A fuzzy logic controller for flight control Through a Weighted Function

A. Bidar Yamchi*

Department of Mathematics, Urmia Branch, Islamic Azad University, Urmia, Iran.

Abstract
Fuzzy systems have gained more and more attention from researchers and practitioners of various fields. In such systems, the output represented by a fuzzy set sometimes needs to be transformed into a scalar value, and this task is known as the defuzzification process. In this study, we suggest a new approach to the problem of defuzzification using the parametric metric between two fuzzy numbers. Some preliminary results on properties of such defuzzification are to be reported. Therefore, by the means of this defuzzification, this article aims to use the concept of parametric symmetric triangular fuzzy number, and introduces a new approach to defuzzify a fuzzy quantity. The basic idea of the new method is to obtain the nearest parametric symmetric triangular fuzzy number which a fuzzy quantity is related to.

Key words: Fuzzy number; Defuzzification; Parametric distance; Regular function.

1 Introduction

For the last few years, many researchers have turned their attention to fuzzy systems when the encountered problems become more and more complex which traditional methods find difficult to solve. It is believed that fuzzy systems theory is more proper to solve complex systems, especially humanistic systems. Fuzzy systems theory was developed based on fuzzy logic and some other related disciplines in such a way that the relationships among system variables are expressed via fuzzy logic. Many successful applications have been reported using fuzzy systems theory to various problems from industrial production process control, refuse incineration plant control, mobile robot control to university enrollments forecast. One of the important steps in applying fuzzy system theory is to transfer the output, in the form of fuzzy sets, into a scalar and this is called defuzzification. Several method have been studied for the defuzzification and the properties of this process have been investigated. The centroid method, for example, is one of the most commonly used. A different method has been used in forecasting university enrollments, where the defuzzification task is performed with a procedure of 3 rules.

* Corresponding Author. Email address: azizeh.bidar@gmail.com
Moreover, in (M. Ming. e.t. 2000) [1], the researchers used the concept of the symmetric triangular fuzzy number, and introduced an approach to defuzzify a fuzzy number based $L_2$-distance. In this paper, the researchers suggest a new approach to the problem of defuzzification using the weighted metric between two fuzzy numbers. In this study some preliminary results on properties of such defuzzification are to be reported. Therefore, by the means of this difuzzification, this article aims to use the concept of symmetric triangular fuzzy number, and introduces a new approach to defuzzify a fuzzy quantity such that fuzziness value of this method is less than fuzziness value obtained by (M. Ming. e.t. 2000) [1]. The basic idea of the new method is to obtain the nearest symmetric triangular fuzzy number which a fuzzy quantity is related to. Unlike other methods, this study defuzzifies the fuzzy number, and at the same time obtains the fuzziness of the original quantity.

The paper is organized as follows: In Section 2, this article recalls some fundamental results on fuzzy numbers. In Section 3, the new defuzzification method is proposed. In this Section some theorems and remarks are proposed and illustrated. Examples and applications of this study are carried out in section 4. The paper ends with conclusions in section 5.

2 Preliminaries

The basic definition of a fuzzy number given in (Grzegorzewski, 2009; Dubois, e.t., 1987; Heilpern, 1992; Kauffman, e.t. 1991; Saneifard, 2009), [2, 4, 5, 6, 8] as follows:

**Definition 1.**

A fuzzy number $A$ is a mapping $A(x): \mathbb{R} \to [0,1]$ with the following properties:

1. $A$ is an upper semi-continuous function on $\mathbb{R}$,
2. $A(x) = 0$ outside of some interval $[a_1, b_2] \subseteq \mathbb{R}$,
3. There are real numbers $a_2, b_1$ such as $a_1 \leq a_2 \leq b_1 \leq b_2$ and

3.1.1 $A(x)$ is a monotonic increasing function on $[a_1, a_2]$,
3.1.2 $A(x)$ is a monotonic decreasing function on $[b_1, b_2]$,
3.1.3 $A(x) = 1$ for all $x$ in $[a_2, b_1]$.

Let $\mathbb{R}$ be the set of all real numbers. The researchers assume a fuzzy number $A$ that can be expressed for all $x \in \mathbb{R}$ in the form

$$A(x) = \begin{cases} g(x) & \text{when } x \in [a, b), \\ 1 & \text{when } x \in [b, c], \\ h(x) & \text{when } x \in (c, d], \\ 0 & \text{otherwise.} \end{cases}$$

(1)

Where $a, b, c$ and $d$ are real numbers such as $a < b \leq c < d$ and $g$ is a real valued function that is increasing and right continuous and $h$ is a real valued function that is decreasing and left continuous.

**Definition 2.**

A fuzzy number $A$ in parametric form is a pair $(\underline{A}, \overline{A})$ of functions $\underline{A}(r)$ and $\overline{A}(r)$ that $0 \leq r \leq 1$, which satisfy the following requirements:

1. $A(r)$ is a bounded monotonic increasing left continuous function,
2. $A(r)$ is a bounded monotonic decreasing left continuous function,
3. $\underline{A}(r) \leq \overline{A}(r), 0 \leq r \leq 1$. 
Definition 3.
The trapezoidal fuzzy number $A = (x_0, y_0, \sigma, \beta)$, with two defuzzifier $x_0, y_0$, and left fuzziness $\sigma > 0$ and right fuzziness $\beta > 0$ is a fuzzy set where the membership function is as

$$A(x) = \begin{cases} 
\frac{1}{\sigma} (x - x_0 + \sigma) & \text{if } x_0 - \sigma \leq x \leq x_0, \\
1 & \text{if } x_0 \leq x \leq y_0, \\
\frac{1}{\beta} (y_0 - x + \beta) & \text{if } y_0 \leq x \leq y_0 + \beta, \\
0 & \text{otherwise}.
\end{cases}$$

If $x_0 = y_0$ and $\sigma = \beta$, a popular fuzzy number is obtain. It is the symmetric triangular fuzzy number $S[x_0, \sigma]$ centered at $x_0$ with basis $2\sigma$ by following form

$$A(x) = \begin{cases} 
\frac{1}{\sigma} (x - x_0 + \sigma) & \text{if } x_0 - \sigma \leq x \leq x_0, \\
1 & \text{if } x = x_0, \\
\frac{1}{\sigma} (x_0 - x + \sigma) & \text{if } x_0 \leq x \leq x_0 + \sigma, \\
0 & \text{otherwise}.
\end{cases}$$

The parametric form of symmetric triangular fuzzy number is

$$\underline{A}(r) = x_0 - \sigma (1 - r), \quad \overline{A}(r) = x_0 + \sigma (r - 1).$$

Definition 4.
A function $f: [0,1] \rightarrow [0,1]$ symmetric around $\frac{1}{2}$, i.e. $f\left(\frac{1}{2} - r\right) = f\left(\frac{1}{2} + r\right)$ for all $r \in [0,\frac{1}{2}]$, which reaches its minimum in $\frac{1}{2}$, is called the bi-symmetrical weighted function. Moreover, the bi-symmetrical weighted function is called regular if

1. $f\left(\frac{1}{2}\right) = 0,$
2. $f(0) = f(1) = 1,$
3. $\int_0^1 f(r) dr = \frac{1}{2}.$

Definition 5.
For fuzzy set $A$ Support function is defined as follows:

$$\text{supp}(A) = \{x | A(x) > 0\}.$$

Where $\overline{\{x | A(x) > 0\}}$ is closure of set $\{x | A(x) > 0\}$. The addition and scalar multiplication of fuzzy numbers are defined by the extension principle and can be equivalently represented as follows. For arbitrary fuzzy numbers $A = (\underline{A}, \overline{A})$ and $B = (\underline{B}, \overline{B})$, this article defines addition $(A + B)$ and multiplication by scalar $k > 0$ as

$$A + B = (\underline{A} + \underline{B}, \overline{A} + \overline{B}), \quad kA = (k\underline{A}, k\overline{A}).$$

To emphasis the collection of all fuzzy numbers with addition and multiplication as defined by (2) and (3) is denoted by $F$, which is a convex cone.
For arbitrary fuzzy numbers $A = (\underline{A}, \overline{A})$ and $B = (\underline{B}, \overline{B})$ the quantity
\[
D(A, B) = \int_0^1 (\underline{A}(r) - \underline{B}(r))^2 \, dr + \int_0^1 (\overline{A}(r) - \overline{B}(r))^2 \, dr,
\]
(4)

is the distance between $A$ and $B$.

Having reviewed the previous methods, another rule for defuzzification introduced in (Ming, M. e.t. 2000) [1]. The authors proposed the nearest symmetric triangular defuzzification approach associated with the metric $D$ in $F$ as follows:

Let $A$ be a fuzzy number and $A = (A(r), \overline{A}(r))$ be its parametric form. To obtain a symmetric triangular fuzzy number $S[x_0, \sigma]$, which was the nearest to $A$, the researchers minimized
\[
D(A, S[x_0, \sigma]) = \int_0^1 (A(r) - S[x_0, \sigma](r))^2 \, dr + \int_0^1 (\overline{A}(r) - \overline{S}[x_0, \sigma](r))^2 \, dr,
\]
with respect to $x_0$ and $\sigma$. If $S[x_0, \sigma]$ minimizes $D$, it provides a defuzzification of $A$ with a defuzzifier $x_0$ and fuzziness $\sigma$. So that to minimize $D$, they solved system of equations
\[
\frac{\partial D(A, S[x_0, \sigma])}{\partial \sigma} = 0, \quad \frac{\partial D(A, S[x_0, \sigma])}{\partial x_0} = 0.
\]

The solution was
\[
\sigma = \frac{3}{2} \int_0^1 [\overline{A}(r) - A(r)](1 - r) \, dr, \quad x_0 = \frac{1}{2} \int_0^1 [A(r) + \overline{A}(r)] \, dr.
\]
(5) (6)

For more details see (M. Ming. e.t. 2000) [1].

Definition 7.
Let $A$ be a fuzzy number and $(A(r), \overline{A}(r))$ be its parametric form. The following values constitute the weighted averaged representative and weighted width, respectively, of the fuzzy number $A$:
\[
I(A) = \frac{1}{2} \int_0^1 [A(r) + \overline{A}(r)] \, dr, \quad (7)
\]
and
\[
D(A) = \int_0^1 [\overline{A}(r) - A(r)] f(r) \, dr.
\]
(8)

were $f: [0, 1] \rightarrow [0, 1]$ is a bi-symmetrical (regular) weighted function.

One can, of course, propose many regular bi-symmetrical weighted functions and hence obtain different bi-symmetrical weighted distances. Further on we will consider mainly a following function
\[
f(r) = \begin{cases} 
1 - 2r & \text{when } r \in \left[0, \frac{1}{2}\right], \\
2r - 1 & \text{when } r \in \left[\frac{1}{2}, 1\right].
\end{cases}
\]
(9)

Definition 8.
For arbitrary fuzzy numbers $A$ and $B$ the quantity
\[
d_p(A, B) = \sqrt{[I(A) - I(B)]^2 + [D(A) - D(B)]^2},
\]
(10)
is called the bi-symmetrical (regular) weighted distance between $A$ and $B$ based on $F$. 
3 Nearest Parametric Symmetric triangular Defuzzification

In this Section, the researchers will propose the nearest weighted symmetric triangular defuzzification approach associated with the weighted metric \( d_p \) in \( F \).

Let \( A \) be a general fuzzy number and \( (\overline{A}(r), \underline{A}(r)) \) be its parametric form. To obtain a parametric symmetric triangular fuzzy number \( S[x_0, \sigma] \), which is the nearest to \( A \), the researchers use the weighted distance (10) and minimize

\[
d_p(A, S[x_0, \sigma]) = \left( \left[ I(A) - I(S[x_0, \sigma]) \right]^2 + \left[ D(A) - D(S[x_0, \sigma]) \right]^2 \right)^{\frac{1}{2}},
\]

with respect to \( x_0 \) and \( \sigma \), where

\[
I(S[x_0, \sigma]) = \frac{1}{2} \int_0^1 \left[ x_0 - \sigma(1-r) + x_0 + \sigma(1-r) \right] dr = x_0,
\]

\[
D(S[x_0, \sigma]) = \int_0^1 \left[ x_0 + \sigma(1-r) - x_0 + \sigma(1-r) \right] f(r) dr
\]

\[
= \int_0^1 2\sigma(1-r)f(r) dr.
\]

In order to minimize it suffices to minimize

\[
\overline{D}_p(A, S[x_0, \sigma]) = d_p^2(A, S[x_0, \sigma])
\]

\[
= \left[ \frac{1}{2} \int_0^1 (\overline{A}(r) + A(r) - 2x_0) dr \right]^2 + \left[ \int_0^1 (\overline{A}(r) - A(r) - 2\sigma(1-r)) f(r) dr \right]^2.
\]

If \( S[x_0, \sigma] \) minimizes \( \overline{D}_p(A, S[x_0, \sigma]) \), then \( S[x_0, \sigma] \) provides a defuzzification of \( A \) with a defuzzifier \( x_0 \) and fuzziness \( \sigma \).

So that to minimize \( \overline{D}_p(A, S[x_0, \sigma]) \), this article has,

\[
\frac{\partial \overline{D}_p(A, S[x_0, \sigma])}{\partial \sigma} = -4 \int_0^1 \left( \overline{A}(r) - A(r) \right) (1-r)f(r) dr + 8\sigma \int_0^1 (1-r)^2 f(r) dr.
\]

and

\[
\frac{\partial \overline{D}_p(A, S[x_0, \sigma])}{\partial x_0} = -2 \int_0^1 (\overline{A}(r) + A(r) - 2x_0) f(r) dr.
\]

By solve system of equation as follows,

\[
\frac{\partial \overline{D}_p(A, S[x_0, \sigma])}{\partial \sigma} = 0, \quad \frac{\partial \overline{D}_p(A, S[x_0, \sigma])}{\partial x_0} = 0.
\]

The solution is

\[
\sigma = \frac{1}{2} \int_0^1 [\overline{A}(r) - A(r)][1-r] f(r) dr,
\]

\[
x_0 = \frac{1}{2} \int_0^1 \left( \overline{A}(r) + A(r) \right) dr.
\]

**Remark 1.**

If we consider \( f(r) = r \), the nearest parametric symmetric triangular defuzzification of \( A \) is given by the defuzzifier

\[
x_0 = \frac{1}{2} \int_0^1 \left( \overline{A}(r) + A(r) \right) dr.
\]
and fuzziness
\[ \sigma_p = 6 \int_0^1 [\overline{A}(r) - \overline{A}(r)](1-r)rdr. \]

The above defuzzification approach can be applied to two fuzzy numbers whenever a single fuzzy quantity is desirable. Let \( A \) and \( B \) be a fuzzy numbers with parametric forms \( A = (\overline{A}(r), \overline{A}(r)) \) and \( B = (\overline{B}(r), \overline{B}(r)) \). To find a parametric symmetric triangular fuzzy number \( S[x_{op}, \sigma_p] \) near both \( A \) and \( B \), this article minimizes
\[
\overline{D}(x_{op}, \sigma_p) = \overline{D}_p(A, S[x_{op}, \sigma_p]) + \overline{D}_p(B, S[x_{op}, \sigma_p])
\]
\[
= \left[ \frac{1}{2} \int_0^1 [(\overline{A}(r) - S[x_{op}, \sigma_p](r)) + (\overline{A}(r) - S[x_{op}, \sigma_p](r))dr \right]^2
\]
\[
+ \left[ \int_0^1 [(\overline{A}(r) - S[x_{op}, \sigma_p](r)) - (\overline{A}(r) - S[x_{op}, \sigma_p](r))f(r)dr \right]^2
\]
\[
+ \left[ \frac{1}{2} \int_0^1 [(\overline{B}(r) - S[x_{op}, \sigma_p](r)) + (\overline{B}(r) - S[x_{op}, \sigma_p](r))dr \right]^2
\]
\[
+ \left[ \int_0^1 [(\overline{B}(r) - S[x_{op}, \sigma_p](r)) - (\overline{B}(r) - S[x_{op}, \sigma_p](r))f(r)dr \right]^2.
\]

Thus, this study must to find a lodger point, \( (x_{op}, \sigma_p) \) for which
\[
\frac{\partial \overline{D}(x_{op, \sigma_p})}{\partial \sigma_p} = 0 , \quad \frac{\partial \overline{D}(x_{op, \sigma_p})}{\partial x_{op}} = 0. \quad (*)
\]

Then
\[
\frac{\partial \overline{D}(x_{op, \sigma_p})}{\partial \sigma_p} = \frac{\partial \overline{D}_p(A, S[x_{op}, \sigma_p])}{\partial \sigma_p} + \frac{\partial \overline{D}_p(B, S[x_{op}, \sigma_p])}{\partial \sigma_p}
\]
\[
= -4 \int_0^1 (\overline{A}(r) - \overline{A}(r))(1-r)f(r)dr + 8 \sigma_p \int_0^1 (1-r)^2f(r)dr
\]
\[
-4 \int_0^1 (\overline{B}(r) - \overline{B}(r))(1-r)f(r)dr + 8 \sigma_p \int_0^1 (1-r)^2f(r)dr = 0,
\]
and
\[
\frac{\partial \overline{D}(x_{op, \sigma_p})}{\partial x_{op}} = \frac{\partial \overline{D}_p(A, S[x_{op}, \sigma_p])}{\partial x_{op}} + \frac{\partial \overline{D}_p(B, S[x_{op}, \sigma_p])}{\partial x_{op}}
\]
\[
= -2 \int_0^1 (\overline{A}(r) + \overline{A}(r) - 2x_{op})dr = -2 \int_0^1 (\overline{B}(r) + \overline{B}(r) - 2x_{op})dr = 0.
\]

Hence, by the solve system of equation (\( * \)), there is
\[
\sigma_p = \int_0^1 [\overline{A}(r) + \overline{A}(r) - \overline{A}(r) - \overline{A}(r) - \overline{B}(r) + \overline{B}(r) + \overline{B}(r)]dr,
\]
and
\[
x_{op} = \frac{1}{4} \int_0^1 [\overline{A}(r) + \overline{A}(r) + \overline{A}(r) + \overline{A}(r)]dr.
\]

If this article assumes \( f(r) = r \), then
\[
\sigma_p = 3 \int_0^1 [\overline{A}(r) + \overline{B}(r) - \overline{A}(r) - \overline{B}(r)](1-r)dr,
\]
and
4 Numerical examples and Application

In this section the researchers present numerical examples to illustrate the difference between the introduced method in this paper and the given method in (Ming, M. et. al. 2000) [1].

Example 1.
Consider a plateau
\[ A(r) = a + (b-a)r, \quad \bar{A}(r) = d - (d-c)r, \]
where \( a \leq b \leq c \leq d \). The nearest parametric symmetric triangular defuzzification procedure yields
\[ x_{op} = \frac{1}{4} \int_{0}^{1} [A(r) + \bar{A}(r)] dr = \frac{1}{4} \int_{0}^{1} [(a + d) + (b-a)r - (d-c)r] dr \]
\[ = \frac{a+b+c+d}{4}, \]
and
\[ \sigma_p = \frac{\int_{0}^{1} [\bar{A}(r) - A(r)](1-r)f(r) dr}{2 \int_{0}^{1} (1-r)^2 f(r) dr} \]
\[ = \frac{1}{24} \int_{0}^{1} [d - (d-c)r - a - (b-a)r](1-r) dr = \frac{c+d-a+b}{2}. \]

In specific case \( b = c \) there is
\[ x_{op} = \frac{1}{4} (a + 2b + d), \quad \sigma_p = \frac{1}{2} (d - a). \]

Example 2.
Consider the Gaussian membership function \( A(x) = e^{-\frac{(x-\mu_0)^2}{\sigma_0^2}} \) which its parametric form is
\[ A(r) = \mu_0 + \sigma_0 \sqrt{-lnr}, \quad \bar{A}(r) = \mu_0 - \sigma_0 \sqrt{-lnr}, \]
then
\[ x_{op} = \frac{1}{2} \int_{0}^{1} [A(r) + \bar{A}(r)] dr = \mu_0, \]
and
\[ \sigma_p = \frac{\int_{0}^{1} [\bar{A}(r) - A(r)](1-r)f(r) dr}{2 \int_{0}^{1} (1-r)^2 f(r) dr} = \frac{\sigma_0 \sqrt{2}}{6} (9\sqrt{2} - 4\sqrt{3}). \]

Furthermore, \( \sigma = \frac{3}{8} \sigma_0 \sqrt{2} (4 - \sqrt{2}) [5] \), it is clear that \( \sigma_p \leq \sigma \).

Example 3.
Let \( A \) be a plateau and \( B \) is a triangular fuzzy number given by
\[ A(r) = r, \quad \bar{A}(r) = 3 - r, \]
\[ B(r) = 2 + r, \quad \bar{B}(r) = 4 - r. \]
The defuzzification procedure yields
\[ x_0 = \frac{1}{4} \int_0^1 \left[ A(r) + B(r) + \bar{A}(r) + \bar{B}(r) \right] dr = \frac{1}{4} \int_0^1 9r dr = \frac{9}{4}, \]
and
\[ \sigma_p = \frac{\int_0^1 \left[ \bar{A}(r) + B(r) - A(r) - B(r) \right] (1 - r) f(r) dr}{4 \int_0^1 (1 - r)^2 f(r) dr} \]
\[ = \frac{1}{4} \int_0^1 (5 - 4r)(1 - r)r dr = \frac{3}{2}. \]

**Example 4.**

In other segment, this article applies the parametric symmetric triangular defuzzification procedure to obtains a fuzzy partition from two extreme values. So, given the extreme values 0 and 1, the researchers define the fuzzy number “medium” \( A^{(1)} \) as
\[ A^{(1)}(r) = \frac{r}{2}, \quad \bar{A}^{(1)}(r) = 1 - \frac{r}{2}. \]

Defuzzify 0 and \( A^{(1)} \), to obtain “lower medium” \( A^{(2,1)} \) for which
\[ x_0 = \frac{1}{4} \int_0^1 \left[ A^{(1)}(r) + 0 + \bar{A}^{(1)}(r) + 0 \right] dr = \frac{1}{4}, \]
\[ \sigma_p = 3 \int_0^1 \left[ \bar{A}^{(1)}(r) + 0 - A^{(1)}(r) + 0 \right] (1 - r) r dr = \frac{1}{4}, \]
Thus \( A^{(2,1)}(r) = 0 + \frac{1}{4} r \) and \( \bar{A}^{(2,1)}(r) = \frac{1}{2} - \frac{1}{4} r \).

This study now defuzzifies \( A^{(1)} \) and 1, to get the “upper medium” \( A^{(2,3)} \) with \( x_0 = \frac{3}{4} \) and \( \sigma_p = \frac{1}{4} \), i.e.
\[ A^{(2,3)}(r) = \frac{1}{2} + \frac{1}{4} r, \quad \bar{A}^{(2,3)}(r) = 1 - \frac{1}{4} r. \]

Update the “medium” by defuzzifying the “lower medium” \( A^{(2,1)} \) and “upper medium” \( A^{(2,3)} \). The result is the “medium” \( A^{(2,2)} \) centered at
\[ x_0 = \frac{1}{4} \int_0^1 \left( \frac{r}{4} + \frac{1}{2} - \frac{r}{4} + \frac{1}{2} + \frac{r}{4} + 1 - \frac{r}{4} \right) dr = \frac{1}{2}, \]
with
\[ \sigma_p = 3 \int_0^1 (1 - r)^2 r dr = \frac{1}{4}, \]
i.e.
\[ A^{(2,2)}(r) = 0.25 + 0.25r, \quad \bar{A}^{(2,2)}(r) = 0.75 - 0.25r. \]
Thus, this article obtains a fuzzy partition with five elements $P = \{0, A^{(2,1)}, A^{(2,2)}, A^{(2,3)}, 1\}$ (Fig. 3).

As the fuzzy partition becomes finer, the fuzziness of its elements obviously decreases.

**Example 5.**

Consider the Gaussian membership function in Example (2) with $\mu_0 = 1$ and $\sigma_0 = \frac{1}{2}$. We apply the parametric symmetric triangular defuzzification procedure to obtain a fuzzy partition from two extreme values 0 and 2. Then

\[
\hat{A}^{(1)}(r) = 1 + \frac{\sqrt{-L_{\text{op}}}}{2}, \quad \tilde{A}^{(1)}(r) = 1 - \frac{\sqrt{-L_{\text{op}}}}{2}.
\]

Defuzzify 0 and $A^{(1)}$, to obtain $A^{(2,1)}$ for which

\[
x_{\text{op}} = \frac{1}{4} \int_0^1 \left[ A^{(1)}(r) + (A^{(1)} - 0)ight] \, dr = \frac{1}{2},
\]

\[
\sigma_p = 3 \int_0^1 \left[ \tilde{A}^{(1)}(r) + 0- \hat{A}^{(1)}(r) + 0 \right] (1-r) \, dr = \frac{1}{2}.
\]

Thus $\hat{A}^{(2,1)}(r) = 0 + \frac{r}{2}$ and $\tilde{A}^{(2,1)}(r) = 1 - \frac{r}{2}$.

We now defuzzifies $A^{(1)}$ and 2, to get the $A^{(2,3)}$ with $x_{\text{op}} = \frac{3}{2}$ and $\sigma_p = \frac{1}{2}$, i.e.

\[
\hat{A}^{(2,3)}(r) = 1 + \frac{r}{2}, \quad \tilde{A}^{(2,3)}(r) = 2 - \frac{r}{2}.
\]

Update the “medium” by defuzzifying the “lower medium” $A^{(2,1)}$ and “upper medium” $A^{(2,3)}$. The result is the “medium” $A^{(2,2)}$ centered at

\[
x_{\text{op}} = 1, \quad \text{with } x_{\text{op}} = 1; \text{ with } \sigma_p = \frac{1}{2},
\]

i.e.

\[
\hat{A}^{(2,2)}(r) = \frac{1}{2} + \frac{r}{2}, \quad \tilde{A}^{(2,2)}(r) = \frac{3}{2} - \frac{r}{2}.
\]

Thus, this article obtains a fuzzy partition with five elements $P = \{0, A^{(2,1)}, A^{(2,2)}, A^{(2,3)}, 1\}$ (Fig. 4).

As the fuzzy partition becomes finer, the fuzziness of its elements obviously decreases.
5 Conclusion

Fuzzy systems have gained more and more attention from researchers and practitioners of various fields. In such systems, the output represented by a fuzzy set sometimes needs to be transformed into a scalar value, and this task is known as the defuzzification process. Several analytic methods have been proposed for this problem, but in this paper, the researchers suggest a new approach to the problem of defuzzification using the weighted metric between two fuzzy numbers. In this study some preliminary results on properties of such defuzzification will be reported.

References
http://dx.doi.org/10.1016/S0165-0114(98)00176-6


http://dx.doi.org/10.1016/0165-0114(87)90028-5

http://dx.doi.org/10.1016/0165-0114(92)90062-9


http://dx.doi.org/10.1016/S0165-0114(00)00114-7


http://dx.doi.org/10.1016/0165-0114(90)90197-E