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Research Article

Traveling Wave Solutions of ZK-BBM Equation Sine–Cosine Method

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Abstract

Travelling wave solutions are obtained by using a relatively new technique which is called sine–cosine method for ZK-BBM equations. Solution procedure and obtained results re-confirm the efficiency of the proposed scheme.

Keywords Nonlinear equations, sine-cosine Method, Traveling Wave Solutions, ZK-BBM equations.

1 Introduction

There has been an unprecedented development in nonlinear sciences [1-44] during the last two decades. In the similar context, several numerical and analytical techniques including Homotopy Analysis (HAM), Perturbation, Modified Adomian's Decomposition (MADM), Variational iteration (VIM), Variation of Parameters, Finite difference, Finite volume, Backlund transformation, inverse scattering, Jacobi elliptic function expansion, tanh function have been developed to solve such equations, see [1-45] and the references therein. Most of these techniques have their inbuilt deficiencies including evaluation of the so-called Adomian's polynomials, divergent results, successive applications of the integral operator, un-realistic assumptions, non-compatibility with the nonlinearity of physical problem and very lengthy calculations. Inspired and motivated by the ongoing research in this area, we apply a relatively new technique which is called Sine–cosine method [43-45] to find travelling wave solutions of ZK-BBM equations. It is worth mentioning that Wazwaz [3, 4] made a detailed study for Compact and noncompact physical structures for the ZK–BBM equation and also calculated exact solutions of compact and noncompact structures for the KP–BBM equation. It is to be highlighted that such equation arises frequently in various branches of physics, applied and engineering sciences, see [43-45] and the references therein. The proposed scheme is fully compatible with the complexity of such problems and is very user-friendly. Numerical results are very encouraging.

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2 Sine–cosine Method for ZK-BBM Equation

The main steps for using sine–cosine method, as following

1. We introduce the wave variable $\xi = x - ct$ into the PDE, we get

$$P(u, u_x, u_t, u_{xx}, u_{xt}, u_{tt}, u_{xxx}....) = 0, \qquad (1)$$

where $u(x,t)$ is traveling wave solution. This enables us to use the following changes

$$\frac{\partial}{\partial t} = -c\frac{\partial}{\partial \xi}, \frac{\partial^2}{\partial t^2} = c^2\frac{\partial^2}{\partial \xi^2}, \frac{\partial}{\partial x} = \frac{\partial}{\partial \xi}, \frac{\partial^2}{\partial x^2} = \frac{\partial^2}{\partial \xi^2},$$
$$\frac{\partial}{\partial y} = \frac{\partial}{\partial \xi}, \frac{\partial^2}{\partial y^2} = \frac{\partial^2}{\partial \xi^2}, \frac{\partial}{\partial z} = \frac{\partial}{\partial \xi}, \frac{\partial^2}{\partial z^2} = \frac{\partial^2}{\partial \xi^2}. \qquad (2)$$

One can immediately reduce the nonlinear PDE (1) into a nonlinear ODE

$$Q(u, u_\xi, u_{\xi\xi}, u_{\xi\xi\xi}....) = 0. \qquad (3)$$

The ordinary differential equation (3) is then integrated as long as all terms contain derivatives, where we neglect integration constants.

2. The solutions of many nonlinear equations can be expressed in the form

$$u(x,t) = \{\lambda \sin^\beta(\mu\xi), |\xi| \le \frac{\pi}{\mu}, \qquad (4)$$

or in the form

$$u(x,t) = \{\lambda \cos^\beta(\mu\xi), |\xi| \le \frac{\pi}{2\mu}, \qquad (5)$$

where λ, μ and β are parameters that will be determined, μ and c are the wave number and the wave speed, respectively, We use

$$u(\xi) = \lambda \sin^\beta(\mu\xi),$$
$$u^n(\xi) = \lambda^n \sin^{n\beta}(\mu\xi),$$
$$(u^n)_\xi = n\mu\beta\lambda^n \cos(\mu\xi)\sin^{n\beta-1}(\mu\xi),$$
$$(u^n)_{\xi\xi} = -n^2\mu^2\beta^2\lambda^n \sin^{n\beta}(\mu\xi) + n\mu^2\lambda^n\beta(n\beta-1)\sin^{n\beta-2}(\mu\xi), \qquad (6)$$

and the derivatives of (5) become

$$u(\xi) = \lambda \cos^\beta(\mu\xi),$$
$$u^n(\xi) = \lambda^n \cos^{n\beta}(\mu\xi),$$
$$(u^n)_\xi = -n\mu\beta\lambda^n \sin(\mu\xi)\cos^{n\beta-1}(\mu\xi),$$
$$(u^n)_{\xi\xi} = -n^2\mu^2\beta^2\lambda^n \cos^{n\beta}(\mu\xi) + n\mu^2\lambda^n\beta(n\beta-1)\cos^{n\beta-2}(\mu\xi), \qquad (7)$$

and so on for the other derivatives.

3. We substitute (6) or (7) into the reduced equation obtained above in (3), balance the terms of the sine functions when (6) is used, or balance the terms of the cosine functions when (7) is used, and solving the resulting system of algebraic equations by using the computerized symbolic calculations. We next collect all terms with same power in $\cos^k(\mu\xi)$ or $\sin^k(\mu\xi)$ and set to zero their coefficients to get a system of algebraic equations among the unknowns λ, μ and β. We obtained all possible value of the parameters λ, μ and β.

3 Solution Procedure

Let us consider the ZK-BBM equation

$$u_t + u_x - a(u^2)_x - (bu_{xt} + ku_{yt})_x = 0, \tag{8}$$

We now employ the sine–cosine method. Using the wave variable $\xi = x + y - ct$, carries (8) into ODE

$$(1-c)u' - a(u^2)' + ((b+k)cu'')' = 0, \tag{9}$$

Integrating (9) gives and by considering the constant of integration to be zero, we get

$$(1-c)u - au^2 + (b+k)cu'' = 0, \tag{10}$$

Substituting (6) into (10) gives

$$\begin{aligned}&(1-c)\lambda \sin^{\beta}(\mu\xi) - a\lambda^2 \sin^{2\beta}(\mu\xi) - (b+k)c\lambda\mu^2\beta^2 \sin^{\beta}(\mu\xi)\\ &+(b+k)c\lambda\mu^2\beta(\beta-1)\sin^{\beta-2}(\mu\xi) = 0,\end{aligned} \tag{11}$$

Equating the exponents and the coefficients of each pair of the sine functions, we find the following system of algebraic equations:

$$\begin{aligned}&\beta - 1 \neq 0,\\ &2\beta = \beta - 2,\\ &-(b+k)c\lambda\mu^2\beta^2 + (1-c)\lambda = 0,\\ &(b+k)c\lambda\mu^2\beta(\beta-1) - a\lambda^2 = 0.\end{aligned} \tag{12}$$

Solving the system (12) yields

$$\begin{aligned}&\beta = -2,\\ &\lambda = \frac{3(1-c)}{2a},\\ &\mu = \frac{1}{2}\sqrt{\frac{1-c}{c(b+k)}}.\end{aligned} \tag{13}$$

The result (13) can be easily obtained if we also use the cosine method (7). Consequently, following periodic solutions for $\frac{1-c}{c(b+k)} > 0$

$$u_1(x,y,t) = \frac{3(1-c)}{2a}\sec^2[\frac{1}{2}\sqrt{\frac{1-c}{c(b+k)}}(x+y-ct)], \qquad |\mu\xi| < \frac{\pi}{2} \tag{14}$$

$$u_2(x,y,t)=\frac{3(1-c)}{2a}\csc^2[\frac{1}{2}\sqrt{\frac{1-c}{c(b+k)}}(x+y-ct)], \qquad 0<\mu\xi<\pi \tag{15}$$

However, for $\frac{1-c}{c(b+k)}>0$ we obtain the soliton solution

$$u_3(x,y,t)=\frac{3(1-c)}{2a}\sec h^2[\frac{1}{2}\sqrt{\frac{c-1}{c(b+k)}}(x+y-ct)], \tag{16}$$

$$u_4(x,y,t)=-\frac{3(c-1)}{2a}\csc h^2[\frac{1}{2}\sqrt{\frac{c-1}{c(b+k)}}(x+y-ct)], \tag{17}$$

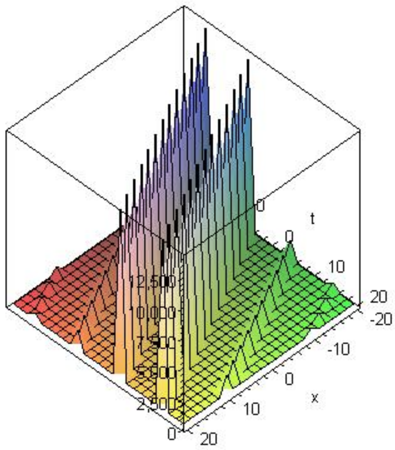

Figure 1: Periodic solution corresponding to $u_1(x,y,t)$ for $c=2, a=-1, b=-3, k=-2.$

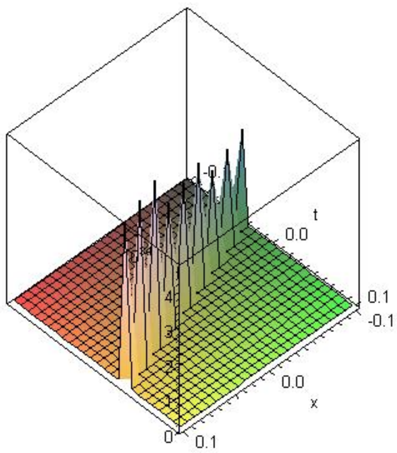

Figure 2: Periodic solution corresponding to $u_2(x,y,t)$ for $c=3, a=-2, b=-5, k=-4.$

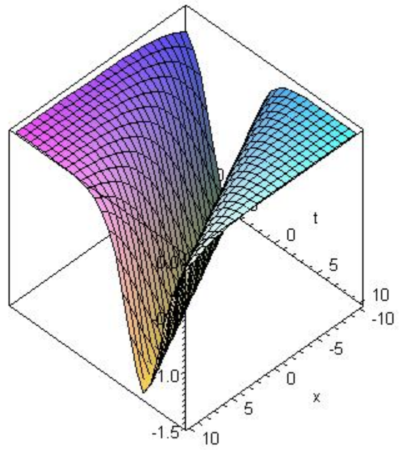

Figure 3: soliton solution corresponding to $u_3(x, y, t)$ for $c = 2, a = 1, b = 3, k = 1.$

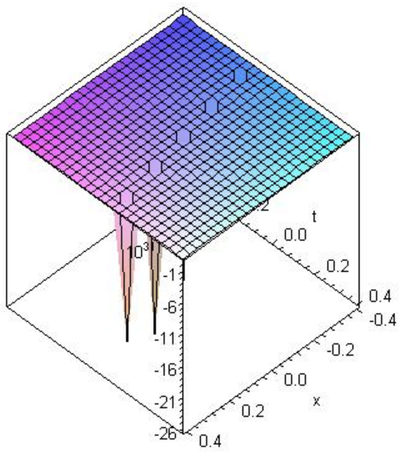

Figure 4: soliton solution corresponding to $u_4(x, y, t)$ for $c = 5, a = 7, b = 5, k = 3.$

4 Conclusion

This study shows that Sin-Cosine method is quite efficient and practically well suited for use in calculating travelling wave solutions for ZK-BBM equations. The reliability of the method and the reduction in the size of computational domain give this method a wider applicability.

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