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Research Article

Residual power series method for fractional Sharma-Tasso-Olever equation

Amit Kumar[1], Sunil Kumar[1*], Mukesh Singh[2]

(1) Department of Mathematics, National Institute of Technology, Jamshedpur- 831014, Jharkhand, India
(2) Department of Basic Science, UP Textile Technology Institute, Kanpur-208001, Uttar Pradesh, India

Abstract

In this paper, we introduce a modified analytical approximate technique to obtain solution of time fractional Sharma-Tasso-Olever equation. First, we present an alternative framework of the Residual power series method (RPSM) which can be used simply and effectively to handle nonlinear fractional differential equations arising in several physical phenomena. This method is basically based on the generalized Taylor series formula and residual error function. A good result is found between our solution and the given solution. It is shown that the proposed method is reliable, efficient and easy to implement on all kinds of fractional nonlinear problems arising in science and technology.

Keywords: Fractional Sharma-Tasso-Olever equation, Approximate solution, Fractional power series method.

1 Introduction

In the past few years, fractional differential equations (FDEs) have been widely investigated due to the fact that its numerous application in different branches of science and engineering such as fluid mechanics, electric network, signal processing, control theory of dynamical system, image processing, optics, viscoelasticity, and many other[1-2]. To find an approximate or analytical solutions of nonlinear fractional partial differential equations various methods are available in our literature like Adomian's decomposition methods [3], Laplace decomposition method [4], homotopy perturbation method [5], homotopy analysis method [6-7], homotopy analysis transform method [8-9] and Differential transform method [10-11].

In this paper, we used a new analytical methods namely Residual power series method. This method is based on constructing power series expansion solution for different nonlinear equations without linearization, perturbation, or discretization [12-16]. With the help of residual error concepts, this method computes the coefficient of the power series by a chain of algebraic equations of one or more variables and finally we get a series solution, in practice a truncated series solution. The main advantage of this method over the other

* Corresponding Author. Email address: skiitbhu28@gmail.com; skumar.math@nitjsr.ac.in; Tel: +91-7870102516

method is it can be applied directly to the given problem by choosing an appropriate initial guess approximation.
In purpose of this article we apply directly RPSM to consider the approximation solution of the nonlinear fractional Sharma-Tasso-Olever equation, which plays an important role in describing the nonlinear wave phenomena. Time-fractional derivative nonlinear fractional Sharma-Tasso-Olever equation written in the following form

$$D_t^{\alpha} u + 3au_x^2 + 3au^2u_x + 3auu_{xx} + au_{xxx} = 0 \quad t > 0 \ , \ 0 < \alpha \le 1, \tag{1.1}$$

where a is an arbitrary constant and α is a parameters describing the order of the fractional time-derivative. Song L et al. [17] has solved the (1.1) by using Variation iteration method, Adomain decomposition and homotopy perturbation method. But the best of author’s knowledge equation (1.1) by using RPSM have not been studied by any scientist and researchers.

2 Preliminaries and notations

Definition 2.1. *A power series expansion of the form [12-16]*

$$\sum_{m=0}^{\infty} c_m (t-t_0)^{m\alpha} = c_0 + c_1(t-t_0)^{\alpha} + c_2(t-t_0)^{2\alpha} + ..., \quad 0 \le n-1 < \alpha \le n, t \ge t_0,$$

is called multiple fractional power series about $t = t_0$, *where* t *is a variable and* c_m *’s are constants of the series.*

Definition 2.2. *A power series of the form [12-16]*

$$\sum_{m=0}^{\infty} f_m (t-t_0)^{m\alpha} = f_0 + f_1(t-t_0)^{\alpha} + f_2(t-t_0)^{2\alpha} + ..., \quad 0 \le n-1 < \alpha \le n, t \ge t_0,$$

is called multiple fractional power series about $t = t_0$, *where* t *is a variable and* f_m *’s are functions of* x *called the coefficients of the series.*

Theorem 2.1. *([16]) suppose that* f *has a fractional power series representation at* $t = t_0$ *of the form*

$$f(t) = \sum_{m=0}^{\infty} c_m (t-t_0)^{m\alpha} \quad 0 \le n-1 < \alpha \le n, t_0 \le t < t_0 + R.$$

If $D^{m\alpha} f(t)$ *are continuous on* $(t_0, t_0 + R)$, $m = 0,1,2,...,$ *then the coefficient* c_m *are given by formula*

$$c_m = \frac{D^{m\alpha} f(t_0)}{\Gamma(m\alpha+1)}, m = 0,1,2,...,$$

where $D^{m\alpha} = D^{\alpha}, D^{\alpha} ... D^{\alpha}$ *(m -times) and* R *is the radius of convergence.*

3 Analysis of residual power series method

To demonstrate the fundamental scheme of RPSM, we consider the following generalized non-linear fractional differential equation

$$D_t^{n\alpha} u(x,t) + R[x]u(x,t) + N[x]u(x,t) = g(x,t), \quad t > 0, \ x \in R, \ n-1 < n\alpha \le n, \tag{3.2}$$

subject to the initial condition

$$f_0(x) = u(x,0) = f(x) \text{ and } f_{(n-1)}(x) = D_t^{(n-1)\alpha} u(x,0) = h(x); \tag{3.3}$$

where $D_t^{n\alpha} = \dfrac{\partial^{n\alpha}}{\partial t^{n\alpha}}$, $R[x]$ is the linear operator in x, $N[x]$ is the general nonlinear operator in x, and $g(x,t)$ are continuous functions.

In case of RPSM method, the solution of equations (3.2) with (3.3) can be expressed as a fractional power series expansion about the initial point $t = 0$. Assume that the solution takes the expansion form

$$u(x,t) = \sum_{n=0}^{\infty} f_n(x) \frac{t^{n\alpha}}{\Gamma(1+n\alpha)}, 0 < \alpha \le 1, x \in I, 0 \le t < R. \tag{3.4}$$

The RPSM provides an analytical approximate solution for Eqs. (3.2) and (3.3) in term of fractional power series. Next, we define $u_k(x,t)$ the k-th truncated series of $u(x,t)$. That is,

$$u_k(x,t) = \sum_{n=0}^{k} f_n(x) \frac{t^{n\alpha}}{\Gamma(1+n\alpha)}, 0 < \alpha \le 1, x \in I, 0 \le t < R, k = 1,2,3,... \tag{3.5}$$

Obviously $u(x,t)$ satisfy the initial condition (3.3), so from Eq. (3.4), we obtain $u(x,0) = f_0(x) = f(x)$. On the other hand, from Eq. (3.5) the initial guess approximation (the 1-st RPS approximate solution) of $u(x,t)$ should be $u_1(x,t) = f(x) + f_1(x) \dfrac{t^{\alpha}}{\Gamma(1+\alpha)}$. Consequently, one can reformulate the expansion of Eq. (3.5) as follows:

$$u_k(x,t) = f(x) + f_1(x) \frac{t^{\alpha}}{\Gamma(1+\alpha)} + \sum_{n=2}^{k} f_n(x) \frac{t^{n\alpha}}{\Gamma(1+n\alpha)}, 0 < \alpha \le 1, x \in I, 0 \le t < R, k = 2,3,4,... \tag{3.6}$$

Subsequently in the RPS technique for finding the values of coefficients $f_n(x), n = 1,2,3,4,..., k$ in the series expansion of Eq. (3.6), we define the residual function as

$$\text{Res}(x,t) = D_t^{n\alpha} u(x,t) + R[x]u(x,t) + N[x]u(x,t) - g(x,t), \tag{3.7}$$

and the $k-th$ residual function, Res$_k$, of the style form

$$\text{Res}_k(x,t) = D_t^{n\alpha} u_k(x,t) + R[x]u_k(x,t) + N[x]u_k(x,t) - g(x,t), \quad k = 1,2,3,4... \tag{3.8}$$

As described in [12-16], it is clear that $\text{Res}(x,t) = 0$ and $\lim_{k\to\infty} \text{Re}\, s_k(x,t) = \text{Re}\, s(x,t)$ for each $x \in I$ and $t \ge 0$. In fact these leads to $D_t^{(n-1)\alpha}$ $\text{Res}_n(x,t_0) = 0, n = 1,2,3,4,..., k$ because the fractional derivative of a constant function in the Caputo's sense is zero. Meanwhile, the fractional derivative of $D_t^{(n-1)\alpha}$ of Res (x,t) and $\text{Res}_n(x,t)$ are matching at $t = 0$ for each $n = 1,2,3,4,..., k$; that is, $D_t^{(n-1)\alpha}$ $\text{Res}(x,0) =$ $D_t^{(n-1)\alpha}$ $\text{Res}_n(x,0) = 0, n = 1, 2,3,4,..., k.$

To obtain the coefficients $f_n(x), n = 1,2,3,..., k$, we apply the following subroutine; substitute n-th truncated series of $u(x,t)$ into Eq. (3.8), apply the fractional derivative formula $D_t^{(n-1)i\alpha}$ on $\text{Res}_n(x,t), n = 1,2,3,4,..., k,$ substitute $t = 0$, in the following formula, equate it to zero, and then lastly solve the obtained algebraic equation to obtain the form of the other coefficients. Any how we need to solve the following algebraic equation:

$$D_t^{(n-1)\alpha} \operatorname{Re} s_k(x,t) = 0, 0 < \alpha \le 1, x \in I, 0 \le t < R, n = 1,2,3,4,\dots,k. \tag{3.9}$$

In this way we can find all the required coefficients of the multiple fractional power series of Eqs. (3.2) and (3.3) are obtained.

4 Residual power series method for time -fractional Sharma-Tasso-Olever equation

Taking the general time -fractional Sharma-Tasso-Olever equation of the form [17]:

$$D_t^{\alpha} u + 3a u_x^2 + 3a u^2 u_x + 3a u u_{xx} + a u_{xxx} = 0 \quad t > 0 \ , \ 0 < \alpha \le 1, \tag{4.10}$$

with initial conditions

$$u(x,0) = \frac{2k\big(w + \tanh(k\,x)\big)}{1 + w\tanh(k\,x)}, \qquad k, w \in C. \tag{4.11}$$

The exact solution of Eq. (4.10) for standard motion i.e. $\alpha = 1$, is given by [17]

$$u(x,t) = \frac{2k\big(w + \tanh(k(x - 4ak^2t))\big)}{1 + w\tanh(k(x - 4ak^2t))} \ . \tag{4.12}$$

Using the RPS method as discussed in section 2, starting with the initial function

$$f_0(x) = u(x,0) = \frac{2k\big(w + \tanh(k\,x)\big)}{1 + w\tanh(k\,x)}, \tag{4.13}$$

and with the $k-th$ residual function for Sharma-Tasso-Olever equation as

$$\operatorname{Res}_k(x,t) = \frac{\partial^{\alpha} u_k}{\partial t^{\alpha}} + 3a\left(\frac{\partial u_k}{\partial x}\right)^2 + 3a u_k^2 \frac{\partial u_k}{\partial x} + 3a u_k \frac{\partial^2 u_k}{\partial x^2} + a\frac{\partial^3 u_k}{\partial x^3}, k = 1,2,3,\dots \ . \tag{4.14}$$

Additionally, taking into account those forms of $f_0(x)$ and based on Eq. (3.6), the $k-th$ truncated series of the multiple FPS expansion of $u(x,t)$ about $t = 0$ should be

$$u_k(x,t) = f(x) + f_1(x)\frac{t^{\alpha}}{\Gamma(1+\alpha)} + \sum_{n=2}^{k} f_n(x)\frac{t^{n\alpha}}{\Gamma(1+n\alpha)}, k = 2,3,4,\dots. \tag{4.15}$$

To determine the first unknown coefficients, $f_1(x)$ in the expansion of (4.15) we should substitute the 1-st truncated series $u_1(x,t)$ into the 1-st residual function, $\operatorname{Res}_1(x,t)$, of Eq. (4.14), to get

$$\operatorname{Res}_1(x,t) = \frac{\partial^{\alpha} u_1}{\partial t^{\alpha}} + 3a\left(\frac{\partial u_1}{\partial x}\right)^2 + 3a u_1^2 \frac{\partial u_1}{\partial x} + 3a u_1 \frac{\partial^2 u_1}{\partial x^2} + a\frac{\partial^3 u_1}{\partial x^3}. \tag{4.16}$$

But since $u_1(x,t) = f(x) + f_1(x)\,\dfrac{t^{\alpha}}{\Gamma(1+\alpha)}$. Then Eq. (4.16) becomes:

$$\begin{aligned}\mathrm{Res}_1(x,t)=&\frac{\partial^\alpha}{\partial t^\alpha}\left(f(x)+f_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)}\right)+3a\left(\frac{\partial}{\partial x}\left(f(x)+f_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)}\right)\right)^2\\&+3a\left(f(x)+f_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)}\right)^2\frac{\partial}{\partial x}\left(f(x)+f_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)}\right)\\&+3a\left(f(x)+f_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)}\right)\frac{\partial^2}{\partial x^2}\left(f(x)+f_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)}\right)\\&+a\frac{\partial^3}{\partial x^3}\left(f(x)+f_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)}\right)\end{aligned} \tag{4.17}$$

Now, depending on the result of Eq. (3.9) for $n=1$, the substitution of $t=0$ through Eq. (4.17) will yields

$$f_1(x)=\frac{8ak^4(-1+w^2)}{(\cosh(kx)+w\sinh(kx))^2}\ . \tag{4.18}$$

Hence, the 1-st RPS approximate solution of Eq. (4.10) expressed as

$$u_1(x,t)=\left(\frac{2k(w+\tanh(kx))}{1+w\tanh(kx)}\right)-\left(\frac{8ak^4(-1+w^2)}{(\cosh(kx)+w\sinh(kx))^2}\right)\frac{t^\alpha}{\Gamma(1+\alpha)}. \tag{4.19}$$

Similarly, to find out the form of the second unknown coefficients, $f_2(x)$ we substitute the 2-nd truncated series $u_2(x,t)=f(x)+f_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)}+f_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}$ of Eq. (4.15) into 2-nd residual functions $\mathrm{Res}_2(x,t)=\frac{\partial^\alpha u_2}{\partial t^\alpha}+3a\left(\frac{\partial u_2}{\partial x}\right)^2+3au_2^2\frac{\partial u_2}{\partial x}+3au_2\frac{\partial^2 u_2}{\partial x^2}+a\frac{\partial^3 u_2}{\partial x^3}$ of Eq. (4.14) to obtained the following discretized form:

$$\begin{aligned}\mathrm{Res}_2(x,t)=&\frac{\partial^\alpha}{\partial t^\alpha}\left(f(x)+f_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)}+f_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right)+3a\left(\frac{\partial}{\partial x}\left(f(x)+f_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)}+f_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right)\right)^2\\&+3a\left(f(x)+f_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)}+f_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right)^2\frac{\partial}{\partial x}\left(f(x)+f_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)}+f_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right)\\&+3a\left(f(x)+f_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)}+f_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right)\frac{\partial^2}{\partial x^2}\left(f(x)+f_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)}+f_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right)\\&+a\frac{\partial^3}{\partial x^3}\left(f(x)+f_1(x)\frac{t^\alpha}{\Gamma(1+\alpha)}+f_2(x)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}\right)\ .\end{aligned} \tag{4.20}$$

Now, operating D_t^α on both sides of Eq. (4.20) gives the α -th time-fractional derivative of $\mathrm{Res}_2(x,t)$ and from Eq. (3.9) for $n=2$ and substituting $t=0$ will yields

$$f_2(x)=\frac{64a^2k^7(-1+w^2)(w\cosh(kx)+\sinh(kx))}{(\cosh(kx)+w\sinh(kx))^3}. \tag{4.21}$$

Therefore, the 2-nd RPS approximation solution of Eq. (4.10) will be of the form

$$u_2(x,t)=\left(\frac{2k\left(w+\tanh(kx)\right)}{1+w\tanh(kx)}\right)-\left(\frac{8ak^4(-1+w^2)}{\left(\cosh(kx)+w\sinh(kx)\right)^2}\right)\frac{t^{\alpha}}{\Gamma(1+\alpha)}$$
$$+\left(\frac{64a^2k^7(-1+w^2)(w\cosh(kx)+\sinh(kx))}{\left(\cosh(kx)+w\sinh(kx)\right)^3}\right)\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}. \quad (4.22)$$

Likewise, the remaining components of $f_n(x),\ n\geq 3$ can be obtained and the final solution of Eq. (4.10) is given of the form

$$u(x,t) = \sum_{n=0}^{\infty} f_n(x)\frac{t^{n\alpha}}{\Gamma(1+n\alpha)}, \quad v(x,t) = \sum_{n=0}^{\infty} g_n(x)\frac{t^{n\alpha}}{\Gamma(1+n\alpha)}. \quad (4.23)$$

5 Numerical simulations and discussions

This section deals with the validity and effectiveness of the proposed method for Sharma-Tasso-Olever equation through the different graphical representation and tabulated data.

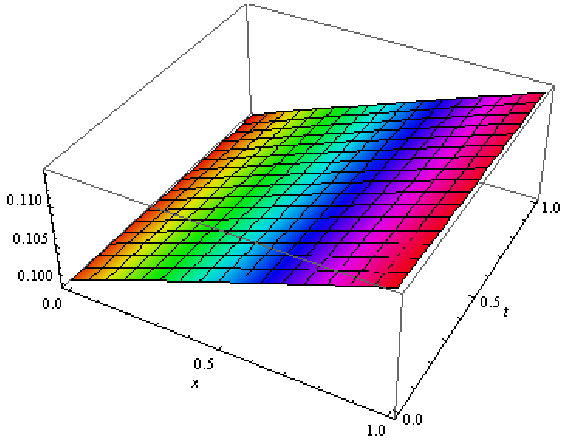

Figure 1: Exact solution of $u(x,t)$ with parameters $w=\frac{1}{2}, k=0.1$ and $a=1$.

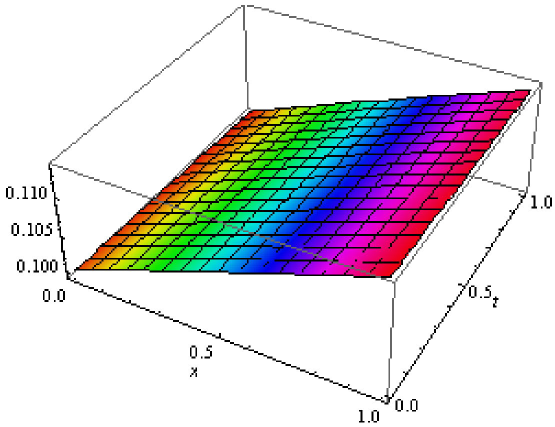

Figure 2: Approximate solution of $u(x,t)$ with parameters $w=\frac{1}{2}, k=0.1$ and $a=1$ at $\alpha=1$.

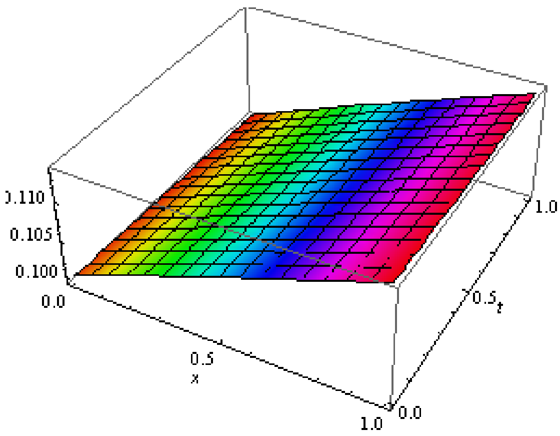

Figure 3: Approximate solution of $u(x,t)$ with parameters $w=\frac{1}{2}, k=0.1$ and $a=1$ at $\alpha=0.9.$

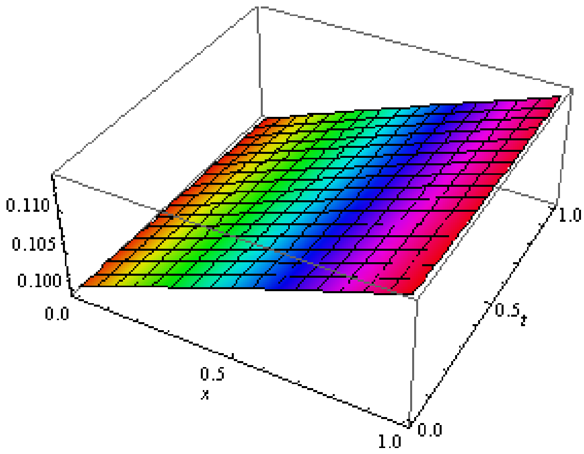

Figure 4: Approximate solution of $u(x,t)$ with parameters $w=\frac{1}{2}, k=0.1$ and $a=1$ at $\alpha=0.8.$

Figs. 1 and 2 shows the graphical judgment of the approximate solution with respect to the exact solution. It is observed from Figs. 1 and 2 that the both are nearly identical in nature. Figs. 3-4 shows the approximate solution for $\alpha=0.9$ and $0.8.$

Further the accuracy of the proposed method can be checked by computing the absolute error $E_5(u)=|u(x,t)-u_5(x,t)|,$ where $u(x,t)$ the exact and $u_5(x,t)$ is approximate solution of (4.10). From the Fig. 5 of absolute error curve, it is observed that it achieves a high level of accuracy i.e., our approximate solution obtained by proposed method quickly converges to the exact solution in only 5th order approximations.

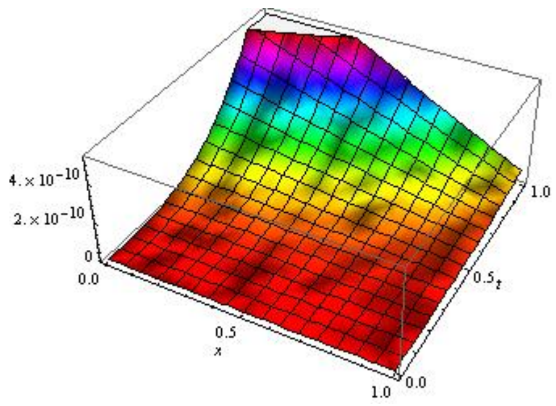

Figure 5: Absolute error $E_5(u)=|u(\mathrm{x},\mathrm{t})-u_5(\mathrm{x},\mathrm{t})|$, with parameters $w=\frac{1}{2}, k=0.1$ and $a=1.$

Table 1: Assessment among the Exact solution, VIM, ADM, HPM and RPSM solution with parameters $w=\frac{1}{2}, k=a=\alpha=1$ and $t=0.001$

x	Exact Solution	VIM	ADM	HPM	RPSM
0	0.938808808	0.938798380	0.938800000	0.938800000	0.933988000
1	1.813631681	1.813642383	1.813642415	1.813642415	1.816015300
2	1.973719022	1.973721044	1.973721044	1.973721044	1.975533572
3	1.996422935	1.996423221	1.996423221	1.996423221	1.996471223
4	1.999515521	1.999515561	1.999515561	1.999515561	1.999549174
5	1.999934426	1.999934431	1.999934431	1.999934431	1.999938981
6	1.999991125	1.999991127	1.999991127	1.999991127	1.999991741
7	1.999998799	1.999998799	1.999998799	1.999998799	1.999998799

Above table shows that numerical values of the obtained solution are also very nearer to the exact solution.

6 Conclusion

In this article, RPSM successfully applied to get the solution of the fractional Sharma-Tasso-Olever equation. Through the above results we found that RPSM is very efficient and more realistic to solve fractional order differential equations like Sharma-Tasso-Olever equation. Therefore we can say that the RPS methodology is very powerful and novel technique for finding the approximate as well as analytical solution of many fractional physical models arising in different branches of science.

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