The interval distance between fuzzy numbers

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Abstract
This metric distance satisfies on some of the properties and we used this metric for similarity measure.

Keywords: Fuzzy numbers; Interval arithmetic, Distance measure.

1 Introduction

Fuzzy systems are used to study a variety of problems. Fussy linear systems [1,2], fuzzy differential equations [3-5], fuzzy linear programming [6,7], particle physics [8,9] and other topics. The concept of fuzzy numbers and arithmetic operations with these numbers were first introduced and investigated by Chang and Zadeh [10] and others. The distance measure is a very essential tool in various fields of study that measures the distance or difference between two points. Many researchers have worked also on distance measure of fuzzy numbers, where more are concluded to be crisp numbers. In [11] tren et al. introduce the distance on the interval numbers and then extend to fuzzy numbers.

2 Background fuzzy

Definition 2.1. \( \hat{A} = (a_1, a_2, a_3, a_4)_{LR} \) is a generalized left right fuzzy number of Dubois and pride, if its membership functions satisfy the following:

\[
\mu_{\hat{A}}(x) = \begin{cases} 
L \frac{(a_2 - x)}{a_2 - a_1} & a_1 \leq x \leq a_2 \\
1 & a_2 \leq x \leq a_3 \\
R \frac{(x - a_3)}{a_4 - a_3} & a_3 \leq x \leq a_4 \\
0 & \text{otherwise}
\end{cases}
\]

Where \( L \) and \( R \) are strictly decreasing functions defined on \([0,1]\) and satisfying conditions.
We used For the ranking of two fuzzy numbers \( \tilde{A} \) and \( \tilde{B} \) we define:

\[
\tilde{A} \preceq \tilde{B} \iff \left[ \int_0^1 A(\alpha)d\alpha , \int_0^1 \bar{A}(\alpha)d\alpha \leq \int_0^1 B(\alpha)d\alpha , \int_0^1 \bar{B}(\alpha)d\alpha \right]
\]

3 Interval distance between fuzzy numbers

Let us consider two fuzzy numbers \( \tilde{A} \) and \( \tilde{B} \) with \( \tilde{A} = [A(r), \overline{A}(r)] \), \( \tilde{B} = [B(r), \overline{B}(r)] \) respectively. The distance between these two fuzzy numbers is defined by \( d(\tilde{A}, \tilde{B}) = [d, \overline{d}] \in \mathbb{I}(R) \) where

\[
d = \min \left\{ \min_{r \in [0,1]} |A(r) - B(r)|, \min_{r \in [0,1]} |\overline{A}(r) - \overline{B}(r)| \right\}
\]

and

\[
\overline{d} = \max \left\{ \max_{r \in [0,1]} |A(r) - B(r)|, \max_{r \in [0,1]} |\overline{A}(r) - \overline{B}(r)| \right\}
\]

3.1. Metric properties

(i) \( d(\tilde{A}, \tilde{B}) \geq 0 \)

(ii) \( d(\tilde{A}, \tilde{A}) = [0, 0] \)

(iii) \( d(\tilde{A}, \tilde{B}) = d(\tilde{B}, \tilde{A}) \)

(iv) \( d(\tilde{A}, \tilde{B}) = [0, 0] \iff \tilde{A} = \tilde{B} \)

(v) \( d(\tilde{A}, \tilde{B}) + d(\tilde{B}, \tilde{C}) \geq d(\tilde{A}, \tilde{C}) \)

Proposition 3.1. If \( \tilde{A}, \tilde{B}, \tilde{C} \in \mathbb{F}(R) \) and \( a, b, \tau \in R \)

1) \( d(\tau \tilde{A}, \tau \tilde{B}) = |\tau|d(\tilde{A}, \tilde{B}) \)

2) \( d(\tilde{A} + \tau, \tilde{B} + \tau) = d(\tilde{A}, \tilde{B}) \)

3) \( d(\tilde{A} + \tilde{C}, \tilde{B} + \tilde{C}) = d(\tilde{A}, \tilde{B}) \)

4) \( d([a, b], [c, d]) = [\min\{|a - c|, |b - d|\}, \max\{|a - c|, |b - d|\}] \)

5) \( d(a, b) = |a - b| \)

Proposition 3.2. If \( \tilde{A} = (a_1, a_2, a_3, a_4) \) and \( \tilde{B} = (b_1, b_2, b_3, b_4) \) are two trapezoidal fuzzy numbers, than \( d(\tilde{A}, \tilde{B}) = [d, \overline{d}] \) so that if

\[
0 < \frac{b_1 - a_1}{a_2 - a_1 - b_2 + b_1} < 1 \quad \text{and} \quad 0 < \frac{a_4 - b_4}{a_4 - a_3 - b_4 + b_3} < 1 \quad \text{then} \quad d = 0 \quad \text{otherwise}
\]

\[
d = \min\{|a_1 - b_1|, |a_2 - b_2|, |a_3 - b_3|, |a_4 - b_4|\}
\]

And

\[
\overline{d} = \max\{|a_1 - b_1|, |a_2 - b_2|, |a_3 - b_3|, |a_4 - b_4|\}
\]
Proposition 3.3. If $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ are two triangular fuzzy numbers then 
\[ d(\tilde{A}, \tilde{B}) = [\frac{a_1 - b_1}{a_2 - a_1 - b_2 + b_1}, \frac{a_3 - b_3}{a_3 - a_2 - b_3 + b_2}] \] 
so that if
\[ 0 < \frac{b_1 - a_1}{a_2 - a_1 - b_2 + b_1} < 1 \quad \text{and} \quad 0 < \frac{a_3 - b_3}{a_3 - a_2 - b_3 + b_2} < 1 \] 
then $d = 0$ otherwise.

Than
\[ d = \min\{|a_1 - b_1|, |a_2 - b_2|, |a_3 - b_3|\} \]
And
\[ d = \max\{|a_1 - b_1|, |a_2 - b_2|, |a_3 - b_3|\} \]

Proposition 3.4. Suppose $d(\tilde{A}, \tilde{B}) = [d, d]$ if $d = 0$ and $d = 0$

4 Similarity measure between fuzzy numbers

The similarity measure is important for presenting degree of similarity between two objects or concepts. Some research introduce the similarity measure between two fuzzy numbers. Chen[10] and Lee[11], proposed a similarity measure for two normal trapezoidal fuzzy numbers $\tilde{A}_1 = (\alpha_1, \beta_1, \gamma_1)$ and $\tilde{A}_2 = (\alpha_3, \beta_2, \gamma_2)$ as follows:

\[ S(\tilde{A}_1, \tilde{A}_2)_{Chen} = 1 - \frac{|(\alpha_1 - \beta_1) - (\alpha_3 - \beta_2)| + |\alpha_1 - \alpha_3| + |\alpha_2 - \alpha_4| + |(\alpha_2 + \gamma_1) - (\alpha_4 + \gamma_2)|}{4} \]

\[ S(\tilde{A}_1, \tilde{A}_2)_{Set} = 1 - \frac{[(\alpha_1 - \beta_2) - (\alpha_3 - \beta_2)]^p + |\alpha_1 - \alpha_3|^p + |\alpha_2 - \alpha_4|^p + |(\alpha_2 + \gamma_1) - (\alpha_4 + \gamma_2)|^p}{\max(U) - \min(U)} \]

Where $U$ is the Universe of discourse.

Here we present a new similarity measure based interval distance measure. The relation which has been used here is real interval.

\[ S(\tilde{A}, \tilde{B}) = \left[ \frac{1}{1 + d}, 1 - \frac{d}{1 + d} \right] \]

Table 1: The sets of fuzzy numbers

<table>
<thead>
<tr>
<th>Set</th>
<th>$\tilde{A}$</th>
<th>$\tilde{B}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set1</td>
<td>(0.1, 0.2, 0.3, 0.4)</td>
<td>(0.1, 0.25, 0.4)</td>
</tr>
<tr>
<td>Set2</td>
<td>(0.1, 0.2, 0.3, 0.4)</td>
<td>(0.4, 0.55, 0.7)</td>
</tr>
<tr>
<td>Set3</td>
<td>(0.3, 0.3, 0.3)</td>
<td>(0.3, 0.3, 0.3)</td>
</tr>
</tbody>
</table>

Table 2: Comparison of the proposed similarity measure whit the other similarity measure.

<table>
<thead>
<tr>
<th>Set</th>
<th>Lee</th>
<th>Chen</th>
<th>New</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set1</td>
<td>0.9167</td>
<td>0.975</td>
<td>[0.952, 1]</td>
</tr>
<tr>
<td>Set2</td>
<td>0.5</td>
<td>0.7</td>
<td>[0.741, 0.815]</td>
</tr>
<tr>
<td>Set3</td>
<td>#</td>
<td>1</td>
<td>[1, 1]</td>
</tr>
</tbody>
</table>

5 Conclusions

Many paper have proposed the fuzzy distance measure. In this paper we developed a distance measure between fuzzy numbers. We used this metric to define a similarity measure on fuzzy number as an interval number.
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