
Influence of deleting some of the inputs and outputs on efficiency status of units in DEA

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Abstract

One of the important issues in data envelopment analysis (DEA) is sensitivity analysis. This study discusses about deleting some of the inputs and outputs and investigates the influence of it on efficiency status of Decision Making Units (DMUs). To this end some models are presented for recognizing this influence on efficient DMUs. Model 2 (Model 3) in section 3 investigates the influence of deleting i(th) input (r(th) output) on an efficient DMU. Thereafter these models are improved for deleting multiple inputs and outputs. Furthermore, a model is presented for recognizing the maximum number of inputs and (or) outputs from among specified inputs and outputs which can be deleted, whereas an efficient DMU preserves its efficiency. Finally, the presented models are utilized for a set of DMUs and the results are reported.

Keywords: Data envelopment analysis; Sensitivity analysis; Efficiency

1 Introduction

Data envelopment analysis (DEA) is a nonparametric approach for evaluating the performance of different organizations that use multiple inputs to produce multiple outputs. For the first time, [3-4] Charnes et al (1978, 1979) introduced CCR model to evaluate relative efficiency of DMUs. DMUs are decision making units that use multiple inputs to produce multiple outputs. The efficiency measure may be changed through modification in the number of inputs and (or) outputs in decision making units. This can be considered as a kind of sensitivity analysis in DEA. Sensitivity analysis of DEA models which is based on the linear programming is both theoretically and practically important. The first DEA sensitivity analysis paper by [2] Charnes et al. (1985) investigated change in a single output. Later, many studies have been conducted in changing the data of the inputs and (or) outputs simultaneously by [18] Seiford et al. (1990), [21] Zhu (2001), [7-8] Cooper et al. (2001, 2007), [19] Valdmanis (1992), [11-12-13] G. R.

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Jahanshahloo et al. (2004, 2005a, 2005b), [9] Grosskopf (1996), [10] Hughes and Yaisawarng (2000) and etc.

Another issue of DEA sensitivity analysis is based on super-efficiency DEA approach in which the test DMU is not included in reference set ([1] Andersen and Petersen, (1993); [17] Seiford and Zhu, (1999)). [5] Charnes et al. (1992), [14] Rousseau and Semple (1995) and [6] Charnes et al. (1996) develop a super-efficiency DEA sensitivity analysis technique for the situation where simultaneous proportional change is assumed in all inputs and outputs for a specific DMU under consideration. This data variation condition is relaxed in [20] Zhu (1996) and [15] Seiford and Zhu (1998a) to a situation where inputs or outputs can be modified separately and the largest stability region which includes that is obtained by [5] Charnes et al. (1992). Also [16] Seiford and Zhu (1998b) generalized the technique in [20] Zhu (1996) and [15] Seiford and Zhu (1998a) to the worst-case scenario where the efficiency of the test DMU is worsening while the efficiencies of the other DMUs are improving.

The issues which has not until now been considered in DEA sensitivity analysis is modification in the number of inputs and (or) outputs. In this paper the number of inputs and outputs has been modified and the result has been presented through some models for deleting the inputs and outputs.

The present study is organized as follows:

Section 2) The basic DEA models and related concepts are reviewed.

Section 3) Some models are presented for recognizing the influence of deleting some inputs and outputs on efficient DMUs. In subsection 3.1, Model 2 (Model 3) investigates the influence of deleting i(th) input (r(th) output) on an efficient DMU. These models are improved for deleting multiple inputs and outputs in subsection 3.2. Furthermore, in subsection 3.3 a model is presented for recognizing the maximum number of inputs and (or) outputs from among specified inputs and outputs which can be deleted, whereas an efficient DMU preserves its efficiency.

Section 4) A set of DMUs are presented and by deleting some of the inputs and (or) outputs, the presented models in section 3 are utilized for this set of DMUs and the results are reported. Finally,

Section 5) The results are synthesized and concluded.

2 Preliminary

Suppose n DMUs are evaluated, each of them consumes m inputs to produce s outputs. Suppose $X_j = (x_{1j}, x_{2j}, \dots, x_{mj})^T$ and $Y_j = (y_{1j}, y_{2j}, \dots, y_{sj})^T$ are as the inputs and outputs of DMU_j for j=1,...,n. For the first time, charnes et al (1978) laid the foundation of DEA through introducing CCR model. The multiplier form of this model is as follows:

$$\text{Max } \sum_{r=1}^s u_r y_{ro} \tag{1}$$

$$\text{S.t. } \sum_{i=1}^m v_i x_{io} = 1$$

$$\sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0 \quad j = 1, \dots, n$$

$$u_r \geq 0 \quad r = 1, \dots, s$$

$$v_i \geq 0 \quad i = 1, \dots, m$$

That O is the index of the evaluated DMU and $U = (u_1, u_2, \dots, u_s) \in R^s$ and $V = (v_1, v_2, \dots, v_m) \in R^m$

Definition 2.1. DMU_o is called CCR efficient if the following conditions are acknowledged:

$$1) \sum_{r=1}^s u_r^* y_{ro} = 1$$

2) At least in one optimal solution of this model $u_r > 0$ for $r = 1, 2, \dots, s$ and $v_i > 0$ for $i=1, 2, \dots, m$.

If both conditions are acknowledged DMU_o is called strong efficient and if just first condition is acknowledged DMU_o is called weak efficient.

Definition 2.2. DMU_o is called CCR inefficient if $\sum_{r=1}^s u_r^* y_{ro} < 1$.

3 Suggested models for deleting some inputs (outputs)

Suppose that DMU_o has been evaluated efficient. Now it is reevaluated through deleting one or multiple inputs and (or) outputs to find out the impact of this modification on its efficiency status. In this section, some models are presented for recognizing the influence of deleting one or multiple inputs (outputs) on efficiency status of DMU_o . Furthermore, a model is presented for recognizing the maximum number of inputs and (or) outputs from among specified inputs and outputs which can be deleted, whereas DMU_o preserves its efficiency.

3.1. Deleting one input or one output

Suppose that DMU_o with m inputs and s outputs has been evaluated inefficient. Now one of the inputs or outputs is deleted. The optimal value of objective function of model (1) will not be better through this deletion, because deleting any input or output is equivalent to deleting a variable in model (1), so it is still preserved its inefficiency. So in current study only the efficient (at least weak efficient) DMUs are considered. In this section it is going to present a model to find out whether the efficient DMU is preserved its efficiency or it has been changed to inefficient with deleting of one input or one output. If at least in one of the optimal solutions of a linear programming, the value of a variable equals zero, then deletion of that variable has no effect on optimality. So, for recognizing the impact of the deletion of one input (one output) on DMU_o efficiency, it is enough to consider the value of corresponding weight with that input (output) in all optimal solutions. So for recognizing the impact of the deletion of input i for $i=1, 2, \dots, m$ on DMU_o efficiency, the following model is presented:

$$\begin{aligned}
 &Min \quad v_i \\
 &S.t \quad \sum_{r=1}^s u_r y_{ro} = 1 \\
 &\quad \quad \sum_{i=1}^m v_i x_{io} = 1 \tag{2} \\
 &\quad \quad \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0 \quad j = 1, \dots, n \\
 &\quad \quad u_r \geq 0 \quad r = 1, \dots, s \\
 &\quad \quad v_i \geq 0 \quad i = 1, \dots, m
 \end{aligned}$$

Model (2) minimizes the weight of i (th) input among all of the optimal solutions of model (1) in evaluating efficient DMU_o . If the minimum value is zero then deleting this input has no effect on the efficiency of DMU_o . Otherwise, DMU_o will become inefficient with deleting i (th) input. The presented model (model (2)) is the modified multiplier form of CCR model which is applied for the first time in order to show the impact of deleting inputs on the efficiency status of efficient DMU_o .

Consecutively, for recognizing the impact of the deletion of output r for $r=1, 2, \dots, s$ on DMU_o efficiency, the following model is presented:

$$\begin{aligned}
 & \text{Min } u_r \\
 & \text{S.t. } \sum_{r=1}^s u_r y_{ro} = 1 \\
 & \quad \sum_{i=1}^m v_i x_{io} = 1 \\
 & \quad \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0 \quad j = 1, \dots, n \\
 & \quad u_r \geq 0 \quad r = 1, \dots, s \\
 & \quad v_i \geq 0 \quad i = 1, \dots, m
 \end{aligned} \tag{3}$$

It is important noting to this point that some of the classic DEA models would not provide an acceptable solution through deleting inputs or outputs. So in this study it is supposed that at the time of deleting one input (one output) at least another input (another output) there exists. It means that $m \geq 2$ ($s \geq 2$) before deleting an input (an output).

Theorem 3.1.

- a) Suppose that DMU_o is efficient (at least weak efficient). With deleting i(th) input, DMU_o still preserves its efficiency if and only if in model (2) $v_i^* = 0$.
- b) Suppose that DMU_o is efficient (at least weak efficient). With deleting r(th) output, DMU_o still preserves its efficiency if and only if in model (3) $u_r^* = 0$.

Proof.

- a) Suppose that with deleting i(th) input, DMU_o still preserves its efficiency (at least weak efficiency). It means that in model (1), after removing the variable v_i the optimal value of objective function equals one. Suppose the optimal solution of model (1) after removing variable v_i is $(v_1^*, \dots, v_{i-1}^*, v_{i+1}^*, \dots, v_m^*, u_1^*, \dots, u_s^*)$. This solution together with $v_i = 0$ is a feasible and also optimal solution for model (2). Now suppose that in model (2), $v_i^* = 0$. It's clear that any optimal solution of model (2) (regardless v_i) will be an optimal solution for model (1) (after removing v_i) with the objective function of (1). So with deleting i(th) input, DMU_o still preserves its efficiency (at least weak efficiency).
- b) Proof is similar to part (a). ■

3.2. Deleting multiple inputs and (or) outputs

In this subsection model (2) and model (3) are generalized for deleting multiple inputs and (or) outputs. Suppose that DMU_o is efficient in presence on m inputs and s outputs. Now the impact of deleting inputs $i_1, i_2, \dots, i_k; 0 \leq k \leq m$ and outputs $r_1, r_2, \dots, r_h; 0 \leq h \leq s$ on efficiency status of DMU_o is investigated. Based on the mentioned issues on deleting one input or one output if there is an optimal solution in evaluated DMU_o in which $v_{i_p}^* = 0$ for each $p=1, \dots, k$ and $u_{r_q}^* = 0$ for each $q=1, \dots, h$ then with deleting all of these k inputs and h outputs, DMU_o is still preserved its efficiency (at least its weak efficiency). Then model (2) and model (3) are generalized as follows for deleting multiple inputs and (or) outputs.

$$\begin{aligned}
 f^* &= \text{Min } \sum_{p=1}^k v_{i_p} + \sum_{q=1}^h u_{r_q} \\
 \text{S.t } \quad & \sum_{r=1}^s u_r y_{r_o} = 1 \\
 & \sum_{i=1}^m v_i x_{i_o} = 1 \\
 & \sum_{r=1}^s u_r y_{r_j} - \sum_{i=1}^m v_i x_{i_j} \leq 0 \quad j = 1, \dots, n \\
 & u_r \geq 0 \quad r = 1, \dots, s \\
 & v_i \geq 0 \quad i = 1, \dots, m
 \end{aligned} \tag{4}$$

In the model mentioned above, the first constraint is the efficiency condition for DMU_o and the rest are as the constraints of CCR model. Deleting of input i_p (output r_q) implies that $v_{i_p}^* = 0$ ($u_{r_q}^* = 0$). Therefore, as these k inputs and h outputs are deleted (While efficiency of DMU_o is preserved), the corresponding weights should be zero. So the sum of weights related to deleted inputs and outputs should be minimized.

Theorem 3.2. Suppose that DMU_o is efficient (at least weak efficient). In model (4) $f^* = 0$ if and only if with deleting the inputs i_1, i_2, \dots, i_k and outputs r_1, r_2, \dots, r_h DMU_o still preserves its efficiency.

Proof.

Proof is similar to theorem 3.1 ■

3.3. Find out the maximum number of deleted inputs and (or) outputs

Suppose that with deleting inputs i_1, \dots, i_k ($1 \leq k \leq m$) and outputs r_1, \dots, r_h ($1 \leq h \leq s$) simultaneously, an efficient DMU_o becomes inefficient. However DMU_o may still preserves its efficiency with deleting some of these inputs and outputs. In this subsection a model is presented which find out the maximum number of inputs and outputs that with deleting them DMU_o still preserves its efficiency.

$$\begin{aligned}
 g^* &= \text{Max } \sum_{i=1}^m d_i + \sum_{r=1}^s d'_r \\
 \text{S.t } \quad & \sum_{r=1}^s u_r y_{r_o} = 1 \\
 & \sum_{i=1}^m v_i x_{i_o} = 1 \\
 & \sum_{r=1}^s u_r y_{r_j} - \sum_{i=1}^m v_i x_{i_j} \leq 0 \quad j = 1, \dots, n \\
 & 0 \leq v_i \leq M(1 - d_i) \quad i = 1, \dots, m \\
 & 0 \leq u_r \leq M(1 - d'_r) \quad r = 1, \dots, s \\
 & d_i \in \{0,1\} \quad i = 1, \dots, m \\
 & d'_r \in \{0,1\} \quad r = 1, \dots, s \\
 & v_i \geq 0 \quad i = 1, \dots, m \\
 & u_r \geq 0 \quad r = 1, \dots, s
 \end{aligned} \tag{5}$$

The model mentioned above is an integer linear programming which can be solved with the integer linear programming methods.

According to theorem 3.1 in model (5) $v_i^*=0$ ($u_r^*=0$) means i(th) input (r(th) output) could be deleted while DMU_o still preserves its efficiency. So $v_i^* > 0$ ($u_r^* > 0$) means remaining i(th) input (r(th) output) is necessary for preserving efficiency of DMU_o . For deciding about deleting or remaining the i(th) input and (or) r(th) output, model (5) considers the following constrains.

$$0 \leq v_i \leq M(1 - d_i) \quad i = 1, \dots, m$$

$$0 \leq u_r \leq M(1 - d'_r) \quad r = 1, \dots, s$$

In these two constraints, d_i and d'_r are binary variables. If $d_i^* = 1$ ($d'_r = 1$) then $v_i^* = 0$ ($u_r^* = 0$). Furthermore if $d_i^* = 0$ ($d'_r = 0$) then $v_i^* > 0$ ($u_r^* > 0$), because of maximization of (5). So $d_i^* = 1$ ($d'_r = 1$) means that i(th) input (r(th) output) could be deleted. Therefore $\sum_{i=1}^m d_i + \sum_{r=1}^s d'_r$ shows the number of deleted inputs and (or) outputs. On the other hand since the point is to maximize the number of deleted inputs and (or) outputs, so the objective function is considered as $Max \sum_{i=1}^m d_i + \sum_{r=1}^s d'_r$. Moreover the first constraint of model (5) is related to the efficiency of DMU_o and the other constraints are related to model (1).

Theorem 3.3. If DMU_o with m input and s output is efficient then model (5) will be feasible.

Proof.

As DMU_o is efficient so model (1) has the optimal solution with objective function equals one. So the feasible solution of model (1) is feasible for the following system.

$$\begin{cases} \sum_{r=1}^s u_r y_{ro} = 1 \\ \sum_{i=1}^m v_i x_{io} = 1 \\ \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0 & j = 1, \dots, n \\ u_r \geq 0 & r = 1, \dots, s \\ v_i \geq 0 & i = 1, \dots, m \end{cases}$$

The feasible solution of above model together with $d_1 = \dots = d_m = d'_1 = \dots = d'_s = 0$ is a feasible solution for model (5). ■

Theorem 3.4. The maximum number of inputs and outputs which could be deleted whereas DMU_o still preserves its efficiency equals g^* .

Proof.

With regard to maximizing the objective function of model (5), it's evident that it is preferred the binary variables d_i and d'_r for $i=1, \dots, m$ and $r=1, \dots, s$, are chosen 1. So by regarding to the constraints

$$0 \leq v_i \leq M(1 - d_i) \quad i = 1, \dots, m$$

$$0 \leq u_r \leq M(1 - d'_r) \quad r = 1, \dots, s$$

If $d_i^* = 1$ ($d'_r = 1$) then v_i^* (u_r^*) should be zero. This means i(th) input (r(th)) output could be deleted. On the other hand if $d_i^* = 0$ ($d'_r = 0$) then $v_i^* > 0$ ($u_r^* > 0$). This means the corresponding input or output

could not be deleted. So $\sum_{i=1}^m d_i + \sum_{r=1}^s d'_r$ indicates the number of deleted inputs and or outputs. With considering this issue and the feasibility of model (5) it can be concluded that g^* equals the maximum number of inputs and (or) outputs which could be deleted in a way that the efficient DMU_o still preserves its efficiency. ■

4 Numerical example

Now the presented models in this paper are used for the data of table1 related to twelve DMUs with two inputs and outputs. These data are extracted from cooper et al (2007). Evaluating these DMUs through CCR model has been revealed that DMUs A, B, and D are efficient.

Table 1: DMU's data (cooper et al,2007)

Hospital	Inputs		Outputs	
	Doctor	Nurse	Outpatient	Inpatient
A	20	151	100	90
B	19	131	150	50
C	25	160	160	55
D	27	168	180	72
E	22	158	94	66
F	55	255	230	90
G	33	235	220	88
H	31	206	152	80
I	30	244	190	100
J	50	268	250	100
K	53	306	260	147
L	38	284	250	120

Now the results of deleting the inputs and outputs on the status of efficiency of the efficient DMUs are presented in tables 2 and 3. The results of table 2 are obtained through using the software DEA-Solver and the results of table 3 are derived by solving models (2), (3) and (4) through using the software Lingo. With comparison table 2 and 3 it can be seen that the obtained results are consistent.

Table 2: Results of extracted from DEA-Solver

Deleted input and (or) output	θ_A^*	θ_B^*	θ_D^*
input1	1	1	1
input2	1	1	0.901961
output1	1	0.640373	0.719048
output2	0.633333	1	0.935714
input1-output1	1	0.640373	0.719048
input1-output2	0.578366	1	0.935714
input2-output1	1	0.584795	0.592593
input2-output2	0.633333	1	0.844444

Table3: Results of solving model (2), model(3), model(4)

	Model(2)		Model(3)		Model(4)			
	Min	Min	Min	Min	Min	Min	Min	Min
DMU_o	v_1	v_2	u_1	u_2	$v_1 + u_1$	$v_1 + u_2$	$v_2 + u_1$	$v_2 + u_2$
DMU_A	0	0	0	$\neq 0$	0	$\neq 0$	0	$\neq 0$
DMU_B	0	0	$\neq 0$	0	$\neq 0$	0	$\neq 0$	0
DMU_D	0	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$	$\neq 0$

Take DMU_D as an example in above table. As it can be seen in table (3), with solving model (2) for $i = 1$ we have $v_1 = 0$, which means with deleting the first input, the efficient DMU_D still preserves its efficiency. It can also be seen in table (2). As it is demonstrated in table 2, with deleting the first input and then evaluating DMU_A , DMU_B and DMU_D through DAE-SOLVER all these efficient units still preserve their efficiency. Moreover, with solving model (2) for DMU_D and $i=2$ we have $v_2 \neq 0$ which means with deleting the second input, the efficient DMU_D will become inefficient (see table (3)). As table 2 shows, with deleting second input and then evaluating units through DAE-SOLVER, DMU_A and DMU_B still preserve their efficiency. However unit D becomes inefficient.

Now through solving model (5) we can determine the maximum number of inputs and outputs that could be deleted in a way that the efficient units still preserves their efficiency. For this purpose we solve model (5) through using the software Lingo and the related results are presented in table 4.

Table4: Results of solving model (5)

	DMU_A	DMU_B	DMU_D
g^*	2	2	1

Take DMU_A as an example in above table. As it can be seen, with solving model (5) the optimal value of objective function equals two which means the maximum number of inputs and (or) outputs of DMU_A can be removed so that DMU_A is still preserves its efficiency equal to two (see table 4). Since $v_1 + u_1 = 0$, $v_2 + u_1 = 0$ and this means with deleting the first input and output simultaneously (or deleting the second input and first output simultaneously), DMU_A still preserves its efficiency.

5 Conclusion

In this paper first a model has been presented that can be used for recognizing that whether efficient DMU_o still preserves its efficiency or it will become inefficient by deleting one input or one output. The mentioned model is the modified multiplier form of CCR model which has been applied for the first time in order to investigate the impact of deleting inputs (outputs) on the efficiency of efficient DMU_o . It may be investigated the impact of deleting a subset of inputs and (or) outputs on efficiency status of DMU_o . So these models have been generalized for deleting multiple inputs and (or) outputs. Then a model has been presented that can find out the maximum number of inputs and outputs which can be deleted whereas efficient DMU_o still preserves its efficiency. Finally the mentioned models have been utilized in a set of DMUs and the results have been presented.

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