Effect of radiation on heat and mass transfer of fluid flow across a horizontal cylinder

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Abstract
The free convective boundary layer flow, heat and mass transfer across an isothermal cylinder in the diffusing species is investigated. The partial differential conservation equations are non-dimensionalized and solved numerically by using an implicit iterative tridiagonal finite-difference method. The effects of radiation parameters on dimensionless velocity, temperature and species concentration distributions are studied, for the case of water (Pr = 7.0), concentration is slightly affected by radiation parameter; Temperatures are however increased by an increase in radiation parameter. Furthermore, the local skin friction, local Nusselt number and local Sherwood number have been analyzed for various parametric conditions. It is found that as the thermal radiation parameter increases the skin-friction coefficient increases and the local Nusselt number decreases. Excellent agreement is obtained with the earlier studies for the purely fluid.

Keywords: Cylinder, Heat and Mass transfer, Radiation.

1 Introduction
The study of coupled heat and mass transfer flows is important in view of several physical problems, such as fluids undergoing exothermic and endothermic chemical reaction, geophysics, chemical engineering, ceramics processing, metallurgy and environmental pollution. Steady thermal convection studies for horizontal cylinders were reported by Sparrow and Lee [1] and Ingham [2]. More recently Sadeghipour and Hannani [3] considered the unsteady free convection from a horizontal cylinder between two vertical planes. Further studies of over cylindrical geometries have been presented by Kaminski et al [4], for the case of convective mass transfer by Sigwalt et al [5], and for the cases of a cavity and forced convection regimes, by Cesini et al [6] and Karniadakis [7]. Heat and mass transfer in fluidized or packed beds of chemical reactors as well as due to several possible mechanisms of mass transfer are discussed from the point of view of applications of the present mathematical approach. Abbassi et al [8] considered...
the case of natural convection from elliptic cross-section cylinders. Sobera et al [9] studied the flow, heat, and mass transfer around a cylinder sheathed by a second, porous cylinder and placed in a perpendicular turbulent air flow, with applications in transport mechanisms in clothing in outdoor conditions. Merkin [10] studied the free convection flow on an isothermal surface of the circular cylinder. He has obtained the results for the local skin-friction coefficient and the local Nusselt number at various positions on the surface of the circular cylinder. The fluid Prandtl number is taken to be Pr = 1.0. Nazar et al. [11] present a numerical study for a free convection boundary layer flow on an isothermal horizontal cylinder considering over the Merkin [10] problem the presence of a micropolar fluid. In this study, the mass conservation equation is solved as a coupled equation with the momentum equation to consider the effect of chemical reaction with the Newtonian fluid which also acts as a radiative substance. The radiative effects have important applications in physics and engineering. The radiative heat transfer effects on different flows are very important in space technology and high temperature processes. But very little is known about the effects of radiation on the boundary layer. Thermal radiation effects may play an important role in controlling heat transfer in industry where the quality of the final product depends on the heat controlling factors to some extent. High temperature plasmas, cooling of nuclear reactors, liquid metal fluids, power generation systems are some important applications of radiative heat transfer processes to conductive gray fluids. The effect of radiation on heat transfer problems have studied by Hossain and Takhar [12], Takhar et. al [13], Hossain et. al. [14], Ogulu, and Makinde [15], Makinde and Ogulu [16]. Duwairi [17] considered thermal radiation effects on mixed convection flow over a non-Isothermal cylinder and a sphere in a Darcian porous material. Recently, Damseh et. al. [18] has discussed the magnetic field and thermal radiation effects on forced convection flow. A new dimension is added to the above mentioned study by considering the effects of porous media Gopi et. al [19], Olajuwon et. al [20].

In this paper, we shall limit our consideration to flows over a horizontal cylinder where the reactive-radiative boundary layer equations is transformed using similarity variables, to a third order and a second order differential equations corresponding to the momentum, thermal and concentration boundary layer equations are obtained. These equations are solved numerically using an implicit iterative tridiagonal finite-difference method. The effects of chemical reaction parameter along with the influence of radiation parameter on velocity and temperature fields are investigated and analyzed with the help of their graphical representations.

2 Mathematical model

Free convective, two-dimensional steady laminar boundary-layer flow of an incompressible heat and mass transfer across an isothermal horizontal circular cylinder of radius, r, in the presence of thermal radiation is considered. A reactive soluble species diffuses in the fluid with sufficiently small concentration so that thermal energy generated during the chemical reaction may be ignored. The surface temperature of the cylinder is Tw and that of the ambient fluid T∞, where Tw > T∞. The physical model is shown in figure 1 below.
The concentration is sustained at a constant magnitude, \( C_w \) at the cylinder surface and the species at ambient fluid is modeled as homogenous with concentration \( C_\infty \). The orthogonal coordinates \( X \) and \( Y \) are measured along the surface of the cylinder, starting with the lower stagnation point, and normal to it, respectively. Assuming constant thermophysical properties, under the Boussinesq approximation, the governing boundary layer equations for momentum, heat and species conservation can be written as [21, 22]:

\[
\begin{align*}
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} &= 0 \tag{2.1} \\
\rho \left( U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} \right) &= \mu \frac{\partial^2 U}{\partial Y^2} + \rho g \beta_T (T - T_\infty) \sin\left(\frac{X}{r}\right) + \rho g \beta_C (C - C_\infty) \sin\left(\frac{X}{r}\right) \tag{2.2} \\
\rho \left( U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} \right) &= k \left( \frac{\partial^2 T}{\partial Y^2} \right) - \frac{1}{c_p} \frac{\partial q_r}{\partial Y} \tag{2.3} \\
\rho \left( U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} \right) &= D \left( \frac{\partial^2 C}{\partial Y^2} \right) \tag{2.4}
\end{align*}
\]

The appropriate boundary conditions for the problem are given by:

\[
\begin{align*}
Y = 0 &: U = 0 \quad V = 0 \quad T = T_w \quad C = C_w \\
Y \to \infty &: U = 0 \quad T = T_\infty \quad C = C_\infty \tag{2.5}
\end{align*}
\]

Here \( U \) and \( V \) are the velocity components along \( X \) and \( Y \) axes; \( T, C, \rho \) and \( \mu \) are the temperature, species concentration, fluid density and coefficient of fluid dynamic viscosity, respectively. \( k \) is the thermal conductivity, \( c_p \) is the specific heat at constant pressure, \( D \) is the species diffusivity, \( q_r \) is the radioactive heat flux.

Using Rosseland approximation for radiation (Brewster [23]) we can write

\[
q_r = -\frac{4\sigma}{3k^*} \frac{\partial T^4}{\partial Y}
\]

where \( \sigma \) is the Stefan-Boltzman constant, \( k^* \) is the absorption coefficient. Assuming that the temperature difference within the flow is such that \( T^4 \) may be expanded in a Taylor series and expanding \( T^4 \) about \( T_\infty \) and neglecting higher orders we get \( T^4 \equiv 4T_\infty^4 T - 3T_\infty^4 \). Therefore, equation (2.3) becomes
\[
\left( \frac{U}{\partial X} + V \frac{\partial T}{\partial Y} \right) = \frac{k}{\rho c_p} \left( \frac{\partial^2 T}{\partial Y^2} \right) + \frac{16\sigma T_0^3}{3\rho c_p k^\gamma} \frac{\partial^2 T}{\partial Y^2} \tag{2.6}
\]

This approximation is valid at points optically far from the boundary surface, and is good only for intensive absorption, that is, for an optically thick boundary layers. Despite these shortcomings, the Rosseland approximation has been used with success in a variety of problems ranging from the transport of radiation through gases of low density to the study of the effects of radiation on blast waves by nuclear explosion, Ali et al. [24].

To solve the above equations, the following non dimensional variables are used:

\[
x = \frac{X}{r}, \quad y = \frac{Y}{Gr^{1/4}}, \quad u = \frac{rU}{vGr^{1/2}}, \quad v = \frac{rV}{vGr^{1/4}},
\]

\[
\theta = \frac{T - T_{\infty}}{T_w - T_{\infty}}, \quad \phi = \frac{C - C_w}{C_w - C_{\infty}}, \quad Gr = \frac{g\beta_r (T_w - T_{\infty}) r^3}{v^2},
\]

\[
N = \frac{\beta_r (C_w - C_{\infty})}{\beta_r (T_w - T_{\infty})}, \quad Pr = \frac{\mu c_p}{k}, \quad Sc = \frac{v}{D}, \quad R_d = \frac{4\sigma T_0^3}{kk^\gamma},
\]

\[
\psi = xf(x, y), \quad u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial y} \tag{2.7}
\]

The corresponding dimensionless equations become:

\[
f'' + ff' - f'^2 + (\theta + N\phi) \sin x = \left( f' \frac{\partial f'}{\partial x} - f'^2 \frac{\partial \phi}{\partial x} \right) \tag{2.8}
\]

\[
\frac{1}{Pr} \left( 1 + \frac{4R_d}{3} \right) \theta'' + f\theta' = x \left( f' \frac{\partial \theta}{\partial x} - \theta' \frac{\partial f}{\partial x} \right) \tag{2.9}
\]

\[
\frac{1}{Sc} \phi'' + f\phi' = x \left( f' \frac{\partial \phi}{\partial x} - \phi' \frac{\partial f}{\partial x} \right) \tag{2.10}
\]

The transformed boundary conditions now reduce to the form:

\[
f'(x, 0) = 0, \quad f(x, 0) = 0, \quad \theta(x, 0) = 1, \quad \phi(x, 0) = 1
\]

\[
f'(x, \infty) = 0, \quad \theta(x, \infty) = 0, \quad \phi(x, \infty) = 0 \tag{2.11}
\]

Where \( \psi \) is the stream function and \( f \) is the dimensionless stream function and the primes denote differentiations with respect to \( y \). Pr is the Prandtl number, Sc is the Schmidt number, N is the buoyancy ratio parameter, Rd is the radiation parameter and Gr is the Grashof number.

### 3 Solution procedure

The non-similar two-point boundary value problem defined by equations (2.8) to (2.11) is strongly nonlinear and an analytical solution is extremely difficult. Self similarity of the conservation equations is achieved when \( x \sim 0 \), which physically corresponds to the lower stagnation point of the cylinder. For such case, the transformed conservation equations of the present model with some simplifications can be reduced to simpler ordinary equations as reported by Nazar et. al [23]. The parameter, \( x \), in fact defines the curvature of the cylinder. At the upper stagnation point, \( x \sim \pi \). The local Nusselt number, local Sherwood number and local skin friction coefficient at the cylinder surface are defined respectively by the following equations:

\[
\]
\[ Nu_x = -\frac{\partial \theta(x,0)}{\partial y}, \quad Sh_x = -\frac{\partial \phi(x,0)}{\partial y}, \quad Cf_x = x \frac{\partial^2 f(x,0)}{\partial y^2} \]  

(3.12)

The above system of governing equations (8-11) are nonlinear partial differential equations depending on curvature parameter \( x \) and the transverse direction \( y \). The present analysis integrates the system (8-11) by the implicit finite difference approximation together with the modified Keller box method of Cebeci and Bradshaw [26, 27].

The equations (8-11) are written in terms of first order equations in \( y \), which are then expressed in finite difference form by approximating the functions and their derivatives in terms of the central differences in both coordinate directions. Denoting the mesh points in the \((x, y)\) plane by \( x_i \) and \( y_j \), where \( i = 1, 2, 3, \ldots, M \) and \( j = 1, 2, 3, \ldots, N \), central difference approximations are made such that the equations involving \( x \) explicitly are centered at \((x_i-1/2, y_j-1/2)\) and the remainder at \((x_i, y_j-1/2)\), where \( y_j-1/2 = (y_j + y_j-1)/2 \), etc. This results in a set of nonlinear difference equations for the unknowns at \( x_i \) in terms of their values at \( x_{i-1} \). These equations are then linearised by the Newton’s quasi-linearization technique and are solved using a block-tridiagonal algorithm, taking as the initial iteration of the converged solution at \( x = x_{i-1} \). Now to initiate the process at \( x = 0 \), we first provide guess profiles for all five variables (arising the reduction to the first order form) and use the Keller box method to solve the governing ordinary differential equations. Having obtained the lower stagnation point solution it is possible to march step by step along the boundary layer. For a given value of \( x \), the iterative procedure is stopped when the difference in computing the velocity and the temperature in the next iteration is less than \( 10^{-4} \), i.e. when \( |\delta f^i| \leq 10^{-4} \), where the superscript denotes the iteration number. The computations were not performed using a uniform grid in the \( y \) direction, but a non uniform grid was used and defined by \( y_j = \sinh ((j-1)/p) \), with \( j = 1, 2, \ldots, 301 \) and \( p = 100 \).

4 Results and discussions

A graphical solutions for heat and mass transfer problem is presented, for the effects of radiation parameter on the velocity, temperature and concentration profiles with distance normal to the cylinder surface, at the upper stagnation point \((x = \pi)\). Additionally we have computed the variation of local skin friction coefficient (dimensionless surface shear stress, \( Cf_x \)), Nusselt number (\( Nux \)) and Sherwood number (\( Shx \)) with coordinate \( x \), (i.e. curvature parameter) along the cylinder surface. To test the accuracy of the present numerical computations, we have compared with the solutions obtained by Merkin [10] and also Nazar et al [11], for the forced convective, purely fluid version of the general model i.e. with \( Rd \rightarrow 0 \), and neglecting the species conservation equation, for \( Pr = 1.0 \) and \( N = 0 \). The comparisons are all shown in Table 1 below.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( Nux )</th>
<th>( Cf_x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0 )</td>
<td>Merkin (30)</td>
<td>0.4214</td>
</tr>
<tr>
<td>( 0.4214 )</td>
<td>0.4214</td>
<td>0.4223</td>
</tr>
<tr>
<td>( \pi/6 )</td>
<td>0.4161</td>
<td>0.4161</td>
</tr>
<tr>
<td>( \pi/3 )</td>
<td>0.4007</td>
<td>0.4005</td>
</tr>
<tr>
<td>( \pi/2 )</td>
<td>0.3745</td>
<td>0.3741</td>
</tr>
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<td>( 2\pi/3 )</td>
<td>0.3364</td>
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</tr>
<tr>
<td>( 5\pi/6 )</td>
<td>0.2825</td>
<td>0.2811</td>
</tr>
<tr>
<td>( \pi )</td>
<td>0.1945</td>
<td>0.1916</td>
</tr>
</tbody>
</table>

Table 1: Values of the local Nusselt number and skin friction coefficient for \( Rd \rightarrow \infty \), \( Pr = 1.0 \) and \( N = 0 \)
In Table 1, we observe that for the case without radiation effect and species transfer, a continuous decrease in Nusselt number, $N_u$, occurs from the lower stagnation point ($x \approx 0$) around the curved surface of the cylinder to the upper stagnation point ($x \approx \pi$). Shear stress function, $C_f$, however increases from $x = 0$ and peaks between $x = \pi/2$ and $\pi/3$, thereafter decreasing markedly toward the upper stagnation point.

Figures 2, 3 and 4 show the effects of radiation parameter $R_d$ on velocity profiles, temperature profiles and concentration profiles for the respective cases of $R_d = 1, 2, 3, 4$. A distinct increase in velocity function is observed with an increase in radiation parameter which in all cases decays from a maximum at the cylinder surface ($y = 0$) to zero in the free stream i.e. at the exterior of the boundary layer. The increase in velocity with an increase in radiation parameter leads the boundary layer thickness to be wider.

From Figs 2 and 3 we have seen that as the radiation parameter $R_d$ increases, both the velocity and the temperature increase. Which means that higher radiation occur for higher values of temperature, which cause the increase of velocity as well. The change of velocity profiles in the $y$ direction reveal the typical velocity profile for natural convection boundary layer flow i.e. the velocity is zero at the boundary wall and then the velocity increases to the peak value as $y$ increases and finally the velocity approaches to zero (the asymptotic value).

The variation of local shear stress function, figure 5, with coordinate $x$ and $C_f$ values are increased with an increase of $R_d$. As such the radiation effect can be seen to exert a significant influence on the momentum in the buoyant flow regime, since it accelerates the flow. The influence of the thermal radiation parameter $R_d$ on the temperature profiles is presented on Fig. 3. From definition of the radiation parameter, the increases in the value of $R_d$ will give the tendency to increase the radiation effect. In turn, this causes the thermal boundary layer to increase and the temperature at every point also will increase. Since the temperature away from the wall will increase, the local Nusselt number will decrease as shown in Fig. 6 due to excessive heating in the fluid adjacent to the cylinder surface.

Figs 5-6 show that skin friction coefficient $C_f$ and local rate of heat transfer ($N_u$) increase for increasing values of radiation parameter $R_d$. It is observed from the Fig 5 that the skin friction increase gradually from zero value at lower stagnation point along the $y$ direction and from figure 6, it reveals that the rate of heat transfer decrease slightly along the $y$ direction from lower stagnation point to the downstream.

From equation (2.10) i.e. the species diffusion equation, it is apparent that there is a coupling between the species field ($\phi$) and the velocity field ($f'$). As such with radiation effects present, the flow is accelerated and this indirectly affects the species diffusion which is lower for higher values of radiation parameter $R_d$. A slightly decrease in concentration function is observed with an increase in radiation parameter which in all cases decays from a maximum at the cylinder surface ($y = 0$) to zero in the free stream i.e. at the exterior of the boundary layer. The local Sherwood number $S_h$ shown in figure 7 conversely is seen to be increased (for the strong radiative case, $R_d = 6$) with distance around the cylinder surface $x$ with an increase in radiation parameter $R_d$ from 1, 2, to maximum radiation parameter of 6. It is imperative to notice here that all the calculations are drawn for constant value of buoyancy ratio parameter $N = 1$. 
Figure 2: Velocity profiles at upper stagnation point $x = \pi$ for various values of radiation parameter $R_d$

Figure 3: Temperature profiles at upper stagnation point $x = \pi$
Figure 4: Concentration profiles at upper stagnation point $x = \pi$

Figure 5: Local skin friction coefficient versus $x$ for various values of radiation parameter $R_d$
Figure 6: Local Nusselt number versus $x$ for various values of radiation parameter $R_d$

Figure 7: Local Sherwood number versus $x$ for various values of radiation parameter $R_d$
5 Conclusions

The present study deals with a mathematical model for steady, two-dimensional free convection heat and mass transfer of a radiative fluid from an isothermal horizontal cylinder. The boundary layer equations have been non-dimensionalized and the resulting nonsimilar partial differential equations shown to be controlled by a number of thermophysical parameters. The implicit iterative tridiagonal finite-difference method has been employed to solve the transformed equations, showing excellent agreement with the earlier non-porous, studies. The resent computations have shown that increasing the radiation parameter (Rd) causes a strong acceleration in the flow regime and boosts velocities and shear stresses, simultaneously increasing temperatures and also reduce concentration function.

Nomenclature

B  dimensionless material parameter
C  concentration
e_p  specific heat of the fluid at constant pressure
C_t  coefficient of friction
D  mass diffusion coefficient
g  magnitude of acceleration due to gravity
Gr_C  mass Grashof number, \( Gr_C = \frac{\rho B_C u(C_w - C_\infty)}{\mu v_w} \)
Gr_T  thermal Grashof number, \( Gr_T = \frac{\rho B_T u(T_w - T_\infty)}{\mu v_w} \)
j  micro-inertia density
k  thermal conductivity
M  dimensionless material parameter
m  dimensionless micro-gyration vector
N  dimensionless micro-rotation variable
Nu  Nusselt number
Pr  Prandtl number, \( \nu / \alpha \)
Q  dimensionless heat generation or absorption parameter
qc  dimensional heat generation or absorption coefficient
R  vortex viscosity parameter
T  temperature
Sc  Schmidt number
Sh  Sherwood number
u  fluid axial velocity
U  dimensionless velocity
v  transverse velocity
x, y  axial and transverse coordinates
X, Y  dimensionless coordinates

Greek symbols:

\( \alpha \)  chemical reaction parameter
\( \beta_C \)  coefficient of concentration expansion
\( \beta_T \)  coefficient of thermal expansion
\( \Lambda \)  spin gradient viscosity
vortex viscosity

λ  dimensionless material parameter

θ  non-dimensional temperature

φ  non-dimensional concentration

γ  dimensionless chemical reaction parameter.

µ  dynamic viscosity

ν  kinematic viscosity

ρ  fluid density

ω  micro-rotation component

Subscripts:

∞  free stream condition

w  condition at the wall

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