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Research Article

An Effect of the Environmental Pollution via Mathematical Model Involving the Mittag-Leffler Function

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Abstract
In the existing condition estimation of pollution effect on environment is big change for all of us. In this study we develop a new approach to estimate the effect of pollution on environment via mathematical model which involves the generalized Mittag-Leffler function of one variable $E^{\gamma_1,\alpha_1}_{\alpha_2,\delta_1;\alpha_3,\delta_2}(z)$ which we introduced here.

Keywords: Mittag-Leffler Function; Non-linear Partial Differential Equation.
AMS Subject Classification: 33C45, 33C60.

1 Introduction

With environment being the most essential element on Earth due to its need in society. Effect of environmental pollution as a topic is very broad that's why many authors have been done remarkably study to carry out pollution effects and factors by various ways (see, for example [8, 9, 10, 11]).
Many times in the world, industries and development activities are focus only on production of the materials by the intensive use of the environment without use of the environmental safety rules. Therefore the resulting pollution effects directly affected the humans' health.
Particular from last few years, all world scientists show their concerned to reduce the environment pollution. Motivate by above novelty reason here, we present new approach to estimate the effect of pollution on environment via mathematical model which involves the generalized Mittag-Leffler function of one variable $E^{\gamma_1,\alpha_1}_{\alpha_2,\delta_1;\alpha_3,\delta_2}(z)$ which we introduce here.

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2 Description of the mathematical model

In the proposed model, we consider the following non-linear partial differential equations (PDF) system:

$$\frac{\partial\theta_i}{\partial t} = \theta_i F_i\big(\theta_1, \theta_2, \mu_i(T), \nu_i(T)\big) + A_i \frac{\partial^2\theta_i}{\partial t^2}, \quad i = 1,2 \tag{2.1}$$

where $\theta_i\,(x,t), (i = 1,2)$ represent the growth of interacting and dispersing biological species of density in a one dimensional linear habitat 0.5 x ≤ L and $F_i\big(\theta_1, \theta_2, \mu_i(T), \nu_i(T)\big)$ determines the interaction function of the species, $\mu_i(T)\ and \nu_i(T)$are the intrinsic growth rate and the carrying capacity of the environment respectively, which are affected by the concentration $T(x,t)$ of pollutant. The positive constant A_i (i = 1, 2) is the dispersion coefficient of the species.
The dynamics of $T(x,t)$ is given by the:

$$\frac{\partial T}{\partial t} = \beta_0 - \alpha T + A_c \frac{\partial^2 T}{\partial x^2} \tag{2.2}$$

where, $\beta_0 > 0$ is the constant which determining the exogenous rate of the input of pollutant into the habitat, α > 0 is represents the first order decay constant as a result of biological, chemical or geological processes. $A_c > 0$ represent the diffusion coefficient of the pollutant.
Here, we assumed that the organismal uptake of the pollutant is proportional to the concentration of the pollutant present in the environment of the population.
Following results are also required for the present study

Definition 2.1.
In 1954, Djrbashyan (see [4, 5, 6] and also see [4]) introduced and studied the two-parametric Mittag-Leffler function defined as:

$$E_{\alpha,\beta}(z) = \sum_{\nu=0}^{\infty} \frac{z^\nu}{\Gamma(\nu\alpha+\beta)} \qquad (\nu > 0) \tag{2.3}$$

For the further details regarding above function we refer the book entitle "Mittag-Leffler functions, related topics and applications" by Gorenflo, Kilbas, Mainardi and Rogosin [7] (see, for examples [1, 2, 3]).

Definition 2.2.
We introduce new generalize Mittag-Leffler-type function of one variable as follows:

$$E^{\gamma_1,\alpha_1}_{\alpha_2,\delta_1;\alpha_3,\delta_2}(z) = \sum_{\nu=0}^{\infty} \frac{(\gamma_1)_{\alpha_1\nu} z^\nu}{\Gamma(\nu\alpha_2+\delta_1)\Gamma(\nu\alpha_3+\delta_2)}, \tag{2.4}$$

Where $(\gamma_1, \delta_1, \delta_2, z \in \mathbb{C}; min[\alpha_1, \alpha_2, \alpha_3] > 0)$.

3 Mathematical Model

In this section we present mathematical model by using Mittag-Leffler function. For our purpose lest us assume T(x,t) in the terms of Mittag-Leffler function as follows:

$$T(x,t) = E_{\alpha,\beta}(zx^\sigma t^\mu) = \sum_{\nu=0}^{\infty} \frac{(zx^\sigma t^\mu)^\nu}{\Gamma(\nu\alpha+\beta)} \qquad (\nu > 0) \tag{3.5}$$

To differentiating (3.5) partially with respect to "t" and "x", we have:

$$\frac{\partial T}{\partial t} = \sum_{\nu=0}^{\infty} \frac{\Gamma(\nu\mu)(zx^\sigma)^\nu t^{\mu\nu-1}}{\Gamma(\nu\alpha+\beta)} \qquad (\nu > 0) \tag{3.6}$$

$$\frac{\partial T}{\partial t} = \frac{1}{t\Gamma(2)} \sum_{\nu=0}^{\infty} \frac{(2)_\nu (zx^\sigma t^\mu)^\nu}{\Gamma(\nu\alpha+\beta)\Gamma(\nu\mu)} \qquad (\nu > 0) \tag{3.7}$$

By applying, Definition 2.2, on (3.7), we get:

$$\frac{\partial T}{\partial t} = \frac{1}{t\Gamma(2)} E^{2,\mu}_{\alpha,\beta;\mu,0}(zx^\sigma t^\mu) \tag{3.8}$$

On the same way, we have:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{x^2} E_{\alpha,\beta;\sigma,-1}^{1,\sigma}(zx^\sigma t^\mu) \tag{3.9}$$

Using (3.5), (3.8) and (3.9) in (2.2) we arrive at

$$\frac{1}{t\Gamma(2)} E_{\alpha,\beta;\mu,0}^{2,\mu}(zx^\sigma t^\mu) = \beta_0 - \alpha E_{\alpha,\beta}(zx^\sigma t^\mu) + A_c \frac{1}{x^2} E_{\alpha,\beta;\sigma,-1}^{1,\sigma}(zx^\sigma t^\mu) \tag{3.10}$$

where the condition of convergent follows given by Definition (2.1 & 2.2).

Concluding Remark:

We conclude our present study by remarking that by choose the suitably values of the parameters involved in (3.10) we can get many cases involving the special functions.

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