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Abstract
In this paper, an Economic Order quantity (E.O.Q) model with unit production cost, time depended holding cost, with-out shortages is formulated and solved. In most real world situation, the objective and constraint function of the decision makers are uncertainty in nature so the coefficients, indices the objective function and constraint goals are imposed here in fuzzy environment. The problem is then solved using both Fuzzy Max-Min Geometric-Programming technique and Fuzzy parametric Geometric-Programming. Sensitivity analysis is also presented here.

Keywords: E.O.Q model, Fuzzy set, Max-Min operator, Geometric Programming, Parametric Geometric Programming Technique.

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1 Introduction

An inventory deals with decision that minimize the cost function or maximize the profit function. For this purpose the task is to construct a suitable mathematical model of the real life Inventory system, such a mathematical model is based on various assumption and approximation.
In ordinary inventory model it considers all parameter like set-up cost, carrying cost, interest cost, shortages etc as a fixed. But in real life situation it will have some little fluctuations. So consideration of fuzzy variables is more realistic.


In this paper we consider crisp inventory model, there after it transformed to fuzzy inventory mode and developed by geometric programming approach. First we solved the model by Fuzzy Max-Min Geometric -Programming technique and then it solved by Fuzzy parametric Geometric -Programming technique. At last it made an example and solved it by both technique.

2 Mathematical Model

An Inventory model is developed under the following notations and assumptions:

2.1. Notations

I(t):Inventory level at any time, t≥0.

D: Demand per unit time, which is constant.

T: Cycle of length of the given inventory.

S: Set-up cost per unit time.

H: Holding cost per item per unit time, which is time depended.

P: Unit demand and set-up cost dependent production cost.

q: Production quantity per batch.

f(D,S): Unit production cost per cycle.

TAC(D,S,q):Total average cost per unit time.

w0: Space area per unit quantity.

W: Total storage space area of the inventory.

2.2. Assumptions

a) The inventory system involves only one item.

b) The replenishment occurs instantaneously at infinite rate.

c) The lead time is negligible.
d) Demand rate is constant.
e) The unit production cost is a continuous function of demand and Set-up cost and take the following form:
\[ p = \theta D^{-x} S^{-1}, \quad \theta, x \in \mathbb{R} (>0). \]
f) Holding cost time depended as an at.

### 2.3. Crisp model

![E.O.Q Model](image)

The differential equation describing \( I(t) \) as follows

\[
\frac{dI(t)}{dt} = -D, \quad 0 \leq t \leq T
\]  

(2.1)

With the boundary condition \( I(0) = q \) and \( I(T) = 0 \).

The solution of (2.1) is obtained as

\[ I(t) = q - Dt \]  

(2.2)

Also there are

\[ T = \frac{q}{D}. \]

Now holding cost = \( H \int_0^T at. I(t) dt = \frac{aHq^3}{6D^2} \)  

(2.3)

Total inventory related cost per cycle = set-up cost + holding cost + production cost

\[ = S + \frac{aHq^2}{6D^2} + pq \]  

(2.4)

i.e., total average cost per cycle is given by

\[ TAC(D, S, q) = \frac{SD}{q} + \frac{aHq^2}{6D^2} + \theta D^{-1-x} S^{-1} \]  

(2.5)

And storage area = \( w_0q \).

So the inventory model can be written as,

\[ \text{Min} \quad TAC(D, S, q) = \frac{SD}{q} + \frac{aHq^2}{6D^2} + \theta D^{-1-x} S^{-1} \]  

subject to \( \omega_0q \leq W \), \( D, S, q > 0 \).  

(2.6)

### 2.4. Fuzzy model

When the objective constraint goals and coefficients become fuzzy sets and fuzzy numbers, then the crisp model (2.6) written to be a fuzzy model, as

\[ \bar{\text{Min}} \quad TAC(D, S, q) = \frac{SD}{q} + \frac{\bar{a}_Hq^2}{\bar{6}D^2} + \bar{\theta} D^{-1-x} S^{-1} \]  

subject to \( \omega_0q \leq W \), \( D, S, q > 0 \).  

(2.7)
3 Some basic concept & definition

3.1. Pre-requisite mathematics

Fuzzy sets first introduced by Zadeh [18] in 1965 as a mathematical way of representing vagueness in every life.

**Definition 3.1.** A fuzzy set $\tilde{A}$ on the given universal set $X$ is a set of order pairs, $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)): x \in X\}$ where $\mu_{\tilde{A}}(x) \rightarrow [0,1]$ is called a membership function.

**Definition 3.2.** The $\alpha$-cut of $\tilde{A}$, is defined by $A_{\alpha} = \{x: \mu_{\tilde{A}}(x) \geq \alpha\}$

![Diagram of fuzzy number with α-cuts](image)

Figure 2: Trapezoidal fuzzy number of $\tilde{A}$ with $\alpha$-cuts.

$A_{\alpha}$ is a non-empty bounded closed interval in $X$ and it can be denoted by $A_{\alpha} = [A_L(\alpha), A_R(\alpha)]$. Where $A_L(\alpha)$ and $A_R(\alpha)$ are the lower and upper bounds of the closed interval respectively. Figure 2 shows a fuzzy number $\tilde{A}$ with $\alpha$-cuts $A_{\alpha_1} = [A_L(\alpha_1), A_R(\alpha_1)]$, $A_{\alpha_2} = [A_L(\alpha_2), A_R(\alpha_2)]$. It is seen that if $\alpha_2 \geq \alpha_1$ then $A_L(\alpha_2) \geq A_L(\alpha_1)$ and $A_R(\alpha_2) \geq A_R(\alpha_1)$.

**Definition 3.3.** $\tilde{A}$ is normal if there exists $x \in X$ such that $\mu_{\tilde{A}}(x) = 1$.

3.2. Mathematical analysis

Consider a non-linear programming (NLP) as follows,

$$\begin{align*}
\text{Min} & \quad g_0(x) \\
\text{Subject to} & \quad g_i(x) \leq 1 \quad (1 \leq i \leq n), \\
& \quad x > 0.
\end{align*}$$

(3.8)

Its objective and constraints of the form

$$g_i(x) = \sum_{k=1}^{T_i} C_{ik} \prod_{j=1}^{m} x_j^{\mu_{ikj}} \quad (0 \leq i \leq n)$$

$x_j > 0, \quad (J=1,2,\ldots,m)$
Here $c_{ik}(>0)$, $(k=1, 2, \ldots, T_i)$ and $\alpha_{ik}$ be any real numbers. When the objective and constraint goals, coefficients and exponents become fuzzy sets and fuzzy numbers respectively, then we transform (3.8) into a fuzzy geometric programming as follows,

$$\text{Min} \quad g_i(x)$$

subject to $g_i(x) \leq 1 \quad (1 \leq i \leq n)$

\[ x > 0, \]

Its objective and constraints of the form

$$g_i(x) = \sum_{k=1}^{T_i} c_{ik} \prod_{j=1}^{m} x_j^{\alpha_{ikj}} \quad (0 \leq i \leq n)$$

are all posynomials of x in which coefficients $\tilde{c}_{ik}$ and indexes $\tilde{\alpha}_{ikj}$ are fuzzy numbers.

### 3.2.1. Some definitions and theorems

**Definition 3.4.** For $n$-th parabolic flat fuzzy number $(a_1,a_2,a_3,a_4)_{PfFN}$ containing the coefficients $\tilde{c}_{ik}$ $(0 \leq i \leq n; 1 \leq k \leq T_i)$, the membership function of $\tilde{c}_{ik}$ is

$$\mu_{\tilde{c}_{ik}}(\tilde{c}_{ik}) = \begin{cases} 
1 - \left(\frac{a_2 - c_{ik}}{a_2 - a_1}\right)^n & \text{for } a_1 \leq c_{ik} \leq a_2 \\
1 & \text{for } a_1 \leq c_{ik} \leq a_2 \\
1 - \left(\frac{c_{ik} - a_3}{a_4 - a_3}\right)^n & \text{for } a_3 \leq c_{ik} \leq a_4 \\
0 & \text{for otherwise.} 
\end{cases} \quad (3.10)$$

Similarly, we can determine the membership function of the indexes $\tilde{\alpha}_{ikj}$ $(0 \leq i \leq n; 1 \leq k \leq T_i; 1 \leq j \leq m)$.

**Note:**

(a) when $n=1$, $\tilde{c}_{ik}$ become Trapezoidal Fuzzy Number (TrFN),
(b) when $n=1$, and $\alpha_{ik}=a_3$, $\tilde{c}_{ik}$ become Triangular Fuzzy Number (TFN),
(c) when $n=2$, $\tilde{c}_{ik}$ become Parabolic flat Fuzzy Number (PfFN),
(d) when $n=2$, and $\alpha_{ik}=a_4$, $\tilde{c}_{ik}$ become Parabolic Fuzzy Number (pFN).

**Definition 3.5.** Here $\delta$-cut of $\tilde{c}_{ik}$ $(0 \leq i \leq n; 1 \leq k \leq T_i)$ is given by

$$\mu_{\tilde{c}_{ik}}^{-1}(\delta) = [\mu_{\tilde{c}_{ikL}}^{-1}(\delta), \mu_{\tilde{c}_{ikR}}^{-1}(\delta)] = [a_1 + \sqrt[\delta]{\tilde{\mu}_{\tilde{c}_{ikL}}^{-1}(\delta)}, a_4 - \sqrt[\delta]{\tilde{\mu}_{\tilde{c}_{ikR}}^{-1}(\delta)}]. \quad (3.11)$$

Similarly, we can determine the $\delta$-cut of $\tilde{\alpha}_{ikj}$ $(0 \leq i \leq n; 1 \leq k \leq T_i; 1 \leq j \leq m)$.

**Proposition 3.1.** When the coefficients and indexes of the fuzzy geometric programming problem are taken as fuzzy numbers, then

$$\text{Min} \quad \sum_{k=1}^{T_i} \tilde{c}_{ik} \prod_{j=1}^{m} x_j^{\tilde{\alpha}_{ikj}}$$

subject to $\sum_{k=1}^{T_i} \tilde{c}_{ik} \prod_{j=1}^{m} x_j^{\tilde{\alpha}_{ikj}} \leq 1 \quad (1 \leq i \leq n), \quad x_j > 0, \quad (j = 1, 2, \ldots, m)$

using $\delta$-cut of fuzzy numbers coefficients and indexes, the above problem is reduces to the following form

$$\text{Min} \quad \sum_{k=1}^{T_i} \mu_{\tilde{c}_{ikL}}^{-1}(\delta) \prod_{j=1}^{m} x_j^{\mu_{\tilde{\alpha}_{ikjL}}^{-1}(\delta)}$$

subject to $\sum_{k=1}^{T_i} \mu_{\tilde{c}_{ikL}}^{-1}(\delta) \prod_{j=1}^{m} x_j^{\mu_{\tilde{\alpha}_{ikjL}}^{-1}(\delta)} \leq 1 \quad (1 \leq i \leq n), \quad x_j > 0,$

Which is equivalent to

$$\text{Min} \quad \sum_{k=1}^{T_i} \mu_{\tilde{c}_{ikL}}^{-1}(\delta) \prod_{j=1}^{m} x_j^{\mu_{\tilde{\alpha}_{ikjS}}^{-1}(\delta)}$$

subject to $\sum_{k=1}^{T_i} \mu_{\tilde{c}_{ikL}}^{-1}(\delta) \prod_{j=1}^{m} x_j^{\mu_{\tilde{\alpha}_{ikjS}}^{-1}(\delta)} \leq 1 \quad (1 \leq i \leq n) \quad (3.13)$
where
\[
\mu_{\tilde{a}_{ikj}}(\delta) = \begin{cases} 
\mu_{\tilde{a}_{ikjL}}^{-1}(\delta) & \text{when } \tilde{a}_{ikjL} > 0, \\
\mu_{\tilde{a}_{ikjR}}^{-1}(\delta) & \text{when } \tilde{a}_{ikjL} < 0,
\end{cases} (1 \leq i \leq n)
\]

**Definition 3.6.** For any \( x \in \mathbb{R}^m \) and feasible index \( d_i \in \mathbb{R} \) (\( \mathbb{R} \) is the real number set), if
\[
g_j(x, \delta) = \sum_{k=1}^{m} \mu_{\tilde{c}_{ikj}}^{-1}(\delta) \prod_{j=1}^{m} x_j^{\mu_{\tilde{a}_{ikj}}^{-1}(\delta)} \leq 1 \quad (1 \leq i \leq n),
\]
then the linear membership function are given by
\[
\begin{aligned}
\mu_0(g_0(x, \delta)) &= \begin{cases} 
1 & \text{if } g_0(x, \delta) \leq z_0, \\
\left(\frac{z_0 + d_0 - g_0(x, \delta)}{d_0}\right) & \text{if } z_0 \leq g_0(x, \delta) \leq z_0 + d_0, \\
0 & \text{if } g_0(x, \delta) \geq z_0 + d_0,
\end{cases} \\
\mu_i(g_i(x, \delta)) &= \begin{cases} 
1 & \text{if } g_i(x, \delta) \leq z_0, \\
\left(\frac{1 + d_i - g_i(x, \delta)}{d_i}\right) & \text{if } 1 \leq g_i(x, \delta) \leq 1 + d_i, \\
0 & \text{if } g_i(x, \delta) \geq 1 + d_i,
\end{cases}
\end{aligned}
\] (3.14) (3.15)

Based on Zimmerman, first finding \( \delta \)-cut of the fuzzy numbers in coefficients and indexes then we build membership functions of both objective and constraints goals and using max-min operator the above problem (3.13) reduced to a fuzzy Non-Linear Programming (FNLP) problem

Max \( \lambda \)

subject to
\[
\begin{aligned}
\mu_i \left( \sum_{k=1}^{m} \mu_{\tilde{c}_{ikj}}^{-1}(\delta) \prod_{j=1}^{m} x_j^{\mu_{\tilde{a}_{ikj}}^{-1}(\delta)} \right) &\geq \lambda \\
x > 0, & \quad \lambda, \delta \in [0,1], (1 \leq i \leq n),
\end{aligned}
\] (3.16)

which is equivalent to a geometric programming problem with parameters \( \lambda, \delta \) variation

Min \( \lambda^{-1} \)

subject to
\[
\begin{aligned}
\mu_i \left( \sum_{k=1}^{m} \mu_{\tilde{c}_{ikj}}^{-1}(\delta) \prod_{j=1}^{m} x_j^{\mu_{\tilde{a}_{ikj}}^{-1}(\delta)} \right) &\geq \lambda \\
x > 0, & \quad \lambda, \delta \in [0,1], (1 \leq i \leq n),
\end{aligned}
\] (3.17)

**Theorem 3.1.** Let the membership function \( \mu_i(g_i(x, \delta)), \mu_{\tilde{c}_{ikj}}(c_{ik}), \mu_{\tilde{a}_{ikj}}(a_{ik}) \) be all continuous and strictly monotone. Then (4.1b.4.4) is equivalent with following form

Min \( \lambda^{-1} \)

subject to
\[
\begin{aligned}
\mu_i \left( \sum_{k=1}^{m} \mu_{\tilde{c}_{ikj}}^{-1}(\delta) \prod_{j=1}^{m} x_j^{\mu_{\tilde{a}_{ikj}}^{-1}(\delta)} \right) &\leq 1 \\
x > 0, & \quad \lambda, \delta \in [0,1], (0 \leq i \leq n, 1 \leq j \leq m).
\end{aligned}
\]


**Corollary 3.1.** Let the membership function \( \mu_i(g_i(x, \delta)), \mu_{\tilde{c}_{ikj}}(c_{ik}), \mu_{\tilde{a}_{ikj}}(a_{ik}) \) be all continuous and strictly monotone and the problem reduce to

Min \( \lambda^{-1} \)

subject to
\[
\begin{aligned}
\mu_i \left( \sum_{k=1}^{m} \mu_{\tilde{c}_{ikj}}^{-1}(\delta) \prod_{j=1}^{m} x_j^{\mu_{\tilde{a}_{ikj}}^{-1}(\delta)} \right) &\leq 1 \\
x > 0, & \quad \lambda, \delta \in [0,1], (0 \leq i \leq n, 1 \leq j \leq m).
\end{aligned}
\] (3.18)

which is a classical posynomial geometric programming (GP) with parameters \( \gamma, \delta \). Its dual form is
Max \[ d(\omega) = \left( \frac{\lambda^{-1}}{\omega_0} \right)^{\omega_{00}} \prod_{i=0}^{n} \prod_{k=1}^{T_i} \left( \frac{\mu_i^{-1}(\delta)/\mu_i^{-1}(\lambda)}{\omega_{ik}} \right)^{\omega_{ik}} \] (3.19) subject to \[ \omega_{00} = 1, \]
\[ \omega_{00} = \sum_{k=1}^{T_0} \omega_{0k}, \]
\[ (\Gamma(\delta))^T \omega = 0, \quad \lambda, \delta \in [0,1], \]
\[ \omega \geq 0 \]

Where \[ \omega_{ik} = \omega_{ik}(\delta, \lambda) \]
\[ = \begin{pmatrix} \tilde{a}_{011}^{-1}(-\delta) & \ldots & \tilde{a}_{011}^{-1}(\delta) & \ldots & \tilde{a}_{01m}^{-1}(\delta) \\ \tilde{a}_{0j3}^{-1}(-\delta) & \ldots & \tilde{a}_{0j3}^{-1}(\delta) & \ldots & \tilde{a}_{0jm}^{-1}(\delta) \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{a}_{pj1}^{-1}(-\delta) & \ldots & \tilde{a}_{pj1}^{-1}(\delta) & \ldots & \tilde{a}_{pjm}^{-1}(\delta) \\ \tilde{a}_{pj3}^{-1}(-\delta) & \ldots & \tilde{a}_{pj3}^{-1}(\delta) & \ldots & \tilde{a}_{pjm}^{-1}(\delta) \end{pmatrix} \]

and \[ \Gamma(\delta) = \begin{pmatrix} \alpha_1 \omega_2 \alpha_3 \end{pmatrix} \]

4 Solution procedure of fuzzy model

4.1. Fuzzy MAX-MIN Geometric Programming Technique on EOQ Model

When coefficient and exponents are taken as a triangular fuzzy number i.e., in general \( \tilde{a} = (\alpha_1, \alpha_2, \alpha_3). \) Then the \( \delta \)-cut of the fuzzy number \( a, \) is given by
\[ a(\delta) = [a_1 + \delta(a_2 - a_3), a_3 - \delta(a_3 - a_2)], \quad \delta \in [0,1]. \]

Taking the membership function as in (3.14) and (3.15) turn the problem (2.7) into (3.18) and Obtained,

Min \[ \lambda^{-1} \]
subject to \[ \frac{-\tilde{a}Q^{-1} - (H^1 + \delta(H^2 - H^1))aq^2d^{-1}6(\theta^1 + \delta(\theta^2 - \theta^1))d^{-1}x_2^{-1}}{(-x_0 + d0) + d\lambda} \leq 1 \]
\[ \frac{(w_0 + \delta(w_0^2 - w_0^4))q}{W + d\lambda} \leq 1 \]
\[ D,S,q > 0, \quad \gamma, \delta \in [0,1]. \]

The dual form of (4.c1.1) is given by
Max \[ d(\omega) = \left( \frac{\lambda^{-1}}{\omega_0} \right)^{\omega_{00}} \left( \frac{1}{x_0 + d0} \right)^{\omega_{01}} \left( \frac{H^1 + \delta(H^2 - H^1)x}{d0} \right)^{\omega_{02}} \left( \frac{(\theta^1 + \delta(\theta^2 - \theta^1))d^{-1}x_2^{-1}}{x_0 + d0} \right)^{\omega_{03}} \frac{w_0^2 + \delta(w_0^2 - w_0^1)}{W + d\lambda} \]
subject to \[ \omega_{00} = 1, \]
\[ \omega_{01} + \omega_{02} + \omega_{03} + \omega_{11} = \omega_{00}, \]
\[ \omega_{01} = \omega_{01}, \]
\[ \omega_{01} - \omega_{02} = 0 \]
\[ \omega_{01} - \omega_{02} + (1 - x)\omega_{03} = 0 \]
\[ -\omega_{01} + 2\omega_{02} + \omega_{11} = 0 \]
\[ \omega_{01} + \omega_{02} + \omega_{03} + \omega_{11} \geq 0. \]

From (4.21) we get \[ \omega_{01} = \frac{1}{4 - x}, \quad \omega_{02} = \frac{2 - x}{4 - x}, \quad \omega_{03} = \frac{1}{4 - x}, \quad \omega_{11} = \frac{2x - 3}{4 - x} \]

Putting the value of the objective function of the problem (4.c1.2), we get
\[ d(\omega) = \lambda^{-1} \left( \frac{4-x}{(x_0+d_0-1)-d_\omega} \right) \frac{1}{4-x} \left( \frac{\left[H^1 + \delta(H^2-H^3)\right]a(x-4)}{6(2-x)} \right) \frac{2-x}{4-x} \]

\[ \times \left( \frac{\left(\theta^2 + \delta(\theta^2-\theta)\right)(4-x)}{(x_0+d_0-1)-d_\omega} \right) \frac{1}{4-x} \left( \frac{\left[H^1 + \delta(H^2-H^3)\right]a(x-4)}{6(x-2)} \right) \frac{2-x}{4-x} \]

\[ \times \left( \frac{\left(\theta^1 + \delta(\theta^2-\theta)\right)(4-x)}{(x_0+d_0-1)-d_\omega} \right) \frac{1}{4-x} \left( \frac{\left[H^1 + \delta(H^2-H^3)\right]a(x-4)}{6(x-2)} \right) \frac{2-x}{4-x} \]

We can obtained \( \lambda \) by the aid of \( d(\omega) = \lambda^{-1} \). Then the above equation is reduces to the following form

\[ \frac{4-x}{(x_0+d_0-1)-d_\omega} \frac{1}{4-x} \left( \frac{\left[H^1 + \delta(H^2-H^3)\right]a(x-4)}{6(x-2)} \right) \frac{2-x}{4-x} \]

\[ \times \left( \frac{\left(\theta^2 + \delta(\theta^2-\theta)\right)(4-x)}{(x_0+d_0-1)-d_\omega} \right) \frac{1}{4-x} \left( \frac{\left[H^1 + \delta(H^2-H^3)\right]a(x-4)}{6(x-2)} \right) \frac{2-x}{4-x} \]

\[ \times \left( \frac{\left(\theta^1 + \delta(\theta^2-\theta)\right)(4-x)}{(x_0+d_0-1)-d_\omega} \right) \frac{1}{4-x} \left( \frac{\left[H^1 + \delta(H^2-H^3)\right]a(x-4)}{6(x-2)} \right) \frac{2-x}{4-x} = 1 \] (4.22)

Solving the above equation of \( \gamma \) for given \( \delta \in [0,1] \) by Newton-Raphson method, we obtain the value of \( \lambda^* \). Putting the value of \( \lambda^* \), we obtained the values of the dual objective function.

Again from the between primal-dual relation, we get

\[ S^* = \frac{6\omega_1^*\omega_2^*}{a(H^1 + \delta(H^2-H^3))} \]

\[ D^* = \frac{6\omega_1^*}{(x_0+d_0-1)-d_\omega} \]

\[ q^* = \frac{6\omega_1 \omega_2}{(x_0+d_0-1)-d_\omega} \]

\[ = \frac{6\omega_1 \omega_2}{(x_0+d_0-1)-d_\omega} \]

4.2. Fuzzy Parametric Geometric Programming Technique on EOQ Model

Taking \( \tilde{H} = H^1 + \delta(H^2 - H^3) \), \( \tilde{\theta} = \theta^1 + \delta(\theta^2 - \theta^1) \), \( \tilde{w}_0 = w_0^1 + \delta(w_0^2 - w_0^3) \), and \( \tilde{w} = W^1 + \delta(W^2 - W^1) \) where \( \alpha \in [0,1] \) in (2.7). The model takes the reduces form as follows

\[ \text{Min } \text{TAC}(D,S,q) = \frac{5D}{x} + \frac{a(H^1 + \delta(H^2-H^3))q^2}{6D} + (\theta^1 + \delta(\theta^2 - \theta^1))D^1 - x \]

subject to \( (w_0^1 + \delta(w_0^2 - w_0^1)) q \leq (W^1 + \delta(W^2 - W^1)) \), \( D,S,q \geq 0 \).

Applying geometric programming GP technique the dual programming of the problem (4.24) is

\[ \text{Max } \text{d}(\omega) = (\frac{1}{\omega_1}) \omega_1 (\frac{a(H^1 + \delta(H^2-H^3))}{6\omega_2}) \omega_2 (\frac{\theta^1 + \delta(\theta^2-\theta^1)}{\omega_3}) \omega_3 (\frac{w_0^1 + \delta(w_0^2 - w_0^3)}{(W^1 + \delta(W^2-W^1))}) \omega_0 \]

subject to \( \omega_1 + \omega_2 + \omega_3 = 1 \),

\[ \omega_1 - \omega_3 = 0, \]

\[ \omega_1 - \omega_2 + (1-x)\omega_3 = 0, \]

\[ -\omega_1 + 2\omega_2 + \omega_0 = 0, \]

\[ \omega_1, \omega_2, \omega_3, \omega_0 \geq 0. \]
i.e., we get $\omega_1 = \frac{1}{4-x}, \omega_2 = \frac{2-x}{4-x}, \omega_3 = \frac{1}{4-x}$, and $\omega_0 = \frac{2x-3}{4-x}$.

Putting the values in (4.24) we get the optimal solution of dual problem. The values of $D, S, q$ is obtained by using the primal dual relation as follows:

From the primal dual relation we have,

\[
\frac{SD}{q} = \omega_1^* \times d^*(\omega),
\]

\[
a(H^1 + \delta(H^2-H^1))q^2 = \omega_2^* \times d^*(\omega),
\]

\[
(\theta^1 + \delta(\theta^2 - \theta^1))D^1 - xS^{-1} = \omega_3^* \times d^*(\omega),
\]

\[
\frac{(w_0^1 + \delta(w_0^2-w_0^3))q}{(W^1 + \delta(W^2-W^3))} = 1.
\]

The optimal solution of the given model through the parametric approach is given by

\[
d^*(\omega) = (4-x)^{\frac{1}{4-x}} \left(\frac{a(H^1 + \delta(H^2-H^1))}{(2-x)^6}\right)^{\frac{2-x}{4-x}} \left((\theta^1 + \delta(\theta^2 - \theta^1))(4-x)\right)^{\frac{1}{4-x}} \times
\]

\[
\frac{(w_0^1 + \delta(w_0^2-w_0^3))(4-x)^{\frac{2x-3}{4-x}}}{(W^1 + \delta(W^2-W^3))(2x-3)^{\frac{2x-3}{4-x}}}.
\]

(4.25)

and

\[
S^* = \frac{6\omega_1^* \omega_2^* d^*(\omega)^2}{a(H^1 + \delta(H^2-H^1))},
\]

\[
D^* = \frac{a(H^1 + \delta(H^2-H^1))q^2}{6\omega_2^* d^*(\omega)},
\]

\[
q^* = \frac{(W^1 + \delta(W^2-W^3))}{(w_0^1 + \delta(w_0^2-w_0^3))}.
\]

5 Numerical example and solution:

A manufacturing company produces a item. It is given that the inventory holding cost of the item is $15 per unit per year. The production cost of the item varies inversely with the demand and set-up cost. From the past experience, the production cost of the item is $120D^{-3}S^{-1}$ where $D$ is the demand rate and $S$ is set-up cost. Storage space area per unit item ($w_0$) and total storage space area ($W$) are 100 sq. ft. and 2000 sq. ft. respectively. Determine the demand rate ($D$), set-up cost ($S$), production quantity ($q$), and optimum total average cost (TAC) of the production system.

Input values of the model (2.6) are

<table>
<thead>
<tr>
<th>a</th>
<th>H</th>
<th>x</th>
<th>θ</th>
<th>w₀</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>15</td>
<td>1.75</td>
<td>120</td>
<td>100</td>
<td>2000</td>
</tr>
</tbody>
</table>

Then the model is of the following form

\[
\text{Min } \quad \text{TAC}(D,S,q) = \frac{SD}{q} + \frac{105q^2}{6D} + 120D^{-0.75}S^{-1}
\]

subject to

\[
100q \leq 2000, \quad D,S,q > 0.
\]

(5.26)

the crisp solution of the given model is in table 2.
Table 2: (Optimal solution of (2.6) for crisp model)

<table>
<thead>
<tr>
<th>Crisp model</th>
<th>S*</th>
<th>D*</th>
<th>q*</th>
<th>TAC*(S*,D*,q*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G.P</td>
<td>0.684</td>
<td>4048</td>
<td>20</td>
<td>140.517</td>
</tr>
<tr>
<td>N.L.P</td>
<td>0.685</td>
<td>4047</td>
<td>20</td>
<td>140.685</td>
</tr>
</tbody>
</table>

For fuzzy Geometric-Programming (FGP) method, let’s consider \( z_0 = 15.5 \), fuzzy objective goal \( d_0 = 1 \) and Total storage space area tolerance \( d_1 = 100 \), also taking \( H = (14,16,18) \), \( \theta = (116,120,124) \), \( w_0 = (96,100,104) \) (as a fuzzy triangular number), \( \delta = 0.5 \), then from (4.c1.3) we get \( \lambda = 0.007 \).

For Fuzzy Parametric Geometric –Programming (FGP) Technique taking \( \alpha = 0.5 \), \( H = (14,16,18) \), \( \theta = (116,120,124) \), \( w_0 = (96,100,104) \) and \( W = (2000,2200,2400) \).

Then corresponding solution is in table 3.

Table 3: (Optimal solution of (2.7) for fuzzy model)

<table>
<thead>
<tr>
<th>Fuzzy model</th>
<th>S*</th>
<th>D*</th>
<th>q*</th>
<th>TAC*(S*,D*,q*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F.G.P(MAX-MIN)</td>
<td>0.677</td>
<td>4236</td>
<td>20.415</td>
<td>142.561</td>
</tr>
<tr>
<td>F.G.P(PARAMETRIC)</td>
<td>0.662</td>
<td>4718</td>
<td>21.428</td>
<td>147.780</td>
</tr>
</tbody>
</table>

6 Sensitivity analysis

6.1. Sensitivity test of fuzzy E.O.Q problem

We now examine to sensitivity analysis of the optimal solution of the given problem for changes of \( \alpha \), keeping the other parameters unchanged. The initial data is given from the above numerical example.

<table>
<thead>
<tr>
<th>Value of ( \alpha )</th>
<th>% of change</th>
<th>F.G.P(MAX-MIN)</th>
<th>F.G.P(PARAMETRIC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>-80</td>
<td>144.939</td>
<td>142.077</td>
</tr>
<tr>
<td>0.2</td>
<td>-60</td>
<td>144.295</td>
<td>142.983</td>
</tr>
<tr>
<td>0.3</td>
<td>-40</td>
<td>143.849</td>
<td>143.570</td>
</tr>
<tr>
<td>0.4</td>
<td>-20</td>
<td>143.108</td>
<td>144.349</td>
</tr>
<tr>
<td>0.5</td>
<td>0</td>
<td>142.561</td>
<td>144.830</td>
</tr>
<tr>
<td>0.6</td>
<td>+20</td>
<td>141.934</td>
<td>145.257</td>
</tr>
<tr>
<td>0.7</td>
<td>+40</td>
<td>141.484</td>
<td>145.542</td>
</tr>
<tr>
<td>0.8</td>
<td>+60</td>
<td>140.802</td>
<td>145.775</td>
</tr>
<tr>
<td>0.9</td>
<td>+80</td>
<td>140.271</td>
<td>145.989</td>
</tr>
</tbody>
</table>

Here we have given a rough graph, which shown how change the value of \( \text{TAC}^*(S^*,D^*,q^*) \) for different values of \( \alpha \).
Figure 3: Change of the value of objective function for change of $\alpha$ by Fuzzy Max-Min Geometric Programming Technique.

Figure 4: Change of the value of objective function for change of $\alpha$ by Fuzzy Parametric Geometric Programming Technique.

6.2. Outcome of sensitivity analysis

Effect, for increment parameters-
1) Fig.3. shows that as $\alpha$ changes increasingly the total average cost of the given problem decreases.
2) Fig.4. shows that as $\alpha$ changes increasingly the total average cost of the given problem increases.

7 Conclusion

In this paper, we have proposed a real life inventory problem in crisp and fuzzy environment and presented solution along with sensitivity analysis approach. The inventory model is developed with unit production cost, time depended holding cost, with-out shortages. This model has been developed for a single item.

In this paper, we first create a model then it transformed as a fuzzy model. At last we give a real example and solved it various methods. In fuzzy we have considered triangular fuzzy number (T.F.N) and solved by fuzzy Max-Min Geometric-Programming and fuzzy Parametric Geometric-Programming Technique. In future, the other type of membership functions such as piecewise linear hyperbolic, L-R fuzzy number,
Trapezoidal Fuzzy Number (TrFN), Parabolic flat Fuzzy Number (PfFN), Parabolic Fuzzy Number (pFN), pentagonal fuzzy number etc can be considered to construct the membership function and then model can be easily solved.

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