Communications in Numerical Analysis 2018 No.1 (2018) 1-20

Available online at www.ispacs.com/cna

Volume 2018, Issue 1, Year 2018 Article ID cna-00330, 20 Pages

doi:10.5899/2018/cna-00330

Research Article

Evs. in Green Grid Era and climate change

F. -H. Li[1]*, Y. Kamata[2], K. Maruyama[2], S. Kanemitsu[2]

(1) *Sanmenxia Suda Communication Group,Sanmenxia 472000,Henan,China.*

(2) *Department of Information Science, Kinki University, Iizuka, Fukuoka 820-8555, Japan.*

Abstract

In this paper we set up foundations for vehicle dynamics and apply them to possible future EVs—wireless in-wheel motor elctric vehicles (wirelsee IWMEVs)—in the coming green grid (GG) era. GG is the most advanced form of the coming smart gird in which electricity is to be generated only through harmless resources, mainly solar energy. All the cars are to be EVs which are wirelessly connected to the GG through a regional aggregator which survey the area and in case of need, the emergency signal is to be sent to the vehicle to make a necessary action for collision avoiding. We also discuss the climate change in general and global warming in particular and state why global warming is a grave issue for the human being through archeo-climatology. We state a plausible waether-cycle conjecture of 1470 years as a combination of conjectures of H. Noshioka and the latest ones based on the Dansgaard-Oeschger (DO) cycles.

Keywords: Collision avoiding system, Vehicle dynamics, Equillibrium of lateral forces, Coriolis force, Climate change.

1 Introduction

It is certain that all fossil fuels will be used up in the near future and any industrial activities will be estimated from the point of view of WTW efficiency **From Well To Wheel efficiency**, i.e. the efficiency of the used material and environmental protection. Only those activities will be allowed to make which have high WTW efficiency and are environmental-friendly. As long as fossil fuels are available, inner combustion engine cars can be driven with high cost of damaging the environment. An **electric vehicle** (EV) is a type of a car which moves by transmitting the power from a battery in a car to wheels. The WTW efficiency for inner combustion cars resp. for EV's is the efficiency of oil dug out from the oil-well, processed and finally used as gasoline or diesel oil. resp. that of oil used to generate electricity. There is a principle in the universe that the simplest is the best. In this respect, EV's are the strongest candidate for the only possible solution for transportation save for railroads. For the principle of their mechanism is so simple that already in 1873, 12 years before the first inner combustion engine cars, EV's were produced for practical use. Indeed, it consists of a battery and a motor in place of a very complicated mechanism of inner combustion engines whose heat efficiency is only 12% since they can use only the explosion energy. However, the cheap oil prevailed the market and the last 100 years were the time of inner combustion engine cars. It is the time to think of the future when there is no more fossil fuel available. Then we need to appeal to other natural resources, esp. solar energy which is the largest of all and is the source for other natural resources including wind, water etc. EV's can move as long as they have electricity and electricity in turn has a great variety of ways to generate. As will be mentioned, **wireless in-wheel EV's** [24] have a possibility of charging during travel on high-ways from the coils set up on the roadside.

*Corresponding author. Email address: kanemitu@fuk.kindai.ac.jp

This is expected to solve the weak point of EV's that they are expensive because of the batteries–lithium ion batteries in particular.
The future cars must be EV's connected to the **Green Grid** (GG) and are being monitored by the control center of GG. Here we mean GG the advanced Smart Grid which does not depend on atomic energy nor fossil fuels and is to be the cleanest of all.
As mentioned in [14], electricity looks clean

Table 1: Defects of electricity

	strong pt	defect
electricity	clean if the generation is so	cannot be stored
fossil fuels	increase of CO_2	can be regulated
Nukes	looks clean	radiative pollution

In this paper, we discuss the emerging importance of EV's in the coming GG Era and also mention the global warming as an aspect of climate change. An EV is a vehicle so that vehicle dynamics applies but with flexibility of in-wheel motors and we may think of a fail-safe guard, an advanced collision avoiding system.
What is proposed is the oblique or even lateral drive to avoid danger ahead. Reaction by humans takes some time and this loss of time may result in catastrophe. There is an elapse of time of total 0.05s for the information transmission through nerves (0.02) together with the mechanical reaction time (0.03) of the vehicle before any action can be taken. Moreover we must recall the startling fact that we live 0.5s past [21] (or more recent 80-millisecond rule). Therefore, in the future situation where all EV's are connected to the GG, each ancillary station (real or virtual) is monitoring the positions of all of the vehicles on the map and can send orders to the driving system directly to take a necessary action as soon as the computer can predict the danger. This includes not only sudden brakes but also shifting to the safer side of the road by lateral driving.

2 Transitional states

Our objective in this section is to pave a road to clarifying both Coriolis force and vehicle dynamics by Theorem 7.1 which describes a transition from one state to another and gives a relation between derivatives in inertia system and rotating system.
We refine arguments given in [**?**, pp. 14-21], [12, p. 40], [27, pp. 62-70], [29, pp. 25-26, 141-149] etc. It turns out that up to a certain point, calculations amount to the Leibniz rule for differentiation of the product of two functions. In vehicle dynamics, analysis of characteristics of dynamics is made using the characteristic equation of the system of differential equations or Laplace transform thereof. Since this system of DEs can be solved explicitly either by linear algebra, Laplace transform, symbolic calculus, etc., we give an explicit solution based on linear algebra in Appendix A. Then based on the form of the solutions, we analyze the relation between steering angles and the radius of steady-state rotation.
The following theorem is fundamental for both Coriolis force and vehicle dynamics in the sense that by taking b to be the position vector resp. velocity vector, (2.2) (=(7.81)) gives the velocity vector resp. acceleration vector, where the reference to different coordinate system is to be noticed.

Theorem 2.1. *Let w_1, w_2, w_3 be fundamental unit vectors constituting the curvilinear orthogonal coordinate system. Let*

$$b = b_1 w_1 + b_2 w_2 + b_3 w_3, \tag{2.1}$$

where

$$b_j = b \cdot w_j.$$

Then we have

$$\left(\frac{\mathrm{d}b}{\mathrm{d}t}\right)_I = \sum_{i=1}^{3}\frac{\mathrm{d}b_i}{\mathrm{d}t}w_i + \sum_{i=1}^{3} b_i\frac{\mathrm{d}w_i}{\mathrm{d}t} = \left(\frac{\mathrm{d}b}{\mathrm{d}t}\right)_R + \sum_{i=1}^{3} b_i\frac{\mathrm{d}w_i}{\mathrm{d}t}, \tag{2.2}$$

where the suffix means that the velocity is observed by the rotating observer (R) *or by a non-rotating inertial observer* (I)*, respectively.*

Proof. The proof follows from the Leibniz rule. □

Remark 2.1. *To express the second term in closed form, we use the expression in Lemma 7.1 below:*

$$\frac{\mathrm{d}w_i}{\mathrm{d}t} = \Omega \times w_i.$$

Hence the 2nd term is

$$\sum_{i=1}^{3} b_i(\Omega \times w_i) = \Omega \times \sum_{i=1}^{3} b_i w_i = \Omega \times b.$$

Theorem 2.2. *We choose the orthogonal curvilinear coordinate system in Theorem 7.1 as follows:*

$$w_1 = \begin{pmatrix}\cos\gamma\\ \sin\gamma\\ 0\end{pmatrix}, w_2 = \begin{pmatrix}-\sin\gamma\\ \cos\gamma\\ 0\end{pmatrix}, w_3 = \begin{pmatrix}0\\0\\1\end{pmatrix} \tag{2.3}$$

and we take

$$\Omega = \gamma' w_3. \tag{2.4}$$

Then for coordinates (2.1)*,* (2.2) *reads*

$$\left(\frac{\mathrm{d}b}{\mathrm{d}t}\right)_I = (b_1' - b_2\gamma')w_1 + (b_2' + b_1\gamma')w_2 + b_3' w_3. \tag{2.5}$$

and its differentiated form is

$$\left(\frac{\mathrm{d}^2 b}{\mathrm{d}t^2}\right)_I = (b_1'' - 2b_2'\gamma' - b_1\gamma'^2 - b_2\gamma'')w_1 + (b_2'' + 2b_1'\gamma' - b_2\gamma'^2 + b_1\gamma'')w_2 + b_3'' w_3. \tag{2.6}$$

In the special case of the rotation of constant angular velocity $\omega > 0$*,* $\gamma = \gamma(t) = \omega t$*, then* (2.5) *amounts to*

$$\left(\frac{\mathrm{d}b}{\mathrm{d}t}\right)_I = (b_1' - \omega b_2)w_1 + (b_2' + \omega b_1)w_2 + b_3' w_3, \tag{2.7}$$

where $\Omega = \omega w_3$*.* (2.6) *reads*

$$\left(\frac{\mathrm{d}^2 b}{\mathrm{d}t^2}\right)_I = (b_1'' - 2\omega b_2' - b_1\omega^2)w_1 + (b_2'' + 2\omega b_1' - b_2\omega^2)w_2 + b_3'' w_3. \tag{2.8}$$

Proof. To express (2.2) in the form of (2.5) we need to use the formula

$$w_1' = \gamma'\begin{pmatrix}-\sin\gamma\\ \cos\gamma\\ 0\end{pmatrix} = \gamma' w_3 \times w_1 = \gamma' w_2, \tag{2.9}$$

$$w_2' = \gamma'\begin{pmatrix}-\cos\gamma\\ -\sin\gamma\\ 0\end{pmatrix} = \gamma' w_3 \times w_2 = -\gamma' w_2.$$

To prove (2.6) we differentiate (2.5) thereby using (2.9). □

The above proof is a little cumbersome because we do not have a rule for transition–differentiation. The following theorem gives an explicit rule for differentiation.

Theorem 2.3. *Let*

$$A_\gamma = \begin{pmatrix} \cos\gamma & -\sin\gamma \\ \sin\gamma & \cos\gamma \end{pmatrix}. \tag{2.10}$$

Then the linear transformation $y = A_\gamma x$ by A_γ means the rotation (in the positive direction, i.e. counterclockwise) w.r.t. the origin by γ. Suppose $\gamma = \gamma(t)$ is a function in time t. Then

$$\frac{\mathrm{d}}{\mathrm{d}t} A_\gamma = \gamma' A_{\gamma-\frac{\pi}{2}} = \gamma' A_\gamma S, \tag{2.11}$$

where

$$S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \tag{2.12}$$

We have

$$\frac{\mathrm{d}}{\mathrm{d}t}(A_\gamma x) = A_\gamma\left(x' + \gamma' S x\right) \tag{2.13}$$

and

$$\begin{aligned} \frac{\mathrm{d}^2}{\mathrm{d}t^2}(A_\gamma x) &= \gamma' A_\gamma\left(Sx' + \gamma' S^2 x\right) + A_\gamma\left(x'' + \gamma' S x' + \gamma'' S x\right) \\ &= A_\gamma\left(x'' + 2\gamma' S x' + \left(-\gamma'^2 + \gamma'' S\right) x\right). \end{aligned} \tag{2.14}$$

In the case where $\gamma = \omega t$ (2.13) *and* (2.14) *read*

$$\frac{\mathrm{d}}{\mathrm{d}t}(A_\gamma x) = A_\gamma\left(x' + \omega S x\right) \tag{2.15}$$

and

$$\frac{\mathrm{d}^2}{\mathrm{d}t^2}(A_\gamma x) = A_\gamma\left(x'' + 2\omega S x' + \left(-\omega^2\right) x\right) \tag{2.16}$$

respectively.

We note that $A_\gamma^{-1} = A_{-\gamma}$ and $A_{-\gamma} A_{\gamma-\frac{\pi}{2}} = A_{-\frac{\pi}{2}} = -S$, so that

$$A_{\gamma-\frac{\pi}{2}} = A_\gamma S. \tag{2.17}$$

By augmenting the matrix $A_\gamma^{(k)}$ $(k = 0, 1)$ as

$$\tilde{A}_\gamma^{(k)} = \begin{pmatrix} A_\gamma^{(k)} & o \\ o & 1 \end{pmatrix} = \begin{pmatrix} \cos^{(k)}\gamma & -\sin^{(k)}\gamma & 0 \\ \sin^{(k)}\gamma & \cos^{(k)}\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{2.18}$$

we may view Theorem 2.2 as a special case of Theorem 2.3. (2.13) corresponds to (2.5). Indeed, writing $\frac{\mathrm{d}^2}{\mathrm{d}t^2}(A_\gamma b) = a_I$, $\frac{\mathrm{d}}{\mathrm{d}t} b = v_R$ etc. for the accleration, velocity in the inertia or rotating system, we may express (2.13) and (2.14) as

$$v_I = A_\gamma\left(v_R + \gamma' S b\right) \tag{2.19}$$

and

$$a_I = A_\gamma\left(a_R + 2\gamma' S v_R + \left(-\gamma'^2 + \gamma'' S\right) b\right) \tag{2.20}$$

which amounts to (2.5) and (2.6), respectively.
For vehicle dynamics we specify $\gamma = \omega t$ and

$$\begin{pmatrix} v_x \\ v_y \end{pmatrix} = b = v_R = V \begin{pmatrix} \cos\beta \\ \sin\beta \end{pmatrix}, \tag{2.21}$$

where $V > 0$ is the maginitude of the velocity and $\beta = \beta(t)$ is a slip angle which is infinitesimal. Then (2.19) reads

$$a_I = A_\gamma \left(a_R + \gamma' S v_R \right). \tag{2.22}$$

Since

$$\frac{\mathrm{d}}{\mathrm{d}t} v_R = V\beta' \begin{pmatrix} -\sin\beta \\ \cos\beta \end{pmatrix}, \tag{2.23}$$

the second factor on the right of (2.22) is

$$\begin{aligned} a_R + \omega S V \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \cos\beta \\ \sin\beta \end{pmatrix} &= V \begin{pmatrix} -(\beta' + \omega)\sin\beta \\ (\beta' + \omega)\cos\beta \end{pmatrix} \\ &= V \begin{pmatrix} -(\beta' + \omega)\beta \\ (\beta' + \omega) \end{pmatrix} + O(\beta^2) \end{aligned}$$

as $\beta \to 0$. Hence

$$a_I = V A_\gamma \begin{pmatrix} -(\beta' + \omega)\beta \\ (\beta' + \omega) \end{pmatrix}. \tag{2.24}$$

(2.24) in the coordinates (2.3) reads

$$a_I = -V\beta(\beta' + \omega) w_1 + V(\beta' + \omega) w_2 + a_3 w_3, \tag{2.25}$$

where a_3 is the third component of a_I and we omitted the error term. Since for a vehicle, we may restrict to the 2-dimensional movement, Newton's equation of motion reads

$$m a_I = -mV\beta(\beta' + \omega) w_1 + mV(\beta' + \omega) w_2. \tag{2.26}$$

3 Vehicle dynamics

During a high speed cornering of a vehicle, as a result of centripetal force, there occurs a discrepancy angle β called the **slip angle** (or sideslip angle) of its actual direction of travel from that of **steering angle** θ (the direction towards which a rolling wheel is pointing). I.e., it is the angle of the vector sum of wheel forward velocity v_x and lateral velocity v_y in (2.21). While moving, there occurs a friction between tires and road surface and the component of frictions perpendicular to the direction of travel is called the **cornering force** (CF). The safe cornering is possible as long as the sum of driving force and inertia force remains within the friction circle, a circle within which the tires keep grip on the road surface, i.e. by the balance of the centripetal force and the sum of cornering forces of all tires.
A yaw rotation with angle γ is a movement around the yaw axis of a vehicle that changes its direction of motion.
The results here are not only modifications but also refinements of those given in [7, pp. 24-28] and [27, pp. 62-70].
Recall Theorem 2.3 and note that $V\sin\beta \sim v\beta$ and that V is a constant. We use the following notation and parameters for a vehicle.

Table 2: Vehicle parameters

m	mass
V	velocity
I	yaw moment of inertia
γ	yaw angular velocity
$\beta = \beta_g$	sideslip angle
β_*	tire sideslip angle
θ_*	tire steering angle
l	wheelbase ($l = l_f + l_r$)
l_*	distance between axle and center of gravity
t_d	axle track ($d = 1/2t_[d]$)
CF_j	cornering forces
K_*	cornering power
R	radius of rotation

Here the suffix $*$ is either f or r as the case may be and the suffices f and r refer to the object related to the front and rear tires. E.g. γ_f and β_r mean the front tire steering angle and rear tire slide slip angle respectively, while $\beta = \beta_g$ means the slip angle with respect to the center of gravity of the vehicle. For intended applications to in-wheel EV's, the distinction between front and rear tires will not be relevant and will be put equal. However, in order to include the case of 2WD front drive vehicles, we write some parameters with suffices.
From the equilibrium of lateral forces we have

$$mv\left(\frac{\mathrm{d}\beta}{\mathrm{d}t}+\gamma\right) = m\left(\frac{\mathrm{d}}{\mathrm{d}t}(V\beta)+v\gamma\right) = ma = \sum_{j=1}^{4} CF_j. \tag{3.27}$$

By geometric consideration we have

$$\beta_* = \theta_* - \left(\beta \pm \frac{l_*}{V}\gamma\right), \tag{3.28}$$

where the sign $\pm$ is to be + in the case of in-wheel EVs. And the cornering force is given by

$$CF_* = K_*\beta_* = K_*\left(\beta + \frac{l_*}{V}\gamma\right) \tag{3.29}$$

with $K_* > 0$ indicating the cornering power.
(3.27) reads

$$mV\left(\frac{\mathrm{d}\beta}{\mathrm{d}t}+\gamma\right) = 2(CF_f + CF_r) = 2K_f\left(\beta + \frac{l_f}{V}\gamma\right) + 2K_r\left(\beta + \frac{l_r}{V}\gamma\right). \tag{3.30}$$

The yawing angular momentum is given by

$$I_\gamma = I\frac{\mathrm{d}\gamma}{\mathrm{d}t}. \tag{3.31}$$

Hence the yawing equation of motion reads

$$I_\gamma = 2l_f CF_f - 2l_r CF_r. \tag{3.32}$$

Combining (3.30) and (3.32), we obtain

$$mV\frac{\mathrm{d}\beta}{\mathrm{d}t} + 2(K_f + K_r)\beta + \left\{mV + \frac{2(l_fK_f - l_rK_r)}{V}\right\}\gamma = 2(K_f\theta_f + K_r\theta_r) \tag{3.33}$$
$$I\frac{\mathrm{d}\gamma}{\mathrm{d}t} + 2(l_fK_f - l_rK_r)\beta + \frac{2({l_f}^2K_f + {l_r}^2K_r)}{V}\gamma = 2(l_fK_f\theta_f - l_rK_r\theta_r).$$

Suppose for simplicity that the steering angles be 0. Then (3.33) amounts to

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\beta\\ \gamma\end{pmatrix} = -B\begin{pmatrix}\beta\\ \gamma\end{pmatrix} \tag{3.34}$$

where

$$B = \begin{pmatrix} \frac{2}{mV}(K_f+K_r) & 1+\frac{2(l_fK_f-l_rK_r)}{mV^2} \\ \frac{2}{I}(l_fK_f-l_rK_r) & \frac{2(l_f{}^2K_f+l_r{}^2K_r)}{VI} \end{pmatrix}. \tag{3.35}$$

There are several methods known to solve (3.34). We shall give the method of linear algebra in Appendix A and below we give basic data for the solution. The eigen-equation for $-B$ is

$$\begin{vmatrix} \lambda+\frac{2}{mV}(K_f+K_r) & 1+\frac{2(l_fK_f-l_rK_r)}{mV^2} \\ \frac{2}{I}(l_fK_f-l_rK_r) & \lambda+\frac{2(l_f{}^2K_f+l_r{}^2K_r)}{VI} \end{vmatrix} = |\lambda E+B| = 0. \tag{3.36}$$

Hence expanding we obtain the eigen-equation

$$\lambda^2+2b\lambda+c=0 \tag{3.37}$$

where

$$b=\frac{1}{mVI}\left(m\left(l_f{}^2K_f+l_r{}^2K_r\right)+I\left(K_f+K_r\right)\right)>0 \tag{3.38}$$

and

$$c=\frac{4l-2}{V^2}\frac{K_fK_r}{mI}(1+\mathfrak{A}V^2) \tag{3.39}$$

and where $\mathfrak{A}$ is the **stability factor** defined by

$$\mathfrak{A}=-\frac{m}{2l^2}\frac{l_fK_f-l_rK_r}{K_fK_r}. \tag{3.40}$$

With steering angle and velocity constant, the vehicle makes a rotation with a fixed radius, called the steady-state rotation. In such a case, the derivatives of the yaw angle and the slip angle may be thought of 0 and (3.33) amounts to a linear equation with determinant given by

$$|B|=-2mV\left(l_fK_f-l_rK_r\right)+\frac{4}{V}K_fK_rl^2=\frac{4}{V}K_fK_rl^2\left(1+\mathfrak{A}V^2\right). \tag{3.41}$$

Solving the system we deduce that

$$\omega=\frac{V}{l}\frac{\theta_f-\theta_r}{(1+\mathfrak{A}V^2)}, \tag{3.42}$$

whence the radius is

$$R=\frac{V}{\omega}=\left(1+\mathfrak{A}V^2\right)\frac{1}{\theta_f-\theta_r}. \tag{3.43}$$

Definition 3.1. *From* (3.43) *we see that with steering angle $\theta_f-\theta_r$ the same and increasing the speed, the radius varies according to the sign of the quantity $I_fK_f-I_rK_r$ or stability factor $\mathfrak{A}$. If $\mathfrak{A}=0$, the radius remains the same and the vehicle is said to have neutral steering property. If $\mathfrak{A}>0$, then R increases as V increases. i.e. there is a deficit of steering angle to keep the radius of the original circle and the vehicle is said to have* **understeer** *property. If $\mathfrak{A}<0$, then the steering angle is excessive and the vehicle is said to have* **oversteer** *property. In practice vehicles are designed to have weak understeer characteristic except for specified use.*

Remark 3.1. *Thus we may assume $\mathfrak{A} \geq 0$ and so $c > 0$ in* (3.37) *and it is often written in th form*

$$\lambda^2 + 2\zeta\omega_n\lambda + \omega_n^2 = 0 \tag{3.44}$$

where

$$\zeta = \frac{1}{\omega_n}\frac{2}{mVI}\left(m\left(l_f{}^2K_f + l_r{}^2K_r\right) + I\left(K_f + K_r\right)\right) \tag{3.45}$$

with ω_n denoting the **natural angular frequency** *defined by*

$$\omega_n = \frac{2l}{V}\sqrt{\frac{K_fK_r}{mI}(1+\mathfrak{A}V^2)}. \tag{3.46}$$

Hence vehicle dynamics is anothe instance of the secod order system with decay cf. [9], [3, pp. 62-65] etc.

The solutions of (3.37) are $\lambda_1, \lambda_2 = -b \pm \sqrt{D}$, $D = b^2 - c$. By Theorem 6.1 the solutions of (3.34) are linear combinations of $e^{(-b+\sqrt{D})t}$ and $e^{(-b-\sqrt{D})t}$ or te^{-bt} and e^{-bt} according as the eigenvalues are different or the same.
In case $c > 0$ and $D \geq 0$, we have $D < b$ and the biggest term $e^{(-b+\sqrt{D})t}$ exponentially decreases.
In case $c > 0$ and $D < 0$, the fundamental solutions may be expressed as $e^{-bt}\cos\sqrt{-D}t$ and $e^{-bt}\sin\sqrt{-D}t$ so that the general solution is an oschillating function with exponential decay.
In case $c < 0$, we have $D > b$ and $e^{(-b+\sqrt{D})t}$ exponentially grows which is discarded as the case of oversteer.
The Laplace transform method can also be used for solving (3.34). Taking the Laplace transform of (3.33) and denoting $\beta(s) = L[\beta](s)$ and $\gamma(s) = L[\gamma](s)$ we obtain on the assumption that $\beta(0) = 0$

$$\begin{aligned}&\begin{pmatrix} mVs + 2(K_f + K_r) & mV + \frac{2(l_fK_f - l_rK_r)}{V} \\ 2(l_fK_f - l_rK_r) & Is + \frac{2(l_f{}^2K_f + l_r{}^2K_r)}{V}\end{pmatrix}\begin{pmatrix}\beta(s) \\ \gamma(s)\end{pmatrix} \\ &= \begin{pmatrix} 2(K_f\theta_f + K_r\theta_r) \\ 2(l_fK_f\theta_f - l_rK_r\theta_r)\end{pmatrix}.\end{aligned} \tag{3.47}$$

The determinant of the coefficient matrix in (3.47) may be expressed as

$$mVIs^2 + 2\left(m\left(l_f{}^2K_f + l_r{}^2K_r\right) + I\left(K_f + K_r\right)\right)s - 2mV\left(l_fK_f - l_rK_r\right) + \frac{4}{V}K_fK_rl^2 \tag{3.48}$$

on writing

$$\left(K_f + K_r\right)\left(l_f{}^2K_f + l_r{}^2K_r\right) = \left(l_fK_f - l_rK_r\right)^2 + K_fK_r\left(l_f + l_r\right)^2.$$

(3.48) amounts to the eigen-equation (3.37) and one may say that it is the characteristic equation for the system (3.37) of DE. Especially with $s = 0$, (3.48) gives the value of $|B|$in (3.41).
There is an extensive exposition of the Laplace transform method in [3] using the partial fraction expansion and we omit further details.

4 Fail-safe guard by GG

It is a world tendency for newly sold vehicles to have a collision avoidance system (CAS) (also known as a precrash system)–the system designed to reduce the severity of a collision–including autonomous emergency braking (AEB) and autonomous emergency steering (AES). It uses radar (all-weather) and sometimes laser (LIDAR) and camera (employing image recognition) to detect an imminent crash. GPS sensors can detect fixed dangers such as approaching the stop signs through a location database. Once the detection is done, these systems either provide a warning to the driver when there is an imminent collision or take action autonomously without any driver input (by AES or AES or both). Collision avoidance by AEB is appropriate at lower speeds (e.g. below 50 km/h), while collision avoidance by AES is appropriate at higher speeds. Cars with collision avoidance may also be equipped with adaptive cruise control, and use the same forward-looking sensors.
A wireless In-Wheel Motor EV (IWM) [24] is a new kind of EV in which power is transferred to motors by wireless

transmission rather than via cables. This will solve the problem of wire disconnection by liquidating the complicated wiring. It also has a remarkable feature that it can be charged while driving.
Indeed, IWM EVs have been known for quite some time [7], [19] etc. Here we take up the **Yonden EV "PIVOT"** [16] which realized two driving modes 20 years ago. In addition to normal drive mode there is a universal drive mode in the negative and positive phase. The negative (resp. positive) phase is the one with the front and rear wheels in opposite (resp. the same) direction. The negative phase enables a pivot rotation including a point rotation and the positive phase enables an oblique drive including a lateral drive. We are more interested in the advanced—wireless IWM version—of Yonden pivot which is to be a model for all possible wireless IWMEVs as the most essential component of the GG. In the near future when fossil fuel is being exhausted, the ideal vehicles will be Yonden pivot type. The present situation looks still "A tail wagging its dog." EVs have been killed once [6] (or the second time counting the moment when they are expelled from the market by gasolin-propelled cars) and although they are being revived [20], the time may not be the most proper. The transient period may last long, up to 40 years, say until all cars will be replaced by EVs because the EVs can compete conventional cars only when the infrastructure is more complete. In the coming society when the infrastructure is fully developed, all cars will be wireless IWM EVs that are connected wireless-wise to the GG through regional aggregators during moving and while driving on high-ways they can be charged by the chargers which are charged by solar energy. Since all cars are monitored by the GG in general and by the regional aggregator in particular, the Grid can check the traffic real time and whenever it detects possilbe collison, it can send orders directly to the driving system to make a necessary action. In this way, GG can realize a real time collision avoidance system, which does avoid collision rather than reducing the damage by collision.

Table 3: Yonden EV universal drive

phase	positive	negative
wheels	the same	opposite
drive	oblique	pivot
special	lateral	sharp turn

The GG control system is to be able to monitor the surroundings of each individual EVs and immediately control the traction force and the steer angle for each wheel by the instruction sent wirelessly from the Aggregate controller.
We illustrate the collison avoidance when there is an obstacle on the lane ahead by oblique or lateral drive by Yonden type.

Figure 3: An EV is under surveillance of Aggregator 1.
Figure 4: The EV moves to the region under surveillance of Aggregator 2.
Figure 5: The EV moves to the region under surveillance of Aggregator 3.

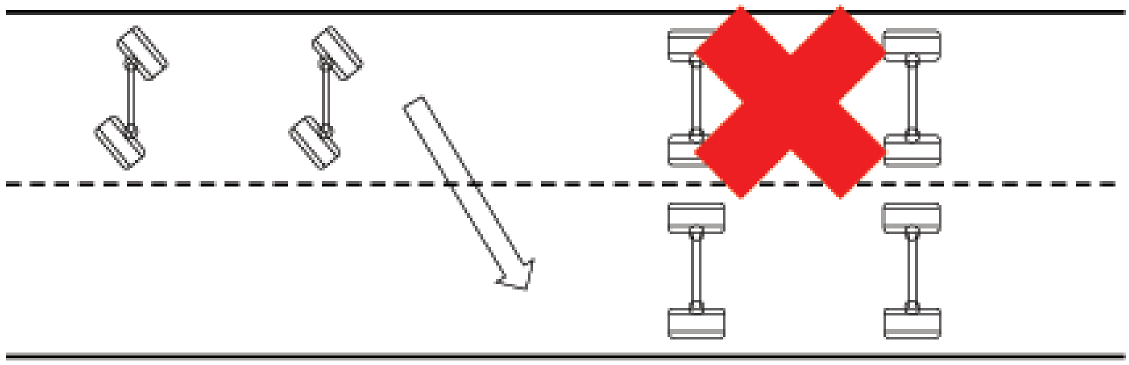

Figure 1: Oblique drive

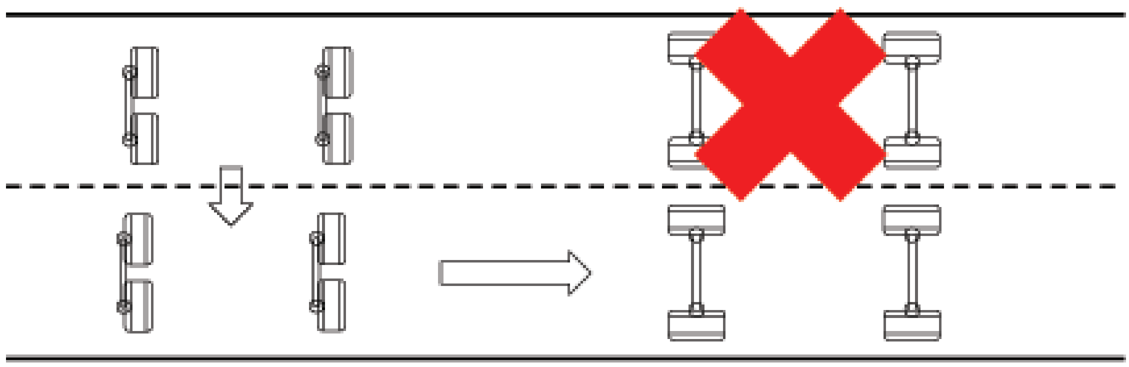

Figure 2: Lateral drive

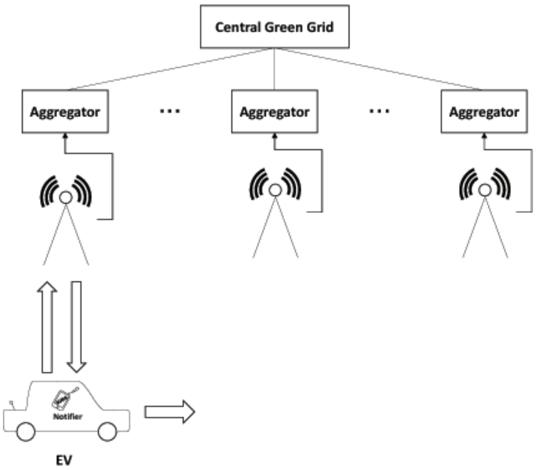

Figure 3: Aggregator 1

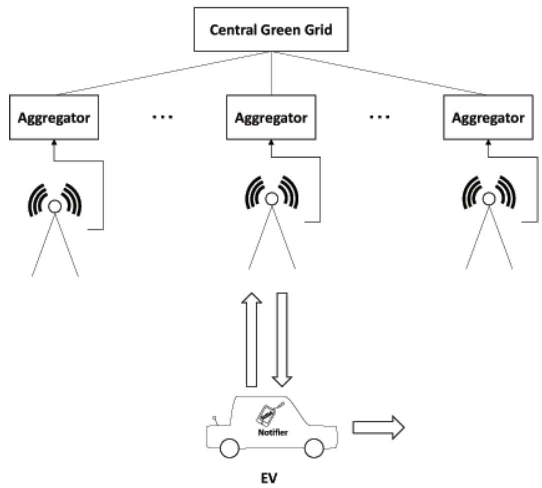

Figure 4: Aggregator 2

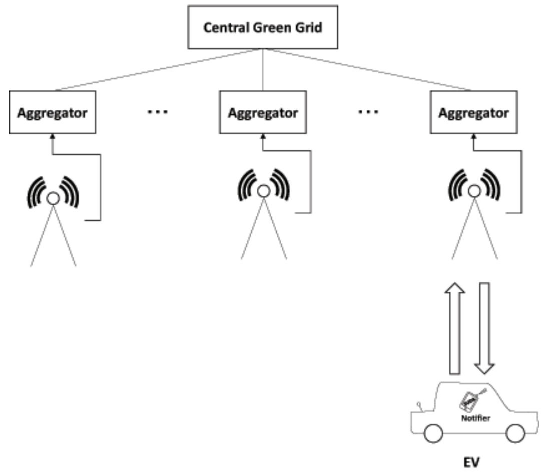

Figure 5: Aggregator 3

5 Climate change

5.1 Change of amount of carbons

Connected with the glacial-interglacial periods, the amount of CO_2 varies. In the transitional time of 6000 years from the **last glacial age** (20,000 years ago) (180ppm) to the **postglacial age** (10,000 years ago) (270 ppm), the amount of carbon dioxide increased by 90 ppm. In the succeeding 10,000 years, it increased by 10 ppm.

(i) The rate of increase of CO_2 in the glacial-interglacial periods: 27×10^{15}g/1000 years.

(ii) The rate of increase of CO_2 by activities of human being: 29×10^{18}g/1000 years.

(iii) The rate of increase of CO_2 by volcanic gases: 3×10^{13}g/1000 years.

Table 4: Increase rate of CO_2

name	CO_2 increase
(i) glacial-interglacial periods	27×10^{15}g/1000 years
(ii) human activities	29×10^{18}g/1000 years
(iii) volcanic gases	3×10^{13}g/1000 years

Comparing (i) and (ii), we see that the **rate of increase of** CO_2 **by human activities is** 1000 **times as much as that by nature**

$$29 \times 10^{18} \div 27 \times 10^{15} \approx 1000/1000$$

years. The issue of increase of carbon dioxide lies not only in the amount but also in its speed of increase. Indeed, the earth used to be rather warm e.g. in the **Cretaceous period** with much higher condensation of carbon dioxide.

The Cretaceous period: $146,000,000 \sim 65,000,000$ years ago belongs to the **Mesozoic era** ($245,000,000 \sim 65,000,000$ years ago). It is said that in the Cretaceous period, the climate was far warmer than now and the density of CO_2 was 10 times as much as the present one.

Table 5: Various periods and climate therein

name	years	climate
Mesozoic period	$245,000,000 \sim 65,000,000$ years ago	very warm
Cretaceous period	$146,000,000 \sim 65,000,000$	10 times as much CO_2
end of the last ice age	$-12,000$	largest area of glacier
best climate period	$-6,000$	Sahara desert with green
medieval warm period	$950 \sim 1320$	marine transregression
little ice age	$1350 \sim 1820$	marine regression

5.2 Global warming

As we have seen in §*5.1*, the extraordinary speed of increase of amount of carbon dioxide is the grave issue. Since ocean is the biggest reservoir of carbons and it is already warmed up, the temperature will keep increasing even if we can stop releasing even a bit carbon dioxide on the ground that warmed water in a large vessel is kept warm than that in a smaller vessel. There is a prediction that even if now all release is stopped of carbon dioxide, the increase will continue in the coming 50 years.

Another concern is that from the geo-data, it is seen that in the dynamical system of climate there are a few stable modes in climate and that by some outer cause, the mode jump occurs, then the new mode will be kept even the outer conditions go back to the previous state, i.e. **mode jumping**.

The third problem is that under drastic climate change, we cannot adopt the old method of human being of moving to a new land. Since the major living area where one can live a cultural life is already exhausted and there is no more new land to move. This makes global warming an international affair that the human race is to confront.

Although global warming should be a *global* problem of all the human race, it is usually the case that the developing countries are releasing more carbon dioxide to develop their industries, following the same track that the developed countries once want along, and there is no global understanding. There are many reasons for this type of let-it-be-as-others'-problem. One is that it cannot be seen very clearly and people tend to leave such things to a later date as needless fear. Secondly there is a misleading information scattered around which ignore the time scale. The mass. comm. reports are often more misleading because they agitate and exaggerate tiny points and mix up real data with false illusion.

The period $950-1250$ is called the **medieval warm period** while $1400-1700$ is called the **little ice age**. We note, however, that between these two periods, the difference of temperature on the whole earth is only 0.4°C! Therefore,

the climate change is rather the change of its spatial pattern than the change of average temperature. I.e. both in medieval warm period and little ice age, there used to be both warmer and colder areas and in general in the latter period, it was colder than in the former.
In conclusion, that at present the carbon flux is 1000 times quicker than that by nature means that by 1000 times quicker the amount of carbon dioxide in the atmosphere will increase and will give rise to a similar hot climate as in the Cretaceous period.

5.3 Dansgaard-Oeschger cycle

In this subsection further study is made on the Dansgaard-Oeschger cycle (D-O cycle) and Heinrich event etc. It is published in Nature in 1993 and refers to a rapid climate fluctuations with extremely large amplitude that occurred 25 times during the last glacial period (75000-20000 years ago).
In the Northern Hemisphere, the course of the D-O cycle takes the form of rapid warming periods whose length are of the order of decades, and each warming period is followed by gradual cooling period which lasts a few hundred years. A 5 °C change over 30-40 years is common in average annual temperatures on the Greenland ice sheet. On the other hand, during the cold period there is an expansion of the polar front, with ice floating further down to the south across the North Atlantic Ocean.
Some scientists say that the D-O cycles occur quasi-periodically with a period length being a multiple of 1,470 years, but this is still speculative. This length of time reminds us of Nishioka's speculation of climate cycles of 700 years [18]. Comparing the graphs , we may conjecture the quasi-periodicity of around 1,470 years.

5.4 Determination of era and temperature

(i) The determination of the era by radio-active isotopes. Cf. [31]. The method of using radio-active isotopes to determine the era is well-known along with that using radio-active material including Polonium. One of the most famous stories is the determination of a picture to be genuine or not.

The method based on radio-active isotope ^{14}C has been widely used and produced important data on climatology. However, it is speculated after the nuclear experiments conducted in the 1950's changed the ratio of the amount of ^{14}C and the method has more deviations than before. Carbon has two stable isotopes ^{12}C, ^{13}C, and the radio-active isotope is ^{14}C. The ratio of their amount in nature is 99 : 1. ^{14}C is generated by heat neutrons released from cosmic rays and falls down to the earth, getting absorbed in the bodies of living organisms. For an application, cf. Yamamoto [32].

(ii) The determination of temperature in the past. Cf. [28], [31]. This is not so well-known as the carbon 14 method. Most of the oxygen is ^{16}O of mass 16. There are other isotopes ^{17}O and ^{18}O which occupy only $\frac{2.4}{100}$% of all isotopes of oxygen. When the temperature is lower, the ratio of the lighter oxygen isotope ^{16}O increases since it vapors into the air and gets frozen in snow form. The ratio of the lighter oxygen isotope in ice-core and the snow crystal is the same. Since the fluctuation of $\delta^{18}O$ is connected with the Milankovitch cycle, one can determine the water temperature.
We consider the phase change

$$ {}^{I}X \to {}^{I}Y, {}^{II}X \to {}^{II}Y, \tag{5.49} $$

where the prefix indicates different atomic numbers. We let the reaction constant to be k_1 and k_2, respectively. Also consider the reverse reaction to (5.49) with reaction constants k_1^- and k_2^-, respectively:

$$ {}^{I}Y \to {}^{I}X, {}^{II}Y \to {}^{II}X, \tag{5.50} $$

and consider a possible reaction

$$ {}^{I}X + {}^{II}Y \to {}^{II}X + {}^{I}Y, \tag{5.51} $$

whose reaction speed to the right is v_1 and v_2 to the left and think of it being in equilibrium, i.e. $v_1 = v_2$. Then

$$ v_1 = k_1[{}^{I}X]k_2^-[{}^{II}Y] = k_1k_2^-[{}^{I}X][{}^{II}Y], v_2 = k_2k_1^-[{}^{II}X][{}^{I}Y] \tag{5.52} $$

with the square bracket indicating the molarity. Recall that the equilibrium constant K is defined by

$$K = \frac{k_2 k_1^-}{k_1 k_2^-}. \tag{5.53}$$

Equating v_1 and v_2 in (5.52), we have

$$K = \frac{k_2 k_1^-}{k_1 k_2^-} = \frac{[^I X][^{II} Y]}{[^{II} X][^I Y]}. \tag{5.54}$$

Recall that the **isotope ratio** r_Y of the substance in the state Y is defined by

$$r_Y = \frac{[^{II} Y]}{[^I Y]}. \tag{5.55}$$

Using (5.55) in (5.54), we may express the fractionation constant $\alpha_{X,Y}$ as

$$\alpha_{X,Y} = K = \frac{r_Y}{r_X}. \tag{5.56}$$

We specify the above reactions with stable isotopes of oxygen. (5.49) reads

$$H_2{}^{16}O_{(aq)} \to H_2{}^{16}O_{(g)}, H_2{}^{18}O_{(aq)} \to H_2{}^{18}O_{(g)} \tag{5.57}$$

with reaction constants k_1, k_1^-, k_2, k_2^- and (5.51) reads

$$H_2{}^{16}O_{(aq)} + H_2{}^{18}O_{(g)} \leftrightarrow H_2{}^{18}O_{(aq)} + H_2{}^{16}O_{(q)} \tag{5.58}$$

with constants v_1, v_2. Note that

$$v_1 = k_1 k_2^- [^I X][^{II} Y] = k_1 k_2^- [^{16}O_{(aq)}][^{18}O_{(g)}], v_2 = k_2 k_1^- [^{II} X][^I Y] = k_1 k_2^- [^{18}O_{(aq)}][^{16}O_{(g)}]. \tag{5.59}$$

Equating $v_1 = v_2$, we have

$$\alpha_{aq,g} = \frac{k_2 k_1^-}{k_1 k_2^-} = k_1 k_2^- [^I X][^{II} Y] = \frac{[^{16}O_{(aq)}][^{18}O_{(g)}]}{[^{18}O_{(aq)}][^{16}O_{(g)}]}, \tag{5.60}$$

where $\alpha_{aq,g}$ is the **isotopic fractionation constant**
(5.55) reads

$$r_{\mathrm{O}_{(g)}} = \frac{[^{18}\mathrm{O}_{(g)}]}{[^{16}\mathrm{O}_{(g)}]}. \tag{5.61}$$

Hence it follows that

$$\alpha_{aq,g} = \frac{r_{\mathrm{O}_{(g)}}}{r_{\mathrm{O}_{(aq)}}} \tag{5.62}$$

In the analysis of Greenland ice-core, the amount of $\delta^{18}O$ was analyzed and determined the temperature at each period.
Stable isotope compositions of low-mass (light) elements such as oxygen, hydrogen, carbon, nitrogen, and sulfur are normally reported as δ values. δ values are reported in units of parts per thousand (denoted as or per mil, or per mil, or per mille – or even recently, per mill) relative to a standard of known composition. δ values are calculated by

$$\delta \quad (\mathrm{in} 0/00) = \left(\frac{r_x}{r_S} - 1\right) \times 1000, \tag{5.63}$$

where r denotes the ratio of the heavy to light isotope (as in (5.55)), and r_x and R_S are the ratios in the sample and standard, respectively.

Figure 6: The temparature record found in Greenland ice-core 1993.
Figure 7: The temparature record mainly in Taiwan found in dendroclimatology due to H. Nishioka 1948.
Figure 8: Comparison of Figures 6 and 7.

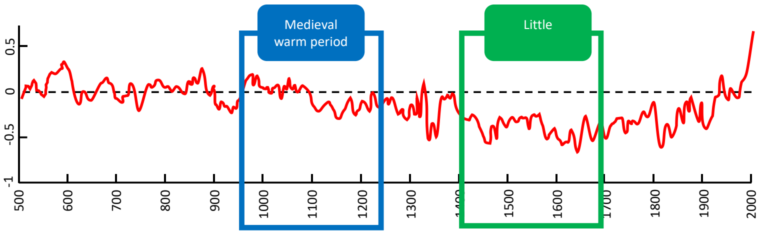

Figure 6: Greenland ice-core graph

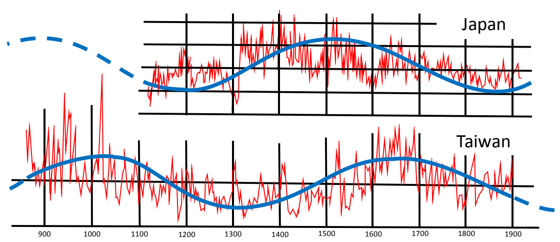

Figure 7: Nishioka graph

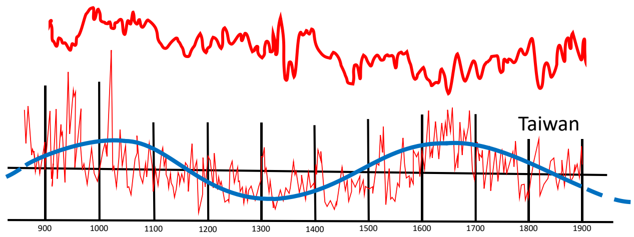

Figure 8: Greenland-Nishioka graph

Looking at Figure 8, we are tempted to speculate a plausible waether-cycle conjecture of 1470 years as a combination of the latest conjectures based on the Dansgaard-Oeschger (DO) cycles and the conjecture of H. Noshioka.

6 Appendix A

Theorem 6.1. *Let* $x(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$ *and let* A *be a* 2×2 *matrix with eigenvalues* λ_1, λ_2. *The solutions of the system of DEs*

$$\frac{\mathrm{d}}{\mathrm{d}x} x(t) = Ax(t) \tag{6.64}$$

under the initial condition $x(0) = \begin{pmatrix} x(0) \\ y(0) \end{pmatrix}$ *are given by linear combinations of* $e^{\lambda_1 t}$ *and* $e^{\lambda_2 t}$ *or* $te^{\lambda t}$ *and* $e^{\lambda t}$ *according as the eigenvalues are different or the same,* $= \lambda$ *say.*

Proof. We distinguish two cases according as the sum of dimension(s) of the eigenspaces is 2 or 1. The latter can occur only when the eigenvalues are a double root.
Suppose eigenvalues are λ, λ (double root) and that the eigenspace is $E_A(\lambda) = \mathbb{C}p_1$, $\dim E_A(\lambda) = 1$, where $p_1 =$

$\begin{pmatrix} p_{11} \\ p_{21} \end{pmatrix}$ and we may discard the cases where one of the entries is 0 since it amounts to the case $A = \begin{pmatrix} a & b \\ 0 & a \end{pmatrix}$. We find p_2 such that

$$P = (p_1, p_2) \in GL_2(\mathbb{C}) \tag{6.65}$$

entails

$$P^{-1}AP = \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix} = J(A). \tag{6.66}$$

By block multiplication, we rewrite $AP = P\begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}$ into

$$\begin{aligned}(\lambda p_1, Ap_2) =& (Ap_1, Ap_2) = A(p_1, p_2) = AP = P\begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix} \\ =& (p_1, p_2)\begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix} = (\lambda p_1, p + \lambda p_2),\end{aligned} \tag{6.67}$$

whence we arrive at the system of equations

$$(A - \lambda E)p_2 = p_1. \tag{6.68}$$

Discarding the case where $a - \lambda = 0$, we may transform the augmented matrix as $(A - \lambda E, p_1) \sim \begin{pmatrix} 1 & \frac{b}{a-\lambda} & \frac{p_{11}}{a-\lambda} \\ 0 & 0 & 0 \end{pmatrix}$ and so the solution is $\begin{pmatrix} \frac{p_{11}}{a-\lambda} \\ 0 \end{pmatrix} + c\begin{pmatrix} -\frac{b}{a-\lambda} \\ 1 \end{pmatrix}$, $c \in \mathbb{C}$. Putting $p_2 = \begin{pmatrix} \frac{p_{11}}{a-\lambda} \\ 0 \end{pmatrix}$, $\{p_1, p_2\}$ are linearly independent. Hence putting P as in (6.65), we obtain (6.66).

We may now solve the DE (6.64). Putting

$$P^{-1}x = y = \begin{pmatrix} X \\ Y \end{pmatrix}, \quad x = Py, \tag{6.69}$$

we see that $\frac{d}{dt}x = Ax$ is equivalent to

$$\frac{d}{dt}y = P^{-1}\frac{d}{dt}x = P^{-1}APy = J(A)y. \tag{6.70}$$

i.e. to

$$\begin{cases} \frac{dX}{dt} = \lambda X + Y \\ \frac{dY}{dt} = \lambda Y. \end{cases} \tag{6.71}$$

From the second equation, we obtain

$$Y = Y(0)e^{\lambda t}. \tag{6.72}$$

Hence, noting that

$$\frac{d}{dt}\left(e^{-\lambda t}X\right) = e^{-\lambda t}\left(\frac{d}{dt}X - \lambda X\right) = e^{-\lambda t}Y = Y(0),$$

we deduce that

$$e^{-\lambda t}X = Y(0)t + c, \ c = X(0).$$

Hence

$$X = (Y(0)t + X(0))\, e^{\lambda t}. \tag{6.73}$$

Substituting (6.72) and (6.73) in (6.69), we obtain

$$x = Py = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix}\begin{pmatrix} (Y(0)t + X(0))\, e^{\lambda t} \\ Y(0)e^{\lambda t} \end{pmatrix} \tag{6.74}$$

$$= \begin{pmatrix} ((p_{11}t + p_{12})Y(0) + p_{11}X(0))\, e^{\lambda t} \\ ((p_{21}t + p_{22})Y(0) + p_{21}X(0))\, e^{\lambda t} \end{pmatrix}. \tag{6.75}$$

To find the initial values $X(0), Y(0)$, we appeal to (6.69). Let $P^{-1} = \begin{pmatrix} q_{11} & q_{12} \\ q_{21} & q_{22} \end{pmatrix}$. Then $X(0) = q_{11}x(0) + q_{11}y(0)$ and $Y(0) = q_{21}x(0) + q_{22}y(0)$. Thus we conclude that

$$x = \begin{pmatrix} -(c_{11}t + c_{12})\, e^{\lambda t} \\ (c_{21}t + c_{11})\, e^{\lambda t} \end{pmatrix}, \qquad \square$$

where

$$c_{11} = p_{11}(q_{21}x(0) + q_{22}y(0)), c_{21} = p_{21}(q_{21}x(0) + q_{22}y(0))$$

In the case where the sum of dimensions of the eigenspaces is 2, we may diagonalize A. If the eigen-equation has a double root λ then $P^{-1}AP = \lambda E$ whence $A = \lambda E$, which we exclude from our consideration and assume that the roots are distinct. We find the basis p_1, p_2 of the eigenspace which is

$$E_A(\lambda_j) = \mathbb{C}p_j, \quad j = 1, 2.$$

Defining P as in (6.65), we apply block multiplication to deduce analogously to (6.67)

$$\begin{aligned} AP = A(p_1, p_2) = (Ap_1, Ap_2) = (\lambda_1 p_1, \lambda_2 p_2) \\ = (p_1, p_2)\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} = P\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \end{aligned} \tag{6.76}$$

,

Correspondingly to (6.66) we have a diagonalization

$$P^{-1}AP = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} = D(A). \tag{6.77}$$

The system (6.64) of DE amounts to

$$\begin{cases} \frac{dX}{dt} = \lambda_1 X \\ \frac{dY}{dt} = \lambda_2 Y. \end{cases} \tag{6.78}$$

As in (6.72), the solutions are immediately obtained:

$$X = X(0)e^{\lambda_1 t}, \quad Y = Y(0)e^{\lambda_2 t}. \tag{6.79}$$

Hence correspondingly to (6.74) we obtain

$$\begin{aligned} x = Py = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix}\begin{pmatrix} X(0)e^{\lambda_1 t} \\ Y(0)e^{\lambda_2 t} \end{pmatrix} \\ = \begin{pmatrix} p_{11}X(0)e^{\lambda_1 t} + p_{12}Y(0)e^{\lambda_2 t} \\ p_{21}X(0)e^{\lambda_1 t} + p_{22}Y(0)e^{\lambda_2 t} \end{pmatrix}. \end{aligned} \tag{6.80}$$

$\square$

7 Appendix B

Theorem 7.1. *Let $b = b(t)$ be an arbitrary vector whose length is not necessarily constant and suppose that the coordinate system is rotating with angular velocity Ω. Then we have*

$$\left(\frac{\mathrm{d}b}{\mathrm{d}t}\right)_I = \left(\frac{\mathrm{d}b}{\mathrm{d}t}\right)_R + \Omega \times b, \tag{7.81}$$

where the suffix means that the velocity is observed by the rotating observer (R) or by a non-rotating inertial observer (I), respectively.

Lemma 7.1. *Let $a = a(t)$ denote an arbitrary vector of* **constant length** *which rotates with angular velocity* Ω. *Then we have*

$$\frac{\mathrm{d}}{\mathrm{d}t}a = \Omega \times a. \tag{7.82}$$

Proof. Let α be the angle between a and Ω.
By the definitin of the angular velocity, we have

$$\frac{\mathrm{d}\gamma}{\mathrm{d}t} = \mid \Omega \mid \tag{7.83}$$

From the definition of the rotation, the infinitesimal change $\frac{\mathrm{d}}{\mathrm{d}t}a$ is perpendicular to Ω . It is also perpendicular to a. For by assumption $0 = \frac{\mathrm{d}}{\mathrm{d}t}|a|^2$ which is $\frac{\mathrm{d}}{\mathrm{d}t}a \cdot a = 2\frac{\mathrm{d}}{\mathrm{d}t}a \cdot a$ by the Leibniz rule.
Hence

$$a \perp \frac{\mathrm{d}}{\mathrm{d}t}a = 0 \quad \text{or} \quad \frac{\mathrm{d}}{\mathrm{d}t}a \perp a.$$

Hence

$$\frac{\mathrm{d}}{\mathrm{d}t}a // \Omega \times a \quad \text{or} \quad \frac{\mathrm{d}}{\mathrm{d}t}a = c(\Omega \times a),$$

where $c > 0$ is a constant.
We determine c. By geometric consideration, the first approximation of

$$\Delta a := a(t + \Delta t) - a(t)$$

is

$$|a|(\sin\alpha)(\sin\Delta\gamma)n,$$

where

$$n = \frac{1}{|\Omega \times a|}\ \Omega \times a$$

is the normal vector. Recalling the Maclaurin expansion

$$\sin\gamma = \gamma + O\left(\gamma^3\right), \gamma \to 0, \tag{7.84}$$

we have

$$\Delta a = |a|(\sin\alpha)(\Delta\gamma)n + O\left((\Delta\gamma)^3\right), \quad \Delta\gamma \to 0. \tag{7.85}$$

From (7.85) it follow that

$$\frac{\mathrm{d}}{\mathrm{d}t}a = \lim_{\Delta t \to 0} \frac{\Delta a}{\Delta t} = |a| \sin\alpha \lim_{\Delta t \to 0} \frac{\Delta\gamma}{\Delta t} n = |a| \sin\alpha \frac{\mathrm{d}\gamma}{\mathrm{d}t} n = |a||\Omega| \sin\alpha n, \tag{7.86}$$

by (7.83). Now in view of $|a||\Omega| \sin\alpha = |\Omega \times a|$, (7.86) reads

$$\frac{\mathrm{d}}{\mathrm{d}t}a = |\Omega \times a| n,$$

which amounts to (7.82). □

References

[1] W. J. Burrows, Weather cycles—real or imaginary?, Cambridge UP, Cambridge, (1992).

[2] The Capitol Net, Smart Gird, Government series, Alexandria, (2009).

[3] K. Chakraborty, S. Kanemitsu, T. Kuzumaki, A quick introduction to complex analysis, World Sci., Singapore etc. (2016).
http://www.worldscientific.com/worldscibooks/10.1142/10003

[4] Committee of the driving system of EVs, Electrical Engineering Society, The latest technologies of EVs, Ohmsha, Tokyo, (1999).

[5] W. Dansgaard, et al; Evidence for general instability of past climate from a 250-kyr ice-core record, Nature, 364 (6434) (1993) 218-220.
https://doi.org/10.1038/364218a0

[6] EC Confidential LLC, Who killed the electric car? (2006).

[7] Newest technology for EVs, Soc. Electr. Engrg, Res. Committee for the driving systems for EVs, Ohmsha, Tokyo, (1999).

[8] X. Feng, T. Ziurntos, K. L. Butler-Purry, Dynamic load management for NG IPS ships, Proc. IEEE Power Engineering Society General Meeting, Minneapolis, (2010).
https://doi.org/10.1109/PES.2010.5589441

[9] F. S. Grodins, Control theory and biological systems, Columbia Univ. Press, New York and London, (1963).

[10] E. Hossain, Z. Han, H. V. Poor, Smart Grid communications and networking, Cambridge UP, Cambridge, (2014).

[11] S. Kanemitsu, Fuhuo Li, Y. Kamata, K. Maruyama, Mathematical foundations of fluid dynamics, Basic Sciences J. Textile Univ, 29 (2016) 275-285.

[12] S. Koide, Physics, Shokabo, Tokyo (1975).

[13] D. Kundurm S, Mashayehk, T. Ziurntos, K. L. Butler-Purry, Cyber-attack impact analysis of smart grid, in H. Hossain, Z. Han and H. V. Poor, Smart Grid communications and networking, Cambridge UP, Cambridge, (2014) 354-372.

[14] F. -H. Li, S. Galhotra, S. Kanemitsu, Emerging importance of EV's in the Green Grid Era, Pure Appl. Math. J. Special issue: Mathematical aspects of engineering disciplines (ed. by H. -L. Li), 4 (5-1) (2015) 38-45.

[15] T. Miyamoto, Acceleration switch on—Making EV yourself, CQ Publishing, Tokyo, (2015).

[16] H. Nasu, H. Higasa, Development of yonden electric vehicle "PIVOT", JSAE Review, 16 (1995) 77-82.
https://doi.org/10.1016/0389-4304(94)00066-3

[17] H. Neunzert, The road vehicle system and related mathematics, Prof. Workshop, Teubner, Stuttgart, (1985).

[18] H. Nishioka, History of cold and hot—a theory of periodic climate change in 700 years, Kogakusha, Tokyo, (1948).

[19] T. Nishiyama, K. Endoo, A. Wada, In-wheel motors—Princiles and design, Kagaku-joho-shuppan, Tokyo, (2016).

[20] Chris Paine (ed.), Revenge of the electric car, (2011).

[21] R. Pollack, The missing moment: How the unconscious shapes modern science Houghton Mifflin Harcourt, New York, (1999).

[22] R. C. Qiu, P. Antonik, Smart Grid and Big Data: Theory and Practice, Wiley, New York, (2014).

[23] P. Rentrop, Numerische Porbleme in der Fahrezeugsdynamik, [17], 44-58.

[24] M. Sato, G. Yamamoto, T. Imura, H. Fujimoto, Efficiency evaluation in wireless power transfer via magnetic resonance coupling to in-wheel motor, IEEJ-MD14094, (2014).

[25] Carol L. Stimmel, Big Data Analytics Strategies for the Smart Grid, CRC Press, New York, (2015).

[26] A. Sumi, The truth of global warming, Wedge, Tokyo, (1999).

[27] K. Suzuki, K. Sogabe, H. Shimosaka, Dynamics of machinery, Jikkyosyuppan, Tokyo, (1984).

[28] R. Tada, Scientific study on climate change, Misuzu-shobo, Tokyo, (2013).

[29] Tamagawa-seiki co. (ed.), Introduction to effective use of gyro technology, Kogyo-chosakai, Tokyo, (2002).

[30] Y. Tanaka, Let's buy an EV at Yamada electricity shop, Random House Japan, Tokyo, (2010).

[31] E. Wada et al, Stable isotopes in the biosphere, Kyoto Univ. Press Kyoto, (1995).

[32] T. Yamamoto, History of Japan as told by climate, Societe, Tokyo, (1976).

[33] Q.-Z. Zong, T. Hornik, Control of power inverters in renewable energy and smart grid integration, Wiley, New York, (2014).