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Research Article

A new DEA approach to rank alternatives in MCDA

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Abstract

One of the principal subjects in multiple criteria decision analysis is ranking alternatives. Here, we present a new method to rank alternatives by using data envelopment analysis. In this paper, one ranking method is proposed based on applying an artificial alternative called aggregate alternative. The method is based on the fact that one efficient alternative with a better performance has stronger effects on the group of other alternatives. That means its deletion forces the remaining alternatives to get smaller efficiency. The described idea in this paper is inspired of Lotfi and et al. (2011). One feature of the proposed method is that it does not need to determine the weight of the prior. Two examples are used to illustrate how the proposed method works in actual practices, and the results are compared with those obtained from the TOPSIS method.

Keywords: Data envelopment analysis, Aggregate alternative, Decision making, Ranking.

1 Introduction

The research area of multiple criteria decision analysis (MCDA) is developed to provide decision aids for complex decision situations. MCDA aims to furnish a set of decision analysis techniques to help decision makers (DMs) that logically identify, compare, and evaluate alternatives according to diversity, usually conflicting, criteria arising from social, economic, and environmental considerations [1–7].

In MCDA, a decision maker (DM) must evaluate alternatives with regard to each criterion, address criteria weights, and select the best result from the generated set of alternatives. The MCDA approach provides an effective way to select among non-commeasurable and conflicting criteria. Some representative methods, such as the simple additive weighting method (SAW), the analytic hierarchy process (AHP), and the technique for order preference by similarity to ideal solution (TOPSIS), have been developed to solve MCDA problems [8, 9, 10]. Also, Kao proposes a measure of relative distance, which involves the calculation of the relative position of an alternative between the anti-ideal and ideal for ranking [11].

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In this paper, one ranking method is proposed based on applying aggregate alternative that will be described later based on data envelopment analysis. The method is based on the fact that one alternative with a better performance has stronger effects on the group of other alternatives. That means that its deletion forces the remaining alternatives to get smaller efficiency score. The rest of the paper is organized as following:

In Section 2, data envelopment analysis is presented.

In Section 3, the data envelopment analysis (DEA) technique for identifying non-dominated alternatives is reviewed.

In Section 4, aggregate alternative with its application, the proposed method is introduced.

In Section 5, by exposing two illustrative examples, the power of our proposal will be examined.

In the last section, concluding remarks are presented.

2 Data envelopment analysis

Data Envelopment Analysis (DEA) as introduced by Charnes et al. [12] (CCR) is a non-parametric performance assessment methodology to measure the relative efficiency of a set of homogeneous Decision Making Units (DMUs) such as bank branches, hospitals, which consume one or more inputs to produce one or more outputs. The main characteristics of DEA are that;

(i) it can be applied to analyze multiple outputs and multiple inputs without pre assigned weights,

(ii) it can be used for measuring a relative efficiency based on the observed data without having information on the production function and

(iii) decision maker preferences can be incorporated in DEA models.

In mathematical terms, consider a set of n DMUs, in which $x_{ij} (i = 1, \dots, m)$ and $y_{rj} (r = 1, \dots, s)$ are inputs and outputs of DMU_j $(j = 1, \dots, n)$. The CCR model, which was suggested by Charnes, Cooper and Rhodes [12], is a fractional linear programming problem and can be solved by transformed into an equivalent linear programming one. The original CCR model in order to maximize the relative efficiency score of DMU_O is as follows:

$$\theta_o^* = \max \frac{\sum_{k=1}^{s} u_k y_{ko}}{\sum_{p=1}^{m} v_p x_{po}}$$

$$\text{S.t.} \qquad \frac{\sum_{k=1}^{s} u_k y_{kj}}{\sum_{p=1}^{m} v_p x_{pj}} \le 1 \qquad j = 1, \dots, n \tag{1}$$

$$u_k \ge \varepsilon, v_p \ge \varepsilon, k = 1, \dots, s; p = 1, \dots, m$$

The model (1) is a fractional model. In order to solve conveniently, its equivalent linear form is used as following:

$$\theta_o^* = \max \sum_{k=1}^{s} u_k y_{ko}$$

$$\sum_{p=1}^{m} v_p x_{po} = 1 \tag{2}$$

$$\sum_{k=1}^{s} u_k y_{kj} - \sum_{p=1}^{m} v_p x_{pj} \le 1 \quad j = 1, \dots, n$$

$$u_k \ge \varepsilon, v_p \ge \varepsilon, k = 1, \dots, s; p = 1, \dots, m$$

DMU_O is efficient if and only if $\theta_o^* = 1$ in the model (2) and otherwise; it is inefficient.

3 The DEA model for ranking alternatives

Assume that there are n alternatives with m criteria to be evaluated. The performance of alternative j in criterion i has the value of Y_{ij}.

When more than one criterion is considered, there will usually be several alternatives which are not dominated by the others; each has at least one criterion which outperforms those of the other alternatives. One of the non-dominated alternatives is chosen for implementation. Charnes et al. [12] proposed the DEA technique to calculate the relative efficiency of a group of decision making units (DMUs) which uses multiple inputs to produce multiple outputs. Each unit is allowed to use different sets of weights to calculate the efficiency. Those with an efficiency value of 1 are non-dominated units, which are called Pareto optimal or efficient units [12]. The MCDA problem can be considered as a DEA problem without inputs, or as a problem in which every alternative has the same amount of input. Hence, the DEA technique can be applied to identify non-dominated alternatives [13].

The DEA model without inputs for calculating the efficiency of the *k*th alternative can be formulated as:

$$\theta_k = \max \sum_{i=1}^{m} w_i Y_{ik}$$

$$S.t. \sum_{i=1}^{m} w_i Y_{ij} \leq 1 \quad j = 1, \ldots, n \tag{3}$$

$$w_i \geq \varepsilon, i = 1, \ldots, m$$

Where $w_i$the importance is associated with the *i*th criterion and ε is a small positive quantity imposed to restrict any criterion from being ignored. The most favorable weights are sought for each DMU in calculating its efficiency. The dual of this model is output-oriented BCC model without inputs.

4 Proposed Method

Definition 4.1

In this section, aggregate alternative which is cornerstone of the proposed method is introduced. This alternative is artificial, and is defined over all alternatives. The performance of aggregates alternative in criterion i has the value of Y_{ia} and shown as:

$$Y_{ia} = \sum_{k=1}^{n} Y_{ik} \qquad i = 1, \ldots, m \tag{4}$$

4.1. The application of aggregate alternative

In this subsection, efficiency score aggregate alternative is computed first as follows:

$$\theta_a^* = \max \sum_{i=1}^{m} w_i Y_{ia}$$

$$S.t. \sum_{i=1}^{m} w_i Y_{ij} \leq 1 \quad j = 1, \ldots, n \tag{5}$$

$$w_i \geq \varepsilon, i = 1, \ldots, m$$

In order to evaluate the efficiency score *p*th alternative, we must clarify the effects of deletion of this alternative from aggregate alternative. In other words, it is necessary to survey that how much change into the efficiency score of aggregate alternative is arisen by deletion of *p*th alternative from the aggregate alternative? To get a response, we delete *p*th alternative of the aggregate alternative and get new aggregate alternative as follows:

$$Y_{ia}^{p} = \sum_{k=1}^{n} Y_{ik} \qquad i = 1, \dots, m \ , i \neq p \tag{6}$$

Then, we calculate efficiency score of new aggregate alternative as:

$$\theta_a^{p*} = \max \sum_{i=1}^{m} w_i Y_{ia}^{p}$$

$$S.t. \sum_{i=1}^{m} w_i Y_{ij} \leq 1 \qquad j = 1, \dots, n \tag{7}$$

$$w_i \geq \varepsilon, i = 1, \dots, m$$

Now, alternatives are ranked based on AR-index and defined as:

$$AR_p = \theta_a^{*} - \theta_a^{p*} \tag{8}$$

The $\mathrm{AR_p}$ indicates the difference between efficiencies of alternative *p*th and aggregate alternative. According to the model (7) is a DEA model with constant input, so just outputs affect the efficiency of the decision making units. Also the constraints of DEA model when calculating the AR index is the same for all alternatives, so AR index is equal for two different alternatives when the objective functions are same and it is not possible according to the formula (6).

5 A numerical example

Now, we illustrate the application of the proposed MCDA models. The proposed models were implemented in MATLAB version 7.8 and were solved using the *linprog* function. The non-Archimedean infinitesimal was set as $\varepsilon = 10^{-4}$.

Example 5.1

Consider a simple example of five alternatives, A, B, C, D, and E. Their performances in two criteria, Y_1 and Y_2, are shown in Table 1. Moreover, the results of the proposed method and DEA model are compared, and organized in it.

Table 1: Data and various performance measures for the given example [11].

Alternative	Y_1	Y_2	DEA efficiency	AR index
A	2	4	0.8(5)	0.6(5)
B	4	5	1(1)	0.9(1)
C	4	4	0.92(4)	0.8(2)
D	5	3	1(1)	0.8(3)
E	5	2	1-ε(3)	0.7(4)

Although, alternatives C and D have equal AR-indexes with one decimal, they have different AR-index with eight decimal.

Example 5.2

Consider an example that appeared in [11]. Ten cars are to be ranked by six criteria: maximum speed (km), horse power (cv), space (m^2), gas consumption in town (1t/100 km), gas consumption at 120 km/h (1t/100 km), and price (francs). Table 2 shows the data. The results of ranking and compared with TOPSIS [17] are shown in table 3.

Table 2: Data for ten cars with sex criteria [10].

No.	Max speed	Horse power	Space	Gas consumption	Gas consumption	price
1	173	10	7.88	11.4	10.01	49.5
2	176	11	7.96	12.3	10.84	46.7
3	142	5	5.65	8.2	7.30	32.1
4	148	7	6.15	10.5	9.61	39.15
5	178	13	8.06	14.5	11.05	64.7
6	180	13	8.47	13.6	10.40	75.7
7	182	11	7.81	12.7	12.26	68.593
8	145	11	8.38	14.3	12.95	55.0
9	161	7	5.11	8.6	8.42	35.2
10	117	3	5.81	7.2	6.75	24.8

Table 3: Rankings for the given example by the TOPSIS and proposed model.

No.	TOPSIS model	Proposed method
	Rc index	AR index
1	0.6329(4)	0.0262(5)
2	0.6783 (2)	0.0265(4)
3	0.6401 (3)	0.0201(9)
4	0.6173 (5)	0.0221(8)
5	0.4979 (7)	0.029(3)
6	0.4359(9)	0.0302(1)
7	0.4847(8)	0.0295(2)
8	0.4164(10)	0.0247(6)
9	0.7447(1)	0.0226(7)
10	0.5564(6)	0.0165(10)

Table 3 shows the results obtained by TOPSIS model and proposed model, respectively. Different methods usually lead to different results. It is inappropriate to say which method is better because every method has a different underlying theory or assertion. However, some methods are more suitable than others for certain cases.

The final ranking from TOPSIS is different from the proposed method. This is not surprising because different methods usually lead to different results. It is inappropriate to say which method is better because every method has a different underlying theory or assertion. However, some methods are more suitable than others for certain cases.

The proposed methods ranked alternative 6 the best while TOPSIS ranked it ninth. In contrast, TOPSIS ranked alternative 9 the best while the proposed methods ranked it seventh. According to the ranking of the alternative 6 and other alternatives it can be concluded that the proposed method is appropriate for problems included in the beneficial criteria only.

6 Conclusion

Multi-Criteria Decision analysis has been one of the fastest growing problem areas in many disciplines. In this paper, a simple method with respect to computational aspect of ranking alternatives in MCDA is offered. The method is based on the effect of each alternative on the performance of aggregate alternative. In order to do this, an index is used to illustrate this effect. Actually, the more influence in the aggregate alternative conclude the larger rank for it. In order to measure its influence, the DEA model is used.
There are two types of weight acquired for representing the importance of each criterion: a priori weights determined by experts and a posteriori weights obtained from the data. This paper adopted the posteriori approach. Hence, it is suitable for cases where no prior information can be used for determining the weights.

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