs under evaluation, we earn new optimal solutions for resources and targets variables when DMU under evaluation changed. So, we unable to reach the individual optimal solutions by model (3).

In addition, we decide to allocate fixed resources and set fixed targets which are preserved the efficiency scores of all DMUs. For this aim we use the concept of benchmarking in IDEA as following three steps:

1- Evaluating n DMUs by solving IDEA model (2).
2- Classifying the DMUs in the two classes; efficient and inefficient DMUs,
3- Determining the λ_j in model (3) for all inefficient units.

By considering the above three steps, we decide to allocate the fixed integer and non-integer values resources and set fixed integer and non-integer values targets among DMUs equitably.
Suppose that n DMUs have been evaluated by solving model (2) and are classified in two classes: efficient and inefficient. The set of efficient and inefficient DMUs are named by 'E' and 'NE' respectively. Now we able to characterize the inefficient DMUs by the convex combination of the DMUs in the reference set. Hence, we consider the concept of benchmarking and create the following system to allocate resources and set targets with selective integer assumption:

$$
\begin{array}{lll}
(Ia) & \sum_{j\in E}\lambda_j^* f_j^I = f_o^I, & o\in NE,\\
(IIa) & \sum_{j\in E}\lambda_j^* f_j^{NI} = f_o^{NI}, & o\in NE,\\
(Ib) & \sum_{j\in E}\lambda_j^* g_j^I = g_o^I, & o\in NE,\\
(IIb) & \sum_{j\in E}\lambda_j^* g_j^{NI} = g_o^{NI}, & o\in NE,\\
(c) & \sum_{j=1}^{n} f_j^I = F_1,\ \sum_{j=1}^{n} f_j^{NI} = F_2, & \\
(d) & \sum_{j=1}^{n} g_j^I = G_1,\ \sum_{j=1}^{n} g_j^{NI} = G_2, & \\
(e) & f_j^I\ , g_j^I \in Z^+, & j=1,\dots,n.
\end{array}
\tag{4}
$$

In the above system, constraints (Ia), (IIa), (Ib) and (IIb) guarantee the efficiency score of each DMU is preserved by defining extra inputs and outputs corresponding to resources allocation and targets setting. The constraints (c) and (d) show the sum of the resources allocation and targets setting are equal to (F_1, F_2) and (G_1, G_2), respectively (subscripts 1 and 2 denote the integer and non-integer indexes, respectively). The constraints (e) state that the non-negative variables related to integer indexes must be integer valued.
Now, we must be solved the system (4) but infeasibility may be occurred for it because of constraints (c) and (d). We use the goal programming (GP) method to solve the above system and introduce some goals for infeasibility of system. (Steuer (1986) [16] and Hwang and Masud (1979) [17]). Hence, we define the extra deviation variables to feasibility of system and propose a mixed integer linear programming (MILP) model based on the benchmarking and GP concepts. Consequently, the resources allocation and targets setting with selective integer assumption can be obtained by solving the following MILP model:

$$
\begin{aligned}
\min \quad & \alpha_1^+ + \alpha_1^- + \alpha_2^+ + \alpha_2^- + \beta_1^+ + \beta_1^- + \beta_2^+ + \beta_2^- \\
s.t. \quad & \sum_{j \in E} \lambda_j^* f_j^I = f_o^I, && o \in NE, \\
& \sum_{j \in E} \lambda_j^* f_j^{NI} = f_o^{NI}, && o \in NE, \\
& \sum_{j \in E} \lambda_j^* g_j^I = g_o^I, && o \in NE, \\
& \sum_{j \in E} \lambda_j^* g_j^{NI} = g_o^{NI}, && o \in NE, \\
& \sum_{j=1}^{n} f_j^I + \alpha_1^+ - \alpha_1^- = F_1, \\
& \sum_{j=1}^{n} f_j^{NI} + \alpha_2^+ - \alpha_2^- = F_2, \\
& \sum_{j=1}^{n} g_j^I + \beta_1^+ - \beta_1^- = G_1, \\
& \sum_{j=1}^{n} g_j^{NI} + \beta_2^+ - \beta_2^- = G_2, \\
& f_j^I, g_j^I \in Z^+ && j = 1, \ldots, n.
\end{aligned}
\tag{5}
$$

4 Numerical example

In this section, we use numerical example to illustrate the applicability of the proposed method for simultaneous resource allocation and target setting.

We decide to determine the optimal resource allocation and target setting for 20 DMUs by the proposed model. The data are for gas companies located in 18 regions in Iran. In this study, we consider three inputs and three outputs. The first input (I1) is capital. The second input (I2) is number of staff and operational costs are as the third input (I3). The first output (O1) is number of subscribers. The second output (O2) is length of gas network and the sold-out gas income is as the third output (O3). Table 1 shows these inputs and outputs. Now we want to allocate one resource as an extra input and to set one target as an extra output. National Gas Company of Iran has 175 experts and would like to dispatch the experts to companies. In lieu of these experts, companies undertake to train 620 trainees. Obviously, the allocated experts and trained employees should be integer valued.

According to the proposed framework, we first calculate the efficiency scores of companies using IDEA model (2) and results are presented in the second column of Table 2. As said the central decision maker allocates 175 resource [units] to the DMUs and subsequently impose 620 units as an output target (i.e., $F_1 = 175$, and $G_1 = 620$). We use the proposed model (5) to obtain the optimal resource allocation as well as target setting for each unit. The results are presented in the last two columns of Table 2.

Table 1. The data for 20 DMUs

DMU$_j$	*I1*	*I2*	*I3*	*O1*	*O2*	*O3*
1	124313	129	198598	30242	565	61836
2	67545	117	131649	14139	153	46233
3	47208	165	228730	13505	211	42094
4	43494	106	165470	8508	114	44195
5	48308	141	180866	7478	248	45841
6	55959	146	194470	10818	230	136513
7	40605	145	179650	6422	127	70380
8	61402	87	94226	18260	182	36592
9	87950	104	91461	22900	170	47650
10	33707	114	88640	3326	85	13410
11	100304	254	292995	14780	318	79883
12	94286	105	98302	19105	273	32553
13	67322	224	287042	15332	241	172316
14	102045	104	155514	18082	441	30004
15	177430	401	528325	77564	801	201529
16	221338	1094	1186905	44136	803	840446
17	267806	1079	1323325	27690	251	832616
18	160912	444	648685	45882	816	251770
19	177214	801	909539	72676	654	443507
20	146324	686	545115	19839	177	341585

Table 2. Resulting efficiency, resource allocation and target setting of the proposed method

DMU$_j$	Efficiency before allocation	Efficiency after allocation	Resource allocation (f_j^I)	Target setting (g_j^I)
1	1.000	1.000	12	40
2	0.711	0.711	4	19
3	0.896	0.896	9	16
4	0.596	0.596	3	14
5	1.000	1.000	11	35
6	1.000	1.000	11	35
7	0.704	0.704	4	19
8	1.000	1.000	9	32
9	1.000	1.000	10	34
10	0.523	0.523	2	10
11	0.668	0.668	3	14
12	1.000	1.000	10	34
13	0.958	0.958	6	25
14	0.994	0.994	7	27
15	1.000	1.000	12	40
16	1.000	1.000	19	63
17	0.942	0.942	7	27
18	1.000	1.000	14	55
19	1.000	1.000	16	56
20	0.891	0.891	6	25
Sum			175	620

In Table 2, company 16 receives the highest resource allocation and target setting with our method because this company is efficient before allocating resource and setting target and this is the reason why it is allocated a much higher fixed resource allocation and target setting. Similarly, in our method company 10 receives the lowest resource allocation and target setting, since this units exhibit the worst performance in the production set. Table 2 shows that resource allocation and target setting values for each company are integer values. As a result, this example shows that the proposed model enables us to achieve our aim which is to preserve efficiency scores of DMUs after incorporating the integer valued of resource allocation and target setting.

5 Conclusion

One of the topics of interests in data envelopment analysis (DEA) is resources allocation and targets setting problems. In some real applications, the input and output variables cannot be satisfied in real conditions and may be taken integer valued. In this paper, we have introduced an IDEA model by the goal programming approach to obtain the optimal selective integer valued of resources allocation and targets setting. The proposed method have some advantages such as: (1) the fixed resources have allocated and fixed targets have set by selective integer assumption based on our new benchmarking method. (2) we have compute the efficiency scores of DMUs based on IDEA model, as well. (3) The major aim in this method is preserving the efficiency scores of DMUs.

The approach in this paper contributes to a field where rich opportunities for modeling are opened in terms of multi-stage evaluation in techno-economic systems such as supply chains. It is plausible that the efficient formulation in this paper may be useful to model these multi-stage systems without exploding in complexity. Other issues concern extensions to dynamic settings to pursue consistent allocation schemes over time, using dynamic decompositions. Finally, the production technology may also be refined to a general case including categorical, environmental, fuzzy and interval data for the input and output parameters.

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