Marichev-Saigo Integral Operators Involving the Product of K-Function and Multivariable Polynomials

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Abstract
The aim of this paper is to establish the Marichev-Saigo-Maeda fractional integration formula to the product of the K-function with the general class of multivariable polynomials. The results are presented in terms of the Wright generalized hypergeometric function. Corresponding assertions in terms of Saigo, Erdélyi-Kober, Riemann-Liouville, and Weyl type of fractional integrals are also presented. Further, we point out also their relevance.

Keywords: Generalized fractional integral operators, K-function, Wright function, general class of multivariable polynomials.

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1 Introduction

Fractional calculus is a branch of classical mathematics, formulated in 1695. After long time, fractional calculus was not very popular among science and engineering community and was regarded as a pure mathematical realm without real applications. Now the field of fractional calculus is undergoing rapid developments with more and more convincing applications in the real world. Various authors Baleanu et al. [2], Kilbas [5], Kiryakova ([6],[7]), Kumar et al. [8], Samko et al. [14] and Suthar et al. [22] etc. investigated on the field of fractional calculus and its applications. Several motivating results which are significant to the present work are also obtained.

A generalization of the hypergeometric fractional integrals for \(a, a', b, b', d, \delta \in \mathbb{C}\) and \(F(\delta) \in \mathbb{C}\), is introduced by Marichev [9] as follows:

\[
\begin{align*}
\left( I^{(a, a', b, b', \delta)}_{0+} f \right)(x) &= \frac{x^{-a}}{\Gamma(\delta)} \int_{0}^{x} (x-t)^{\delta-1} t^{-a} F_{3}(a, a', b, b'; \delta; 1-(t/x), 1-(t/x)) f(t) \, dt, \\
\left( I^{(a, a', b, b', \delta)} f \right)(x) &= \frac{x^{-a'}}{\Gamma(\delta)} \int_{x}^{\infty} (t-x)^{\delta-1} t^{-a} F_{3}(a, a', b, b'; \delta; 1-(x/t), 1-(x/t)) f(t) \, dt,
\end{align*}
\]

In (1.1) and (1.2), \(F_{3}(.)\) denotes the Appell function (also known as Horn function) which is introduced by Srivastava and Karlson [21]

\[
pF_{q}(\alpha, \alpha', \beta, \beta'; \delta; x, y) = \sum_{m, n=0}^{\infty} \frac{\left( \alpha \right)_{m} \left( \alpha' \right)_{m} \left( \beta \right)_{n} \left( \beta' \right)_{n}}{\left( \delta \right)_{m+n} m! n!} x^{m} y^{n}, \max\{|x|, |y|\} < 1,
\]

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An interesting further generalization of the generalized hyper-geometric function is due to Wright [23] in a series representation of the form

$$\psi_q(z) = \sum_{n=0}^{\infty} \frac{(a_1)_n; \ldots; (a_p)_n (b_1)_n; \ldots; (b_q)_n}{(\mu n + \xi)_n n!} z^n,$$

where \(z, a_i, b_j \in \mathbb{C}, A_i, B_j \in \mathbb{R}^+, A_i \neq 0, B_j \neq 0; i = 1, \ldots, p; j = 1, \ldots, q\)

$$1 + \sum_{j=1}^{q} (\sum_{i=1}^{p} A_i) \geq 0.$$  

We recall, The K-function defined by [17] as:

$$\mu, \xi, \gamma \in \mathbb{R}, (\mu) > 0, (a_j)_n \text{ and } (b_j)_n \text{ are the Pochhammer symbols.}$$

If any numerator parameter \(a_m\) is a negative integer or zero, then the series terminates to a polynomial in \(x\). The series (1.8) is defined when none of parameters \(b_j, j = 1, 2, \ldots, q\) is a negative integer or zero. From the ratio test it is evident that the series is convergent for all \(x\) if \(p > q + 1, p = q + 1\) and \(|x| = 1, the series can converge in some cases.

The corresponding Wright generalized hypergeometric function (1.6) of the generalized K-function is given by:

$$\psi_q(z) = \sum_{n=0}^{\infty} \frac{(a_j)_n (b_j)_n}{(\mu n + \xi)_n n!} z^n.$$

As per Srivastava and Garg([20], pp. 686, eq.(14)), the definition of multivariable generalization of the polynomial \(S_n^m (x)\) is:

$$S_n^m (y_1^{h_1}, \ldots, y_s^{h_s}) = \frac{h_1 + \ldots + h_s \leq L}{k_1 \ldots k_s = 0} (-L)^{h_1 k_1 + \ldots + h_s k_s} A(L, k_1, \ldots, k_s) \frac{x_1^{k_1}}{k_1!} \ldots \frac{x_s^{k_s}}{k_s!}.$$  

In which, \(h_1, \ldots, h_s \in \mathbb{Z}^+\) where as the coefficients \(A(L, k_1, \ldots, k_s), (L, h_i \in \mathbb{N}_0, i = 1, \ldots, s)\) are arbitrarily. Chosen constants real or complex, As Srivastava [19] defined; by \(s = 1\) on the above polynomial we obtain a polynomial of the form \(S_n^m (x)\).

2 Main Results

Here, we establish two image formulas for the K-function involving left and right sided operators of Marichev-Saigo-Maeda fractional integral operators defined in (1.1) and (1.2), results are in term of the generalized Wright function. These formulas are given by the following theorems.
Theorem 2.1. Let $\alpha, \alpha', \beta, \beta', \delta, \rho \in \mathbb{C}$ and $\mu, \xi, \gamma \in \mathbb{C}, \Re(\mu) > 0$. Further let the constants satisfy the condition $a \in \mathbb{C}$. If the condition (1.7) is satisfied, then the fractional integration $I^{(\alpha, \alpha', \beta, \beta', \delta)}_{\rho}$ of the product of $K$-function $\mu \cdot \xi \gamma$ and multivariable polynomial $S^a_{\mu \xi \gamma}$ exists, under the condition

$$\Re(\delta) > 0, \Re(v) > 0, \Re(\rho) > \max\{0, \Re(\alpha + \alpha' + \beta - \delta), \Re(\alpha' - \beta')\},$$

then there hold the following formula:

$$\left(I^{(\alpha, \alpha', \beta, \beta', \delta)}_{\rho}\left(\prod_{j=1}^{n} \Gamma(b_j) \sum_{k_1, \ldots, k_n=0} h_{k_1} \ldots h_{k_n} \leq \mathbb{L} \sum_{k_1, \ldots, k_n=0} (-L) h_{k_1} \ldots h_{k_n} A(L; k_1, \ldots, k_n) \frac{\lambda_{k_1}^{v_1}}{k_1!} \ldots \frac{\lambda_{k_n}^{v_n}}{k_n!} \chi_{\Lambda_{k_1}^{v_1} \ldots \Lambda_{k_n}^{v_n}} \right) \left(\sum_{n=0}^{\infty} (a_1)_n \ldots (a_p)_n \gamma_n \alpha^n \left(I^{(\alpha, \alpha', \beta, \beta', \delta)}_{\rho}(t)^{v_1} \ldots \chi_{\Lambda_{k_1}^{v_1} \ldots \Lambda_{k_n}^{v_n}}(x)\right) \right) \right)\times \prod_{j=1}^{p+4} \Psi_{\rho, \gamma, \xi, \epsilon}(t) = 0.$$

Proof. Let $\Theta$ be the left-hand side of (2.11), using (1.8) and (1.10) then changing the order of integration and summation, we obtain

$$\Theta = \sum_{k_1, \ldots, k_n=0} (-L) h_{k_1} \ldots h_{k_n} A(L; k_1, \ldots, k_n) \frac{\lambda_{k_1}^{v_1}}{k_1!} \ldots \frac{\lambda_{k_n}^{v_n}}{k_n!} \chi_{\Lambda_{k_1}^{v_1} \ldots \Lambda_{k_n}^{v_n}} \left(\sum_{n=0}^{\infty} (a_1)_n \ldots (a_p)_n \gamma_n \alpha^n \left(I^{(\alpha, \alpha', \beta, \beta', \delta)}_{\rho}(t)^{v_1} \ldots \chi_{\Lambda_{k_1}^{v_1} \ldots \Lambda_{k_n}^{v_n}}(x)\right) \right),$$

Applying (1.4) on the above equation (2.12), it becomes

$$\Theta = \frac{x^{\rho-\alpha-\alpha'+\beta-\delta-1}}{\Gamma(\gamma)} \sum_{n=0}^{\infty} (a_1)_n \ldots (a_p)_n \gamma_n \alpha^n \left(I^{(\alpha, \alpha', \beta, \beta', \delta)}_{\rho}(t)^{v_1} \ldots \chi_{\Lambda_{k_1}^{v_1} \ldots \Lambda_{k_n}^{v_n}}(x)\right).$$

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Interpreting the right-hand side of the above equation, in view of the definition (1.9), we arrive at the result (2.11). If we set $S^{\gamma}_{\mu \xi \gamma}[x]$ reduce to unity, i.e. $S^{\gamma}_{\mu \xi \gamma}[x] \rightarrow 1$ in (2.11), then Theorem 2.1 takes the following form:

Corollary 2.1. Let $\alpha, \alpha', \beta, \beta', \delta, \rho \in \mathbb{C}$ and $\mu, \xi, \gamma \in \mathbb{C}, \Re(\mu) > 0$ then the fractional integral formula $I^{(\alpha, \alpha', \beta, \beta', \delta)}_{\rho}$ of the $K$-function $\mu \cdot \xi \gamma$ exists, and the following integral holds true:

$$\left(I^{(\alpha, \alpha', \beta, \beta', \delta)}_{\rho}(\prod_{j=1}^{n} \Gamma(b_j) \sum_{k_1, \ldots, k_n=0} h_{k_1} \ldots h_{k_n} \leq \mathbb{L} \sum_{k_1, \ldots, k_n=0} (-L) h_{k_1} \ldots h_{k_n} A(L; k_1, \ldots, k_n) \frac{\lambda_{k_1}^{v_1}}{k_1!} \ldots \frac{\lambda_{k_n}^{v_n}}{k_n!} \chi_{\Lambda_{k_1}^{v_1} \ldots \Lambda_{k_n}^{v_n}} \right)\times \prod_{j=1}^{p+4} \Psi_{\rho, \gamma, \xi, \epsilon}(t) = 0.$$
Remark 2.1. If we set \( \gamma = 1 \) and make some suitable adjustment of the parameters in (2.14), we arrive at the known result given by Chohan et al. ([4], p. 91, eq. (12)).

Theorem 2.2. Let \( \alpha, \alpha', \beta, \beta', \delta, \rho \in \mathbb{C} \) and \( \mu, \xi, \gamma \in \mathbb{C} \), \( R(\mu) > 0 \). Further let the constants satisfy the condition \( a \in \mathbb{C} \). If the condition (1.7) is satisfied, then the fractional integration \( I^{(\alpha, \alpha', \beta, \beta', \delta)} \) of the product of K-function \( pK_q(\cdot) \) and multivariable polynomial \( S_{1, \ldots, k}^{(h)}(\cdot) \) exists, then under the condition

\[
R(\delta) > 0, \, R(\nu) > 0, \, R(\rho) < 1 + \min \{ R(-\beta), R(\alpha + \alpha' - \delta), R(\alpha - \beta' - \delta) \}
\]

therehold the following formula:

\[
\left( I^{(\alpha, \alpha', \beta, \beta', \delta)} \right) \left( \psi_{\mu, \xi, \gamma} \left( \frac{a^{\rho - \alpha - \alpha' - \delta - 1}}{\Gamma(\gamma)} \right) \right) = \frac{\psi_{\mu, \xi, \gamma} \left( \frac{a^{\rho - \alpha - \alpha' - \delta - 1}}{\Gamma(\gamma)} \right)}{\Gamma(\gamma)} \times \sum_{n=0}^{\infty} \prod_{j=1}^{\nu} \Gamma(b_j) \psi_{\mu, \xi, \gamma} \left( \frac{a^{\rho - \alpha - \alpha' - \delta - 1}}{\Gamma(\gamma)} \right) \prod_{j=1}^{\nu} \Gamma(b_j)
\]

\[
\times \sum_{n=0}^{\infty} \prod_{j=1}^{\nu} \Gamma(b_j) \psi_{\mu, \xi, \gamma} \left( \frac{a^{\rho - \alpha - \alpha' - \delta - 1}}{\Gamma(\gamma)} \right) \prod_{j=1}^{\nu} \Gamma(b_j)
\]

\[
\times \sum_{n=0}^{\infty} \prod_{j=1}^{\nu} \Gamma(b_j) \psi_{\mu, \xi, \gamma} \left( \frac{a^{\rho - \alpha - \alpha' - \delta - 1}}{\Gamma(\gamma)} \right) \prod_{j=1}^{\nu} \Gamma(b_j)
\]

Proving Let \( \Theta \) be the left-hand side of (2.15), using (1.8) and (1.10) and then changing the order of integration and summation, we obtain

\[
\Theta = \sum_{n=0}^{\infty} \prod_{j=1}^{\nu} \Gamma(b_j) \psi_{\mu, \xi, \gamma} \left( \frac{a^{\rho - \alpha - \alpha' - \delta - 1}}{\Gamma(\gamma)} \right) \prod_{j=1}^{\nu} \Gamma(b_j)
\]

Applying (1.5) on the above equation (2.16), it becomes

\[
\frac{x^{\rho - \alpha - \alpha' - \delta - 1}}{\Gamma(\gamma)} \sum_{n=0}^{\infty} \prod_{j=1}^{\nu} \Gamma(b_j) \psi_{\mu, \xi, \gamma} \left( \frac{a^{\rho - \alpha - \alpha' - \delta - 1}}{\Gamma(\gamma)} \right) \prod_{j=1}^{\nu} \Gamma(b_j)
\]

Interpreting the right-hand side of the above equation, in view of the definition (1.9), we arrive at the result (2.15).

Also if we set \( S^h(y) \rightarrow 1 \) in (2.15), then Theorem 2.2 takes the following form:

Corollary 2.2. Let \( \alpha, \alpha', \beta, \beta', \delta, \rho \in \mathbb{C} \) and \( \mu, \xi, \gamma \in \mathbb{C} \), \( R(\mu) > 0 \) then the fractional integral formula \( I^{(\alpha, \alpha', \beta, \beta', \delta)} \) of the K-function \( pK_q(\cdot) \) exists, and the following integral holds true:

\[
\left( I^{(\alpha, \alpha', \beta, \beta', \delta)} \right) \left( \psi_{\mu, \xi, \gamma} \left( \frac{a^{\rho - \alpha - \alpha' - \delta - 1}}{\Gamma(\gamma)} \right) \prod_{j=1}^{\nu} \Gamma(b_j) \right)
\]

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multivariable polynomial $Sk(\cdots, (\gamma, 1), (1 + \alpha + \alpha' - \delta - \rho, v),
(b1, 1), \cdots, (bq, 1), (\xi, \mu), (1 - \rho, v),$
\(1 + \alpha + \beta' - \delta - \rho, v), (1 - \beta - \rho, v);
(1 + \alpha + \alpha' + \beta' - \delta - \rho, v), (1 + \alpha - \beta - \rho, v); \ ax^{-v}\right]. \quad (2.18)

**Remark 2.2.** If we set $\delta = 1$ and make suitable adjustment of the parameters in (2.18), we arrive at the known results given by Chohan et al. [4, p. 91, eq. (14)].

3 Special Cases

In this section, we consider some special cases of the main results derived in the preceding section. If we set $\alpha' = 0$ in the operators (1.1) and (1.2), then we have the following identities:

\[
\left(\binom{\alpha+\beta,0,-\eta,\beta',\alpha}{0+1}f\right)(x) = \left(\binom{\alpha,\beta,\eta}{0+1}f\right)(x),
\]

(3.19)

\[
\left(\binom{\alpha+\beta,0,-\eta,\beta',\alpha}{-1}f\right)(x) = \left(\binom{\alpha,\beta,\eta}{-1}f\right)(x).
\]

(3.20)

where the hypergeometric operators, appeared in the right hand side are due to Saigo [12], defined as:

\[
\left(\binom{\alpha,\beta,\eta}{0}f\right)(x) = \frac{x^{-\alpha-\beta}}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} \frac{1}{1-(t/x)} f(t) dt.
\]

(3.21)

\[
\left(\binom{\alpha,\beta,\eta}{-1}f\right)(x) = \frac{1}{\Gamma(\alpha)} \int_x^\infty (t-x)^{\alpha-1} t^{-\alpha-\beta} \frac{1}{1-(t/x)} f(t) dt.
\]

(3.22)

Therefore, if we set $\alpha' = 0, \beta = -\eta, \delta = \alpha$ and replace $\alpha$ by $\alpha + \beta$ in (1.1) and (1.2), we get the following results, involving the left and right hand sided Saigo type integral operators:

**Corollary 3.1.** Let $\alpha, \beta, \eta, \rho \in \mathbb{C}$ and $\mu, \xi, \gamma \in \mathbb{C}, \Re(\mu) > 0$. Further let the constants satisfy the condition $a \in \mathbb{C}$.

If the condition (1.7) is satisfied, then the fractional integration $I^\mu_{0+}\{\alpha,\beta,\eta\}$ of the product of K-function $\left(\binom{\alpha,\beta,\gamma}{0}S_L^{k_1\cdots k_r}(\cdots)\right)$ exists, then under the condition

\[
\Re(\gamma) > 0, \Re(\nu) > 0, \Re(\rho) > \max\{0, \Re(\beta - \eta)\}
\]

there hold the following formula:

\[
\binom{\alpha,\beta,\eta}{0+1} \left(p^{-1} \binom{h_1! \cdots h_n!}{h_1! \cdots h_n!} \left(\binom{\mu,\gamma}{1}S_L^{k_1\cdots k_r}(ar^v)\right)\right)(x) = \frac{x^{p-\beta-1}}{\Gamma(\gamma)} \sum_{n=0}^\infty \prod_{j=1}^n a_j \left(\binom{b_j}{0+1}\right).
\]

(3.23)

**Corollary 3.2.** Let $\alpha, \beta, \eta, \rho \in \mathbb{C}$ and $\mu, \xi, \gamma \in \mathbb{C}, \Re(\mu) > 0$. Further let the constants satisfy the condition $a \in \mathbb{C}$.

If the condition (1.7) is satisfied, then the fractional integration $I^\mu_{-1}\{\alpha,\beta,\eta\}$ of the product of K-function $\left(\binom{\alpha,\beta,\gamma}{0}S_L^{k_1\cdots k_r}(\cdots)\right)$ exists, then under the condition

\[
\Re(\gamma) > 0, \Re(\nu) > 0, \Re(\rho) < 1 + \min\{\Re(\eta), \Re(\beta)\}
\]
there hold the following formula:

\[
\left( \frac{\rho-1}{\Gamma(y)} \left( \frac{\rho-1}{\Gamma(y)} \left( y_{\lambda_1}, \ldots, y_{\lambda_k} \right) \right) \right) (x) = \frac{x^{\rho-1} \sum_{n=0}^{\infty} \prod_{j=1}^{n} \Gamma(b_j)}{\sum_{n=0}^{\infty} \prod_{j=1}^{n} \Gamma(a_j)}
\]

\[
\times \sum_{k_1, \ldots, k_s \geq 0} (-L)_{h_1 k_1 + \ldots + h_s k_s} A(L; k_1, \ldots, k_s) \frac{y_{k_1}}{k_1!} \ldots \frac{y_{k_s}}{k_s!} \lambda_1 k_1 + \ldots + \lambda k_s
\]

\[
\times \prod_{j=1}^{\infty} \Psi_{q+2} \left[ \frac{(a_1, 1), \ldots, (a_p, 1), (\gamma, 1), (1 - \rho - \sum_{i=1}^{p} \lambda_i k_i + \eta, v)}{(b_1, 1), \ldots, (b_q, 1), (\xi, \mu), (1 - \rho - \sum_{i=1}^{q} \lambda_i k_i, v)}, (1 - \rho - \sum_{i=1}^{p+q} \lambda_i k_i + \alpha + \beta + \eta, v), ax^{-v} \right] .
\] (3.24)

**Remark 3.1.** If we set \( S_{\rho}^b [x] \) to reduce to unity, i.e. \( S_{\rho}^b [x] \to 1 \) in (3.23) and (3.24), make suitable adjustment in the parameters in Corollary 3.1 and Corollary 3.2, we arrive at the known result given by Sharma ([18], p. 42-43 eq.(2.1, 3.1)).

Further, if we follow results of Corollaries 3.1 and 3.2, when \( \beta = -\alpha \), we arrive at the following results involving left and right sided Riemann-Liouville and Weyl fractional type integrals of the generalized Gauss hypergeometric function given by the following corollaries.

**Corollary 3.3.** Let \( \alpha, \rho \in \mathbb{C} \) and \( \mu, \xi, \gamma \in \mathbb{C}, \Re(\mu) > 0 \). Then the following formula holds true:

\[
\left( \frac{\rho-1}{\Gamma(y)} \left( \frac{\rho-1}{\Gamma(y)} \left( y_{\lambda_1}, \ldots, y_{\lambda_k} \right) \right) \right) (x) = \frac{x^{\rho+1} \sum_{n=0}^{\infty} \prod_{j=1}^{n} \Gamma(b_j)}{\sum_{n=0}^{\infty} \prod_{j=1}^{n} \Gamma(a_j)}
\]

\[
\times \sum_{k_1, \ldots, k_s \geq 0} (-L)_{h_1 k_1 + \ldots + h_s k_s} A(L; k_1, \ldots, k_s) \frac{y_{k_1}}{k_1!} \ldots \frac{y_{k_s}}{k_s!} \lambda_1 k_1 + \ldots + \lambda k_s
\]

\[
\times \prod_{j=1}^{\infty} \Psi_{q+2} \left[ \frac{(a_1, 1), \ldots, (a_p, 1), (\gamma, 1), (\rho + \sum_{i=1}^{p} \lambda_i k_i, v)}{(b_1, 1), \ldots, (b_q, 1), (\xi, \mu), (\rho + \sum_{i=1}^{q} \lambda_i k_i, v)}, (\alpha x)^{v} \right] .
\] (3.25)

**Corollary 3.4.** Let \( \alpha, \rho \in \mathbb{C} \) and \( \mu, \xi, \gamma \in \mathbb{C}, \Re(\mu) > 0 \). Then the following formula holds true:

\[
\left( \frac{\rho-1}{\Gamma(y)} \left( \frac{\rho-1}{\Gamma(y)} \left( y_{\lambda_1}, \ldots, y_{\lambda_k} \right) \right) \right) (x) = \frac{x^{\rho+1} \sum_{n=0}^{\infty} \prod_{j=1}^{n} \Gamma(b_j)}{\sum_{n=0}^{\infty} \prod_{j=1}^{n} \Gamma(a_j)}
\]

\[
\times \sum_{k_1, \ldots, k_s \geq 0} (-L)_{h_1 k_1 + \ldots + h_s k_s} A(L; k_1, \ldots, k_s) \frac{y_{k_1}}{k_1!} \ldots \frac{y_{k_s}}{k_s!} \lambda_1 k_1 + \ldots + \lambda k_s
\]

\[
\times \prod_{j=1}^{\infty} \Psi_{q+2} \left[ \frac{(a_1, 1), \ldots, (a_p, 1), (\gamma, 1), (\rho + \sum_{i=1}^{p} \lambda_i k_i, v)}{(b_1, 1), \ldots, (b_q, 1), (\xi, \mu), (\rho + \sum_{i=1}^{q} \lambda_i k_i, v)}, (\alpha x)^{v} \right] .
\] (3.26)

**Corollary 3.5.** Let \( \alpha, \rho \in \mathbb{C} \) and \( \mu, \xi, \gamma \in \mathbb{C}, \Re(\mu) > 0 \). Then the following formula holds true:

\[
\left( \frac{\rho-1}{\Gamma(y)} \left( \frac{\rho-1}{\Gamma(y)} \left( y_{\lambda_1}, \ldots, y_{\lambda_k} \right) \right) \right) (x) = \frac{x^{\rho+1} \sum_{n=0}^{\infty} \prod_{j=1}^{n} \Gamma(b_j)}{\sum_{n=0}^{\infty} \prod_{j=1}^{n} \Gamma(a_j)}
\]

\[
\times \sum_{k_1, \ldots, k_s \geq 0} (-L)_{h_1 k_1 + \ldots + h_s k_s} A(L; k_1, \ldots, k_s) \frac{y_{k_1}}{k_1!} \ldots \frac{y_{k_s}}{k_s!} \lambda_1 k_1 + \ldots + \lambda k_s
\]

\[
\times \prod_{j=1}^{\infty} \Psi_{q+2} \left[ \frac{(a_1, 1), \ldots, (a_p, 1), (\gamma, 1), (1 - \rho - \sum_{i=1}^{p} \lambda_i k_i - \alpha, v)}{(b_1, 1), \ldots, (b_q, 1), (\xi, \mu), (1 - \rho - \sum_{i=1}^{p} \lambda_i k_i, v)}, (\alpha x)^{v} \right] .
\] (3.27)

**Corollary 3.6.** Let \( \alpha, \rho \in \mathbb{C} \) and \( \mu, \xi, \gamma \in \mathbb{C}, \Re(\mu) > 0 \). Then the following formula holds true:

\[
\left( \frac{\rho-1}{\Gamma(y)} \left( \frac{\rho-1}{\Gamma(y)} \left( y_{\lambda_1}, \ldots, y_{\lambda_k} \right) \right) \right) (x) = \frac{x^{\rho+1} \sum_{n=0}^{\infty} \prod_{j=1}^{n} \Gamma(b_j)}{\sum_{n=0}^{\infty} \prod_{j=1}^{n} \Gamma(a_j)}
\]

\[
\times \sum_{k_1, \ldots, k_s \geq 0} (-L)_{h_1 k_1 + \ldots + h_s k_s} A(L; k_1, \ldots, k_s) \frac{y_{k_1}}{k_1!} \ldots \frac{y_{k_s}}{k_s!} \lambda_1 k_1 + \ldots + \lambda k_s
\]

\[
\times \prod_{j=1}^{\infty} \Psi_{q+2} \left[ \frac{(a_1, 1), \ldots, (a_p, 1), (\gamma, 1), (1 - \rho - \alpha, v)}{(b_1, 1), \ldots, (b_q, 1), (\xi, \mu), (1 - \rho, v)}, (\alpha x)^{v} \right] .
\] (3.28)
4 Conclusion

The K-function, expressed in this paper, is relatively basic in nature. Therefore, on some suitable adjustment of the parameters on function, we may obtain other special functions such as M-series, Mittag-Leffler function, Bessel-Maitland function (see, e.g., ([1], [3], [10], [11], [15], [16]) as its special cases, and therefore, various unified fractional integral presentations can be obtained as special cases of our results. Moreover, the results obtained in this paper also corresponds to Saigo hypergeometric fractional integrals, Weyl and Riemann-Liouville fractional integral operators as special cases.

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