Hydromagnetic flow and heat transfer of an upper-convected Maxwell fluid in a parallel plate channel with stretching walls

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Abstract
A study of an upper-convected Maxwell (UCM) fluid flow and heat transfer in a parallel plate channel with stretching walls in the presence of an applied magnetic field is carried out. The governing non-linear coupled equations with appropriate boundary conditions are initially cast into dimensionless form by similarity transformations. Then the resulting non-dimensional equations are solved analytically by Optimal Homotopy Analysis Method (HAM). The effects of the pertinent parameters on the velocity and temperature fields are analyzed graphically. The analysis reveals that the effect of the magnetic field is to decrease the velocity normal to the wall, but the opposite effect is observed for the temperature field. The present study is relevant to the haemodynamic flow of blood in the cardiovascular system in the presence of uniform magnetic field.

Keywords: Hydromagnetic flow, haemodynamic flow, heat transfer, Maxwell fluid, stretching sheet, skin friction, Nusselt number.

1 Introduction
The study of flow and heat transfer in a viscous fluid over a stretching sheet is of interest because of its industrial applications. These applications include, the cooling of an infinite metallic plate in a cooling bath, the extrusion of polymers involving the cooling of a molten liquid, the drawing and tinning of copper wires, paper production, glass blowing, and the heat treatment of materials travelling on conveyor belts. In order to obtain the quality desired for such processes, two considerations must be made, the selection of the liquid to be used to cool the object of interest and the rate of stretching applied to the material. Processes involving sudden solidification focus heavily on the rate of stretching. After the initiation of the problem by Sakiadis [1], many papers have been written in regards to the boundary layer flow over a solid surface. Crane [2] extended the work of Sakiadis [1] by introducing the
stretching sheet component to the problem. Several researchers [3-8] extended Crane’s [2] problem. All those authors restricted their investigations to flow and heat transfer in the absence of an applied magnetic field. However, a study on flow and heat transfer in the presence of a magnetic field is important due to its industrial applications, such as: the cooling of continuous strips, filaments drawn through a quiescent electrically conducting fluid subject to a magnetic field, and the purification of molten metals from nonmetallic inclusions. One such study was performed by Chakrabarti and Gupta [9]. They analyzed the temperature distribution in the MHD boundary layer flow over a stretching sheet in the presence of suction. Also, Vajravelu and Nayfeh [10] analyzed the hydrodynamic flow of a dusty fluid over a stretching sheet. Problems relating to the hydro-magnetic, three-dimensional flow on a stretching surface have been studied by Chamka [11] and Abo-Eldlahab [12]. Recently, Misra et al. [13] studied the flow and heat transfer of a MHD viscoelastic fluid in a channel with linear stretching walls due to its application to the field of haemodynamics. Since blood is electrically conducting, its flow in the cardiovascular system is likely to be influenced by a magnetic field.

In the above studies, researchers considered Newtonian/non-Newtonian fluids for different physical models. Several fluids of practical interest exhibit dynamic deviation from Newtonian behavior depending upon the flow configuration and/or the rate of deformation. For example, visco-elastic and Walters’ models often obey non-linear constitutive equations and the complexity in the equations is the main culprit for the lack of exact analytical solutions (refer for details Andersson [14], and Vajravelu and Rollins [15]). These two models are well known to infringe on convinced and set rules of thermodynamics. Hence the significance of the results analyzed in the above works is restricted as far as the polymer industry is concerned. Evidently for the theoretical results to be of any industrial importance, more general visco-elastic fluid models, such as upper-convected Maxwell model (UCM fluid) or Oldroyd B model, should be invoked. In recent times, these fluid models are being used to study the viscoelastic fluid flow over a stretching /non-stretching sheet with/without heat transfer (Hayat et al. [16], Aliakbar et al. [17], Vajravelu et al. [18], Hayat and Qasim [19], Prasad et al. [20], Vajravelu et al. [21]). Recently, Vajravelu et al. [22] investigated the MHD flow and heat transfer of a dusty non-Newtonian UCM fluid over a stretching sheet numerically and established that the Maxwell fluid reduces the wall-shear stress.

In view of the above studies, in the present paper, we analyze MHD flow of a UCM fluid in a parallel plate channel permeated by a uniform, transverse magnetic field where the surface velocity of the channel varies linearly with distance from the origin. The motivation of this study is to analyze the flow of blood in arteries whose walls are stretchable and the flow field is governed by the non-Newtonian behavior depicted by non-Newtonian viscoelastic fluid model. The governing equations are a coupled system of nonlinear partial differential equations which are transformed into a system of nonlinear ordinary differential equations. The transformed equations are solved analytically using the Optimal Homotopy Analysis Method (OHAM) and the results are displayed graphically. The values of the skin friction for different sets of physical parameters are also tabulated and discussed.

2 Flow analysis

Let us consider the steady, laminar flow of an incompressible and electrically conducting UCM fluid in a parallel plate channel bounded by the planes \( y = \pm a \). The flow is
driven by stretching the channel walls such that the surface velocity of each wall is proportional to the
distance from the origin (see Fig. 1).

A uniform magnetic field of strength \( B_0 \) is imposed normal to the channel walls i.e., parallel to
the \( y \)-axis, the electrical conductivity \( \sigma \) is assumed to be a constant. The steady, two-dimensional
boundary layer equations for the flow in usual notation are (see Sadeghy et al. [23])

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2.1}
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \lambda \left( u \frac{\partial^2 u}{\partial x^2} + v \frac{\partial^2 v}{\partial y^2} + 2uv \frac{\partial^2 u}{\partial x \partial y} \right) = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} \left( u + \lambda \frac{\partial u}{\partial y} \right) \tag{2.2}
\]

where \( u \) and \( v \) are the fluid velocities in the \( x \) and \( y \) directions, respectively. \( \lambda \) is the Maxwell
parameter, \( \sigma \) is the electrical conductivity, and \( \rho \) is the density of the fluid. The corresponding boundary
conditions for the flow problem are as follows:

\[
u = bx, \quad v = 0 \quad \text{at} \quad y = a,
\]

\[
u \frac{\partial u}{\partial y} = 0, \quad v = 0 \quad \text{at} \quad y = 0 \tag{2.3}
\]

where \( a \) and \( b \) are constants of proportionality. Equations (2.1) and (2.2) admit a self-similar solution of
the form

\[
u = bx f'(\eta), \quad v = -ab f(\eta) \quad \text{and} \quad \eta = \frac{y}{a}. \tag{2.4}
\]

We introduce the following dimensionless variables

\[
x^* = \frac{x}{a}, \quad \eta = \frac{y}{a}, \quad u^* = \frac{u}{ab}, \quad v^* = \frac{v}{ab}, \tag{2.5}
\]

\[
u^* = x^* f'(\eta), \quad v^* = -f(\eta). \tag{2.6}
\]

It can be seen that \( u \) and \( v \) satisfy the continuity equation defined in equation (2.1). Also, equation (2.2)

\[
f'''' - f'^2 + f f'' + \beta \left( 2f f' f'' - f'^2 f'' \right) - Mn (f'' - \beta f f'') = 0 \tag{2.7}
\]

where \( \beta = \frac{\lambda b}{a} \) is the Maxwell parameter and \( Mn = \frac{\sigma B_0^2}{\rho b} \) is the magnetic parameter. Using the

\[
f'(\eta) = 1, \quad f(\eta) = 0 \quad \text{at} \quad \eta = 1,
\]

\[
f(\eta) = 0 \quad \text{at} \quad \eta = 0. \tag{2.8}
\]
3 Heat transfer analysis

The energy equation with the standard boundary layer approximation is written as:

\[ u \frac{\partial T}{\partial x} + V \frac{\partial T}{\partial y} = \alpha \frac{\partial^3 T}{\partial y^3} \]  

(3.9)

where \( \alpha \) is the temperature dependent thermal diffusivity. The relationship between \( \alpha \) and the temperature will be defined shortly. The corresponding boundary conditions for this equation are:

\[ T = T_w \text{ at } y = a, \]
\[ \frac{\partial T}{\partial y} = 0 \text{ at } y = 0 \]  

(3.10)

where \( T_w \) is a constant. Now, introducing the dimensionless temperature variable as

\[ \theta = \frac{T}{T_w} \]  

(3.11)

and using (2.5) – (2.6) and (3.11), equation (3.9) can be rewritten as

\[ (1 + \varepsilon \theta) \theta'' + \varepsilon \theta'^2 + \text{Pr} f \theta' = 0 \]  

(3.12)

where \( \text{Pr} = \frac{v}{\alpha_0} \) is the Prandtl number and \( \varepsilon \) is the thermal conductivity parameter. Similarly, equation (3.10) can be rewritten as

\[ \theta' (\eta) = 0 \text{ at } \eta = 0, \]
\[ \theta (\eta) = 1 \text{ at } \eta = 1. \]  

(3.13)

The temperature dependent thermal diffusivity can now be defined as \( \alpha = \alpha_0 (1 + \varepsilon \theta) \).

4 Homotopy analysis method

4.1. Basic ideas

To begin, assume a non-linear differential equation in the following form

\[ N[u(\tau)] = 0 \]  

(4.14)

where \( N \) is a nonlinear operator, \( \tau \) is an independent variable, and \( u(\tau) \) is the solution to the equation. The function \( \phi(\tau, p) \) is defined as follows

\[ \lim_{p \to 0} \phi(\tau, p) = u_0(\tau) \]

where \( p \in [0, 1] \) and \( u_0(\tau) \) is the initial guess which satisfies the initial or boundary conditions and

\[ \lim_{p \to 1} \phi(\tau, p) = u(\tau). \]

By using the generalized homotopy method, Liao’s [24] so-called zero-order deformation equation for (4.14) turns out to be

\[ (1 - p) L \left[ \phi(\tau, p) - u_0(\tau) \right] = ph H(\tau) N \left[ \phi(\tau, p) \right] \]  

(4.15)

Where \( h \) is the auxiliary parameter, deemed to be the convergence control parameter (for details see Vajravelu and Van Gorder [25]), \( H(\tau) \) is the auxiliary function and \( L \) is the linear operator. It should be noted that there is a great freedom to choose the auxiliary parameter \( h \), the auxiliary function \( H(\tau) \), the
initial guess \( u_0(\tau) \) and the auxiliary linear operator \( L \). This freedom to choose \( h \) plays an important role in establishing the validity and flexibility of HAM as shown in this paper. Thus, when \( p \) increases from 0 to 1, the solution \( \phi(\tau, p) \) changes between the initial guess \( u_0(\tau) \) and the solution \( u(\tau) \). The Taylor series expansion of \( \phi(\tau, p) \) with respect to \( p \) is:

\[
\phi(\tau, p) = u_0(\tau) + \sum_{m=1}^{\infty} u_m(\tau) p^m
\]

and

\[
U_0^{(m)}(\tau) = \left. \frac{\partial^m \phi(\tau, p)}{\partial p^m} \right|_{p=0}
\]

where \( U_0^{(m)}(\tau) \) is called the \( m^\text{th} \) order of deformation, which leads to

\[
u_m(\tau) = u_0^m \frac{m!}{m!} = \left. \frac{\partial^m \phi(\tau, p)}{\partial p^m} \right|_{p=0}
\]

Now, the vector \( \overline{u}_m \) is defined as:

\[
\overline{u}_m = \{u_0, u_1, u_2, \ldots, u_m\}
\]

According to the definition in equation (4.16), the governing equation and corresponding initial conditions of \( u(\tau) \) can be deduced from the zero-order deformation equation (4.15). Differentiating equation (4.15) \( m \) times with respect to the embedding parameter \( p \) and setting \( p = 0 \), and finally dividing by \( m! \), we will have the so-called \( m^\text{th} \)-order deformation equation in the form

\[
L[u_m(\tau) - \chi_m u_{m-1}(\tau)] = h H(\tau) R\left(\overline{u}_{m-1}\right),
\]

where

\[
R\left(\overline{u}_{m-1}\right) = \frac{1}{(m-1)!} \left. \frac{\partial^{m-1} N\phi(\tau, p)}{\partial p^{m-1}} \right|_{p=0}
\]

and

\[
\chi_m = \begin{cases} 0, & m \leq 1 \\ 1, & m > 1 \end{cases}
\]

By applying the inverse linear operator to both sides of the linear equation (4.17), it can easily be solved and the constant of integration can be computed by applying the boundary conditions.

4.2. HAM Solution of the problem

Using equations (2.7) and (3.12), along with their respective boundary conditions (2.8) and (3.13), an accurate analytical approximation of the problem can be obtained. Here, the initial guesses are as follows:

\[
f_0(\eta) = \eta^2 - \eta, \quad \theta_0(\eta) = 1 - \frac{\Pr \eta^3}{12} + \frac{\Pr \eta^4}{6} - \frac{\Pr \eta^5}{12}
\]

while the auxiliary linear operators are chosen as:

\[
L_1[f(\eta)] = f''(\eta), \quad L_2[\theta(\eta)] = \theta''(\eta) + \Pr (\eta^2 - \eta),
\]

with

\[
L_1(c_1 + c_2 \eta + c_3 \eta^2) = 0, \quad L_2(c_4 + c_5 \eta) = 0
\]
where \( c_i \), \( i = 1, 2, \ldots, 5 \) are constants.

Using these definitions, the zero-order deformation equations become

\[
(1 - p)L_1[f - f_0] = p h N_1[f] \\
(1 - p)L_2[\theta - \theta_0] = p h N_2[f, \theta]
\]

where

\[
N_1[f] = f''(\eta, p) - f''(\eta, p) + f(\eta, p)f''(\eta, p) + \beta f(\eta, p) f'(\eta, p) f''(\eta, p) \\
- f''(\eta, p) f''(\eta, p) - M n(f'(\eta, p) - \beta f(\eta, p) f''(\eta, p))
\]

\[
N_2[f, \theta] = [(1 + \varepsilon \theta(\eta, p)) \theta'(\eta, p)] + Pr f(\eta, p) \theta'(\eta, p)
\]

\[
f(1, p) = 0, f'(1, p) = 1, f(0, p) = 0
\]

\[
\theta'(0, p) = 0, \theta(1, p) = 1
\]

where \( p \in [0,1] \) is an embedding parameter and \( h \) is the non-zero auxiliary parameter. As \( p \) varies from 0 to 1, \( f(\eta, p) \) and \( \theta(\eta, p) \) vary from their respective initial guesses \( f_0(\eta) \), \( \theta_0(\eta) \) to their solutions \( f(\eta), \theta(\eta) \). Using Taylor’s Theorem, we get

\[
f(\eta, p) = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta) p^m, \quad f_m(\eta) = \left. \frac{\partial^m f(\eta, p)}{\partial p^m} \right|_{p=0}
\]

\[
\theta(\eta, p) = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta) p^m, \quad \theta_m(\eta) = \left. \frac{\partial^m \theta(\eta, p)}{\partial p^m} \right|_{p=0}
\]

In order for these series’ to converge, \( h \) must be selected in such a way that it will minimize any residual error. Assume that \( h \) is chosen so the series (4.26) are convergent at \( p = 1 \) Equation (4.26) then become

\[
f = f_0(\eta) + \sum_{m=1}^{\infty} f_m(\eta),
\]

\[
\theta = \theta_0(\eta) + \sum_{m=1}^{\infty} \theta_m(\eta)
\]

Similarly, the \( m^{th} \) order deformation equations can be found by differentiating equations (4.26) \( m \) times with respect to \( p \). Then setting \( p = 0 \), and finally dividing by \( m! \). The results are as follows:

\[
L_1[f_m(\eta) - \chi_m f_{m-1}(\eta)] = h N_1^m(\eta),
\]

\[
L_2[\theta_m(\eta) - \chi_m \theta_{m-1}(\eta)] = h N_2^m(\eta)
\]

where

\[
N_1^m(\eta) = f''_{m-1} - M n f'_{m-1} + \sum_{k=0}^{m-1} \left\{ f_{m-1-k} f''_{k} - f'_{m-1-k} f'_k + \beta f_{m-1-k} \sum_{i=0}^{k} \left\{ 2 f'_{k-i} f''_{i} + M n f''_{i} - f'_{i} f''_{k} \right\} \right\}
\]

\[
N_2^m(\eta) = \theta''_{m-1} + \sum_{k=0}^{m-1} \left\{ \varepsilon \left\{ \theta'_{m-1-k} \theta''_{k} + \theta''_{m-1-k} \theta'_k \right\} + Pr f_k \theta'_{m-1-k} \right\}
\]

and
\[ X_m = \begin{cases} 0, & m \leq 1 \\ 1, & m > 1 \end{cases} \]

The corresponding boundary conditions are as follows
\[ f_m'(1) = 0, \quad f'_m(1) = 0, \quad f_m(0) = 0 \]
\[ \theta'_m(0) = 0, \quad \theta'_m(1) = 0 \]

Now the general solutions to equation (4.28) can be written as
\[ f_m(\eta) = f^*_m(\eta) + c_1 + c_2\eta + c_3\eta^2, \]
\[ \theta_m(\eta) = \theta^*_m(\eta) + c_4 + c_5\eta \]

in which \( f^*_m(\eta) \) and \( \theta^*_m(\eta) \) are the specific solutions of equations (4.28), and \( c_1 - c_5 \) are constants that can be found using the boundary conditions in equation (4.31). With these general solutions now available, it is possible to analytically solve the linear equations (4.28) for \( m = 1, 2, 3, ... \) using mathematical computational software such as Maple.

5 Results and discussion

In order to understand the mathematical model, results are presented graphically for the horizontal velocity field \( f' \) and the temperature profile \( \theta \), with respect to \( \eta \). The results are obtained for different values of the parameters \( \beta, Mn, Pr \) and \( \epsilon \) and are presented in Fig. 2-7. Values of skin friction \( f'^*(0) \) are presented in Table 1.

Figures 2-5 present the variation of the dimensionless velocity components \( u^* = x^* f'(\eta) \) and \( v^* = -f(\eta) \) for a given value of \( x' \). It can be seen from Figure 2 that the axial velocity component is positive near the vessel wall (\( \eta = 1 \)), negative towards the centerline (\( \eta = 0 \)), and tends towards zero with increasing magnetic parameter \( Mn \). This implies that a flow reversal takes place near the centerline of the channel, which is the result of the stretching of the channel walls. Figure 3 shows the velocity component normal to the channel wall for increasing values of \( Mn \). It can be seen that the velocity decreases as the magnetic field strength increases. Figure 4 shows the axial velocity component for different values of \( Mn \) and \( \beta \). Again the velocity is positive near the vessel wall (\( \eta = 1 \)), negative towards the centerline (\( \eta = 0 \)), and tends towards zero with increasing magnetic parameter \( Mn \). However the increase in Maxwell parameter \( \beta \) also forces the velocity to zero. Figure 5 shows the velocity field normal to the channel wall for different values of \( Mn \) and \( \beta \). As in Figure 4, as \( Mn \) increases, the velocity component decreases. The effect of increasing Maxwell’s parameter is to decrease the velocity field. Figures 6 and 7 provide the spatial variation of temperature for different values of the magnetic parameter \( Mn \) and the Prandtl number \( Pr \). In Figure 6, it can be seen that the temperature increases with increase in the magnetic field and the viscoelastic parameter. Thus, under the action of a magnetic field in an electrically conducting fluid, there develops a resistive force (Lorentz force) causing the impedance of flow and enhancement of temperature. Additionally, Figure 7 shows as the Prandtl number increases the temperature distribution decreases.

The effects of the physical parameters on the skin friction \( f'^*(0) \) are presented in Table 1. As the values of \( Mn, \epsilon \) and \( Pr \) increase, \( f'^*(0) \) increases. However, \( f'^*(0) \) increases as \( \beta \) decreases.
Figure 2: Variation of $f'$ with $\eta$ for different values of $Mn$ and $\beta = 0.001$, $Pr = 0.72$, $\varepsilon = 0.1$

Figure 3: Variation of $f$ with $\eta$ for different values of $Mn$ and $\beta = 0.001$, $Pr = 0.72$, $\varepsilon = 0.1$

Figure 4: Variation of $f'$ with $\eta$ for different values of $Mn$ and $\beta$ with $Pr = 0.72$, $\varepsilon = 0.1$
Figure 5: Variation of $f$ with $\eta$ for different values of $Mn$ and $\beta$ with $Pr = 0.72, \varepsilon = 0.1$

Figure 6: Variation of $\theta$ with $\eta$ for different values of $Mn$ and $\beta = 0.1, Pr = 7.0, \varepsilon = 0.1$

Figure 7: Variation of $\theta$ with $\eta$ for different values of $Pr$ and $\beta = 0.001, Mn = 15.0, \varepsilon = 0.001$
Table 1: Skin friction values at 14th approximation for various combinations of parameter

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6 Conclusion

The present study analyzes two dimensional boundary layer flow of a viscous, incompressible, electrically conducting, UCM fluid in the presence of transverse magnetic field in a horizontal channel with stretching walls.

- The effect of increasing Magnetic parameter is to reduce the axial velocity component, and to increase the temperature field.
- The effect of increasing Prandtl number is to reduce the temperature field.
- The effect of increasing Maxwell parameter is to reduce the axial velocity component.
- Skin friction increases with increasing Magnetic parameter, Prandtl number, and thermal conductivity parameter, but the opposite effect is observed with increasing Maxwell parameter.
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