On Soft $\mu$-Compact Soft Generalized Topological Spaces

Sunil Jacob John$^1$, Jyothis Thomas$^1$

(1) Department of Mathematics, National Institute of Technology Calicut, Calicut Pin-673 601, India.

Abstract

The main purpose of this paper is to introduce soft $\mu$-compact soft generalized topological spaces as a generalization of compact spaces. A soft generalized topological space $(F_\lambda, \mu)$ is soft $\mu$-compact if every soft $\mu$-open soft cover of $F_\lambda$ admits a finite soft sub cover. We characterize soft $\mu$-compact space and study their basic properties.

Keywords: Soft sets, generalized topology, soft generalized topology, subspace soft generalized topology, soft basis, soft $\mu$-compact, soft $(\mu, \eta)$-continuous functions.

1 Introduction

Molodtsov [11] in 1999, initiated the concept of soft set theory as a mathematical tool for modeling uncertainties. A soft set is a collection of approximate descriptions of an object. Later other researchers like Maji et al. [9] have further improved the theory of soft sets. Naim Cagman et al. [4] modified the definition of soft sets which is similar to that of Molodtsov. Csaszar [6] in 2002, introduced the concept of generalized topology and also studied some of its basic properties. Let $X$ be a nonempty set and $\xi$ be a collection of subsets of $X$. Then $\xi$ is called a generalized topology (briefly GT) on $X$ if and only if $\emptyset \in \xi$ and $G_i \in \xi$ for $i \in J$ implies $\bigcup_{i \in J} G_i \in \xi$. Jyothis and Sunil [7] introduced the notion of soft generalized topology on a soft set and studied basic concepts of soft generalized topological spaces such as soft $\mu$-interior, soft $\mu$-closure, soft $\mu$-boundary, soft $\mu$-exterior, soft $\mu$-neighborhood, soft basis, soft $\mu$-limit point, soft continuity of soft functions etc.

In this paper, firstly we give some basic definitions and important results related to soft set theory which are useful for subsequent sections. We then give the definitions and basic theorems of soft generalized topology on an initial soft set. Finally we introduce the concept of soft $\mu$-compact spaces and study their basic properties. We also give equivalent conditions for a soft $\mu$-compact space. We can say that a soft $\mu$-compact soft generalized topological space gives a parametrized family of $\mu$-compact generalized topological spaces on the initial universe.

Production and hosting by ISPACS GmbH.

*Corresponding author. Email address: sunil@nitc.ac.in, Tel:+91 495 2287 250
2 Preliminaries

In this section we recall some definitions and results defined and discussed in [4, 7, 8, 9, 11]. Throughout this paper $U$ denotes initial universe, $E$ denotes the set of all possible parameters, $P(U)$ is the power set of $U$ and $A$ is a nonempty subset of $E$.

**Definition 2.1.** A soft set $F_A$ on the universe $U$ is defined by the set of ordered pairs $F_A = \{(e, f_A(e)) : e \in E, f_A(e) \in P(U)\}$, where $f_A : E \rightarrow P(U)$ such that $f_A(e) = \emptyset$ if $e \notin A$. Here $f_A$ is called an approximate function of the soft set $F_A$. The value of $f_A(e)$ may be arbitrary. Some of them may be empty, some may have nonempty intersection. The set of all soft sets over $U$ with $E$ as the parameter set will be denoted by $S(U)$. 

**Definition 2.2.** Let $F_A \in S(U)$. If $f_A(e) = \emptyset$ for all $e \in E$, then $F_A$ is called an empty soft set, denoted by $F_\emptyset$. $f_A(e) = \emptyset$ means that there is no element in $U$ related to the parameter $e$ in $E$. Therefore we do not display such elements in the soft sets as it is meaningless to consider such parameters.

**Definition 2.3.** Let $F_A \in S(U)$. If $f_A(e) = U$ for all $e \in E$, then $F_A$ is called an $A$-universal soft set, denoted by $F_A$. If $A = E$, then the $A$-universal soft set is called an universal soft set, denoted by $F_E$.

**Definition 2.4.** Let $F_A, F_B \in S(U)$. Then $F_B$ is a soft subset of $F_A$, denoted by $F_B \subseteq F_A$, if $f_B(e) \subseteq f_A(e)$, for all $e \in E$.

**Definition 2.5.** Let $F_A, F_B \in S(U)$. Then $F_B$ and $F_A$ are soft equal, denoted by $F_B = F_A$, if $f_B(e) = f_A(e)$, for all $e \in E$.

**Definition 2.6.** Let $F_A, F_B \in S(U)$. Then, the soft union of $F_A$ and $F_B$, denoted by $F_A \cup F_B$, is defined by the approximate function

$$f_{A \cup B}(e) = f_A(e) \cup f_B(e).$$

**Definition 2.7.** Let $F_A, F_B \in S(U)$. Then, the soft intersection of $F_A$ and $F_B$, denoted by $F_A \cap F_B$, is defined by the approximate function

$$f_{A \cap B}(e) = f_A(e) \cap f_B(e).$$

**Definition 2.8.** Let $F_A, F_B \in S(U)$. Then, the soft difference of $F_A$ and $F_B$, denoted by $F_A \setminus F_B$, is defined by the approximate function

$$f_{A \setminus B}(e) = f_A(e) \setminus f_B(e).$$

**Definition 2.9.** Let $F_A \in S(U)$. Then, the soft complement of $F_A$, denoted by $(F_A)^c$, is defined by the approximate function

$$f_A^c(e) = (f_A(e))^c,$$

where $(f_A(e))^c$ is the complement of the set $f_A(e)$, that is, $(f_A(e))^c = U \setminus f_A(e)$ for all $e \in E$. Clearly $(F_A)^c = F_A$ and $(F_\emptyset)^c = F_E$.

**Definition 2.10.** Let $F_A \in S(U)$. The soft power set of $F_A$, denoted by $P(F_A)$, is defined by $P(F_A) = \{F_i : F_A \subseteq F_i, i \in J \subseteq N\}$.

**Theorem 2.1.** Let $F_A, F_B, F_C \in S(U)$. Then,

1. $F_A \cup F_A = F_A$.
2. $F_A \cap F_A = F_A$.
3. $F_A \cup F_\emptyset = F_A$.
4. $F_A \cap F_\emptyset = F_\emptyset$.
5. $F_A \cup F_E = F_E$.
6. $F_A \cap F_E = F_A$. 

International Scientific Publications and Consulting Services
Theorem 2.2.

Definition 2.11. [8] Let \( S(U)_E \) and \( S(V)_K \) be the families of all soft sets over \( U \) and \( V \) respectively. Let \( \varphi : U \to V \) and \( \chi : E \to K \) be two mappings. The soft mapping \( \varphi \chi : S(U)_E \to S(V)_K \) is defined as:

1. Let \( F_A \) be a soft set in \( S(U)_E \). The image of \( F_A \) under the soft mapping \( \varphi \chi \) is the soft set over \( V \), denoted by \( \varphi \chi(F_A) \) and is defined by
   \[
   \varphi \chi(F_A)(k) = \bigcup_{e \in \chi^{-1}(k) \cap A} \varphi(\chi(e)), \quad \text{if} \quad \chi^{-1}(k) \cap A \neq \emptyset; \quad \text{and} \quad \varphi \chi(F_A)(k) = \emptyset \quad \text{otherwise}, \quad \text{for all} \quad k \in K.
   \]
2. Let \( G_B \) be a soft set in \( S(V)_K \). The inverse image of \( G_B \) under the soft mapping \( \varphi \chi \) is the soft set over \( U \), denoted by \( \varphi \chi^{-1}(G_B) \) and is defined by
   \[
   \varphi \chi^{-1}(G_B)(e) = \varphi^{-1}(G_B(\chi(e))), \quad \text{if} \quad \chi(e) \in B; \quad \text{and} \quad \varphi \chi^{-1}(G_B)(e) = \emptyset, \quad \text{otherwise}, \quad \text{for all} \quad e \in E.
   \]

The soft mapping \( \varphi \chi \) is called injective, if \( \varphi \) and \( \chi \) are injective. The soft mapping \( \varphi \chi \) is called surjective, if \( \varphi \) and \( \chi \) are surjective.

Definition 2.12. Let \( \varphi : S(U)_E \to S(V)_K \) and \( \tau : S(V)_K \to S(W)_L \), then the soft composition of the soft mappings \( \varphi \chi \) and \( \tau \), denoted by \( \varphi \chi \circ \tau \), is defined by \( \varphi \chi \circ \tau = (\varphi \circ \tau)(\chi \circ \sigma) \).

Theorem 2.2. [8] Let \( S(U)_E \) and \( S(V)_K \) be the families of all soft sets over \( U \) and \( V \) respectively. Let \( F_A, F_B, F_A, G_B, G_B, G_B, G_B \in S(U)_E \) and \( G_A, G_B, G_B, G_B \in S(V)_K \). For a soft mappings \( \varphi \chi : S(U)_E \to S(V)_K \) and \( \tau : S(V)_K \to S(W)_L \), the following statements are true:

1. If \( F_B \subseteq F_A \), then \( \varphi \chi(F_B) \subseteq \varphi \chi(F_A) \).
2. \( \varphi \chi(\bigcup_{i \in J} F_A_i) = \bigcup_{i \in J} \varphi \chi(F_A_i) \).
$3. \quad \varphi_X(\bigcap_{i \in J} F_A) \subseteq \bigcap_{i \in J} \varphi_X(F_A)$, equality holds if $\varphi_X$ is injective.

$4. \quad F_A \subseteq \varphi_X^{-1}(\varphi_X(F_A))$, equality holds if $\varphi_X$ is injective.

$5. \quad \varphi_X(\varphi_X^{-1}(F_A)) \subseteq F_A$, equality holds if $\varphi_X$ is surjective.

$6. \quad G_B \subseteq G_A$, then $\varphi_X^{-1}(G_B) \subseteq \varphi_X^{-1}(G_A)$.

$7. \quad \varphi_X^{-1}(G_B^c) = (\varphi_X^{-1}(G_B))^c$.

$8. \quad \varphi_X^{-1}(\bigcup_{i \in J} G_B) = \bigcup_{i \in J} \varphi_X^{-1}(G_B)$.

$9. \quad \varphi_X^{-1}(\bigcap_{i \in J} G_B) = \bigcap_{i \in J} \varphi_X^{-1}(G_B)$.

$10. \quad (\tau_\sigma \circ \varphi_X)^{-1} = \varphi_X^{-1} \circ \tau_\sigma^{-1}$.

**Definition 2.13.** A collection $\mathcal{I}$ of soft subsets of a soft set $F_A$ is said to have the finite intersection property (abbreviated FIP) if for every finite sub collection $\{F_{H_1}, F_{H_2}, \ldots, F_{H_n}\}$ of $\mathcal{I}$, the soft intersection $F_{H_1} \cap F_{H_2} \cap \ldots \cap F_{H_n}$ is non soft empty.

### 3 Soft Generalized Topological Spaces

**Definition 3.1.** [7] Let $F_A \in S(U)$. A Soft Generalized Topology (SGT) on $F_A$, denoted by $\mu$ or $\mu_{F_A}$ is a collection of soft subsets of $F_A$ having the following properties:

1. $F_\emptyset \in \mu$

2. $\{F_A : i \in J \subseteq N\} \subseteq \mu \Rightarrow \bigcup_{i \in J} F_A \in \mu$

The pair $(F_A, \mu)$ is called a Soft Generalized Topological Space (SGTS)

Observe that $F_A \in \mu$ must not hold.

**Definition 3.2.** [7] A soft generalized topology $\mu$ on $F_A$ is said to be strong if $F_A \in \mu$.

**Definition 3.3.** [7] Let $(F_A, \mu)$ be a SGTS. Then every element of $\mu$ is called a soft $\mu$-open set.

Note: clearly $F_\emptyset$ is a soft $\mu$-open set.

**Definition 3.4.** [7] Let $(F_A, \mu)$ be a SGTS. A sub family $\mathcal{R}$ of $\mu$ is said to be a soft basis for $\mu$ if every member of $\mu$ can be expressed as the soft union of some members of $\mathcal{R}$.

**Theorem 3.1.** [7] Let $(F_A, \mu)$ be a SGTS and $\mathcal{R} \subseteq \mu$. Then $\mathcal{R}$ is a soft basis for $\mu$ if and only if for each soft $\mu$-open set $F_G$, and each $\alpha \in F_G$, there exists $F_B \in \mathcal{R}$ such that $\alpha \in F_B$ and $F_B \subseteq F_G$.

**Theorem 3.2.** [7] Let $F_A$ be a soft set and $\mathcal{R}$ be a family of its soft subsets. Then there exists a SGTS on $F_A$ with $\mathcal{R}$ as a soft basis.

**Definition 3.5.** [7] Let $(F_A, \mu)$ be a SGTS and $F_B \subseteq F_A$. Then $F_B$ is said to be a soft $\mu$-closed set if its soft complement $(F_B)^c$ is a soft $\mu$-open set.
Theorem 3.3. [7] Let \((F_A, \mu)\) be a SGTS and \(F_B \subseteq F_A\). Then the following conditions hold:

1. The universal soft set \(F_B\) is soft µ-closed.
2. Arbitrary soft intersections of the soft µ-closed sets are soft µ-closed.

Definition 3.6. [7] Let \((F_A, \mu)\) be a SGTS and \(F_B \subseteq F_A\). Then the collection \(\mu_{F_B} = \{F_B \cap F_D : F_D \in \mu\}\) is called a Subspace Soft Generalized Topology (SSGT) on \(F_B\). The pair \((F_B, \mu_{F_B})\) is called a Soft Generalized Topological Subspace (SGTSS) of \(F_A\).

Theorem 3.4. [7] Let \((F_A, \mu)\) be a SGTS and \(F_B \subseteq F_A\). Then a SSGT on \(F_B\) is a SGT.

Theorem 3.5. [7] Let \((F_A, \mu)\) be a SGTS and \((F_B, \mu_{F_B})\) a SGTSS of \(F_A\). Then,

1. \(F_G\) is soft \(\mu_{F_B}\)-open if and only if \(F_G = F_H \cap F_B\) for some soft µ-open set \(F_H\).
2. \(F_G\) is soft \(\mu_{F_B}\)-closed if and only if \(F_G = F_H \cap F_B\) for some soft µ-closed set \(F_H\).

Definition 3.7. [7] Let \((F_A, \mu)\) be a SGTS and \(\alpha \in F_A\). If there is a soft µ-open set \(F_B\) such that \(\alpha \in F_B\), then \(F_B\) is called a soft µ-open neighborhood or soft µ-nbd of \(\alpha\). The set of all soft µ-nbds of \(\alpha\), denoted by \(\psi(\alpha)\), is called the family of soft µ-nbds of \(\alpha\), i.e. \(\psi(\alpha) = \{F_B : F_B \in \mu, \alpha \in F_B\}\).

Theorem 3.6. [7] Let \((F_A, \mu)\) be a SGTS and \(F_G, F_H \subseteq F_A\). Then

1. if \(F_G \in \psi(\alpha)\), then \(\alpha \in F_G\).
2. if \(F_G \in \psi(\alpha)\) and \(F_G \subseteq F_H\), then \(F_H \in \psi(\alpha)\).
3. \(F_G\) is soft µ-open if and only if \(F_G\) contains a soft µ-nbd of each of its points.

Definition 3.8. [7] Let \((F_A, \mu)\) be a SGTS, \(F_B \subseteq F_A\) and \(\alpha \in F_A\). If every soft µ-nbd of \(\alpha\) soft intersect \(F_B\) in some point other than \(\alpha\) itself, then \(\alpha\) is called soft µ-limit point of \(F_B\). The set of all soft µ-limit points of \(F_B\) is denoted by \((F_B)^{'\mu}\). In other words, if \((F_A, \mu)\) is a SGTS, \(F_B, F_G \subseteq F_A\) and \(\alpha \in F_A\), then \(\alpha \in (F_B)^{'\mu}\) if and only if \(F_G \cap (F_B \setminus \alpha) \neq \emptyset\) for all \(F_G \in \psi(\alpha)\).

Theorem 3.7. [7] Let \((F_A, \mu)\) be a SGTS. Then the collection \(\mu_e = \{F_B(e) : \exists F_B \in \mu\text{ such that } (e, F_B(e)) \in F_B\}\) for each \(e \in E\), is a generalized topology on \(U\).

The above theorem shows that corresponding to each parameter \(e \in E\), we have a GT \(\mu_e\) on \(U\). Thus a SGTS on \(F_A\) gives a parameterized family of GTs on \(U\). The converse of the above theorem does not hold.

Example 3.1. [7] Let \(U = \{h_1, h_2, h_3, h_4\}, E = \{e_1, e_2, e_3\}, A = \{e_1, e_2\} \subseteq E\) and \(F_A = \{(e_1, \{h_1, h_2, h_3, h_4\}), (e_2, \{h_2, h_3, h_4\})\}\). Let \(\mu = \{F_{\phi}, F_{A_1}, F_{A_2}, F_{A_3}\}\), where \(F_{A_1} = \{(e_1, \{h_3\}), (e_2, \{h_2\})\}, F_{A_2} = \{(e_1, \{h_2, h_3\}), (e_2, \{h_2, h_4\})\}, F_{A_3} = \{(e_1, \{h_1, h_2, h_3\}), (e_2, \{h_1, h_2, h_4\})\}\). Then \(\mu\) is not a SGT on \(F_A\), because \(F_{A_1} \cup F_{A_2} = \{(e_1, \{h_2, h_3, h_4\}), (e_2, \{h_2, h_4\})\} \notin \mu\). Also \(\mu_{e_1} = \{\phi, \{h_3\}, \{h_2, h_4\}, \{h_2, h_3, h_4\}\}\) and \(\mu_{e_2} = \{\phi, \{h_2\}, \{h_2, h_3, h_4\}\}\) are GTs on \(U\). The above example shows that any collection of soft sets need not to be a SGT on \(F_A\), even if the collection corresponding to each parameter defines a GT on \(U\).

Theorem 3.8. [7] Let \((F_A, \mu)\) be a SGTS and \(F_B \subseteq F_A\). Then \((\mu_{F_B})_e\) is a subspace of the GT \(\mu_e\) for each \(e \in E\).

Definition 3.9. [7] Let \((F_A, \mu)\) and \((F_B, \eta)\) be two SGTSs. A soft function \(\varphi : (F_A, \mu) \to (F_B, \eta)\) is said to be soft \((\mu, \eta)\)-continuous (briefly, soft continuous), if for each soft \(\eta\)-open subset \(F_G\) of \(F_B\), the inverse image \(\varphi^{-1}_A(F_G)\) is a soft \(\mu\)-open subset of \(F_A\).
4 Soft $\mu$-Compact Soft Generalized Topological Spaces

Throughout this section we assume that SGTS’s are strong SGTS’s.

Definition 4.1. Let $F_H \subseteq F_A$. A collection $\mathcal{S}$ of soft subsets of $F_A$ is said to be a soft cover (soft covering) of a soft set $F_H$ if $F_H$ is contained in the soft union of members of $\mathcal{S}$. A soft sub cover of $\mathcal{S}$ is a sub family $\mathcal{S}_0$ of $\mathcal{S}$ which itself is a soft cover. A soft cover is said to be soft $\mu$-open cover if all of its members are soft $\mu$-open.

Definition 4.2. A soft subset $F_H$ of a SGTS $(F_A, \mu)$ is said to be soft $\mu$-compact subset of $F_A$ if every soft $\mu$-open soft cover of $F_H$ admits a finite soft sub cover. A SGTS $(F_A, \mu)$ is said to be soft $\mu$-compact if $F_A$ is a soft $\mu$-compact subset of itself.

Theorem 4.1. If $(F_A, \mu)$ is a finite SGTS, then $F_A$ is soft $\mu$-compact.

Proof. Let $F_A = \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ where $\alpha_i = (e_i, f_A(e_i))$. Let $\mathcal{S}$ be a soft $\mu$-open soft cover of $F_A$. Then each element in $F_A$ belongs to one of the members of $\mathcal{S}$, say, $\alpha_1 \in F_{A_1}, \alpha_2 \in F_{A_2}, \ldots, \alpha_n \in F_{A_n}$ where $F_{A_1} \subseteq F_A$ and $F_{A_i} \in \mathcal{S}, \forall i$. Since each $F_{A_i}$ is a soft $\mu$-open set, the collection $\{F_{A_1}, F_{A_2}, \ldots, F_{A_n}\}$ is a finite sub collection of soft $\mu$-open sets which soft cover $F_A$. Hence $F_A$ is soft $\mu$-compact.

Theorem 4.2. Let $(F_A, \mu)$ be a SGTS, where $\mu = \{F_A \subseteq F_A : F_A \setminus F_H$ is either finite or is all of $F_A\}$. Then $F_A$ is soft $\mu$-compact.

Proof. Let $F_A$ be a soft $\mu$-open soft cover of $F_A$. Let $F_G$ be an arbitrary member of $\mu$. Then $F_A \setminus F_G$ is finite. Suppose $F_A \setminus F_G = \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ since $F_A$ is a soft $\mu$-open soft cover of $F_A$, each $\alpha_i$ belongs to one of the members of $\mathcal{S}$, say, $\alpha_1 \in F_{A_1}, \alpha_2 \in F_{A_2}, \ldots, \alpha_n \in F_{A_n}$, where $F_{A_i} \subseteq F_A$ and $F_{A_i} \in \mathcal{S}, \forall i$. Then the collection $\{F_{A_1}, F_{A_2}, \ldots, F_{A_n}\}$ is a finite sub collection of $\mathcal{S}$ of soft $\mu$-open sets whose soft union contains $F_A \setminus F_G$. Then the collection $\{F_{A_1}, F_{A_2}, \ldots, F_{A_n}, F_G\}$ is a finite sub collection of $\mathcal{S}$ of soft $\mu$-open sets soft covering $F_A$. Hence $F_A$ is soft $\mu$-compact.

Theorem 4.3. Let $(F_A, \mu)$ be a SGTS. Then finite soft union of soft $\mu$-compact set is soft $\mu$-compact.

Proof. Let $F_G$ and $F_H$ be any two soft $\mu$-compact subsets of $F_A$. Let $\mathcal{S}$ be a soft $\mu$-open soft cover of $F_G \cup F_H$. Then $\mathcal{S}$ will also be a soft $\mu$-open soft cover of both $F_G$ and $F_H$. So by assumption, there exists finite sub collections of $\mathcal{S}$ of soft $\mu$-open sets, say, $\{F_{G_1}, F_{G_2}, \ldots, F_{G_n}\}$ and $\{F_{H_1}, F_{H_2}, \ldots, F_{H_n}\}$ soft covering $F_G$ and $F_H$ respectively. Then clearly the collection $\{F_{G_1}, F_{G_2}, \ldots, F_{G_n}, F_{H_1}, F_{H_2}, \ldots, F_{H_n}\}$ is a finite sub collection of $\mathcal{S}$ of soft $\mu$-open sets soft covering $F_G \cup F_H$. Hence $F_G \cup F_H$ is soft $\mu$-compact. By induction, every finite soft union of soft $\mu$-compact sets is soft $\mu$-compact.

Theorem 4.4. Every infinite soft subset $F_G$ of a soft $\mu$-compact space $(F_A, \mu)$ has at least one soft $\mu$-limit point in $F_A$.

Proof. Suppose $(F_A, \mu)$ is a soft $\mu$-compact space and let $F_G$ be an infinite soft subset of $F_A$. Assume that $F_G$ has no soft $\mu$-limit point in $F_A$. Then by the definition of soft $\mu$-limit point, for each $e \in F_A$, there exists a soft $\mu$-open set $F_{G_e}$ containing $e$ such that $F_{G_e} \cap F_G = \{e\}$. Then the collection $\{F_{G_e} : e \in F_A\}$ is a soft $\mu$-open soft cover of $F_A$. Since $F_A$ is soft $\mu$-compact, there exists points $\alpha_{(1)}, \alpha_{(2)}, \ldots, \alpha_{(n)}$ in $F_A$ such that the soft union $\cup^n_{i=1} F_{G_{e(i)}} = F_A$. Then $(F_{G_{e(1)}} \cap F_G) \cup (F_{G_{e(2)}} \cap F_G) \cup \ldots \cup (F_{G_{e(n)}} \cap F_G))$ is a finite soft set or the empty soft set $F_G$. But then $(F_{G_{e(1)}} \cap F_G) \cup (F_{G_{e(2)}} \cap F_G) \cup \ldots \cup (F_{G_{e(n)}} \cap F_G) = (F_{G_{e(1)}} \cup F_{G_{e(2)}} \cup \ldots \cup F_{G_{e(n)}}) \cap F_G = F_A \cap F_G = F_G$ is a finite soft set or the empty soft set $F_G$, which contradicts the assumption that $F_G$ is an infinite soft subset of $F_A$.

Theorem 4.5. A SGTS $(F_A, \mu)$ is soft $\mu$-compact if and only if there exists a soft basis $\mathcal{B}$ for it such that every soft cover of $F_A$ by members of $\mathcal{B}$ has a finite soft sub cover.

Proof. The necessity of the condition is obvious. For sufficiency, assume that $\mathcal{B}$ is a soft basis for $F_A$ with the property that every soft cover of $F_A$ by members of $\mathcal{B}$ has a finite soft sub cover. Let $\mathcal{S}$ be any soft $\mu$-open soft cover of $F_A$, not necessarily by members of $\mathcal{B}$. For each soft set $F_C \in \mathcal{S}$, there exists a sub family $\mathcal{F}_{F_C}$ of $\mathcal{B}$ such that $F_C = \cup_{F_C \in \mathcal{F}_{F_C}} F_C$. Let $\mathcal{B} \subseteq \mathcal{F}_{F_C}$. Clearly $\mathcal{B}$ is a soft $\mu$-open soft cover of $F_A$ by members of $\mathcal{B}$ since $\mathcal{S}$ is a soft $\mu$-open soft cover and more over $\mathcal{B} \subseteq \mathcal{B}$. By hypothesis, there is a finite sub collection, say, $\{F_{D_1}, F_{D_2}, \ldots, F_{D_n}\}$ of $\mathcal{B}$ which soft cover $F_A$. For each $j = 1, 2, \ldots, n$, there exists $F_{C_j} \in \mathcal{S}$ such that $F_{D_j} \subseteq F_{C_j}$. But then clearly $F_{D_j} \subseteq F_{C_j}$ and so $\{F_{C_1}, F_{C_2}, \ldots, F_{C_n}\}$ is a finite sub collection of $\mathcal{S}$ of soft $\mu$-open sets which soft covers $F_A$. Thus $F_A$ is soft $\mu$-compact.
Theorem 4.6. Let \((F_E, \mu)\) be a SGTS. Then the following statements are equivalent:

1. \(F_E\) is soft \(\mu\)-compact

2. For every collection \(\mathcal{S}\) of soft \(\mu\)-closed subsets of \(F_E\), the soft intersection of all the elements of \(\mathcal{S}\) is an empty soft set, then the collection \(\mathcal{S}\) contains a finite sub collection with empty soft set soft intersection.

Proof. (1) \(\Rightarrow\) (2) Suppose \((F_E, \mu)\) is a soft \(\mu\)-compact space. Let \(\mathcal{S} = \{F_i\}_{i \in J}\) be a collection of soft \(\mu\)-closed subsets of \(F_E\). Assume that the soft intersection of all the elements of \(\mathcal{S}\) is an empty soft set. i.e. \(\cap_{i \in J} F_i = F_E \Rightarrow \cap_{i \in J} (F_i^c)^c = F_E \Rightarrow \{(F_i^c)^c\}_{i \in J}\) is a soft \(\mu\)-open soft cover of \(F_E\). Since \(F_E\) is soft \(\mu\)-compact, there is a finite sub collection, say, \(\{(F_{i_1}^c)^c, (F_{i_2}^c)^c, \ldots, (F_{i_n}^c)^c\}\) soft covering \(F_E\). i.e. \(\cup_{i=1}^n (F_{i_i}^c)^c = F_E \Rightarrow \cap_{i=1}^n F_{i_i} = F_\emptyset\).

(2) \(\Rightarrow\) (1) Assume that for every collection \(\mathcal{S}\) of soft \(\mu\)-closed subsets of \(F_E\), the soft intersection of all the elements of \(\mathcal{S}\) is an empty soft set implies the collection \(\mathcal{S}\) contains a finite sub collection with empty soft set soft intersection. Let \(\varphi = \{F_i\}_{i \in J}\) be a soft \(\mu\)-open soft cover of \(F_E\). Then \(\cup_{i \in J} F_i = F_E \Rightarrow \cap_{i \in J} (F_i^c)^c = F_\emptyset\). Then by hypothesis, \(\cap_{i=1}^n (F_i^c)^c = F_\emptyset \Rightarrow \cup_{i=1}^n F_i = F_E\). That is the collection \(\{F_{i_1}, F_{i_2}, \ldots, F_{i_n}\}\) is a finite sub collection of \(\varphi\) which soft covers \(F_E\). Hence \(F_E\) is soft \(\mu\)-compact. \(\square\)

Remark 4.1. If \(F_E\) is replaced by a soft set \(F_A\), \((A \subset E)\), then the above theorem is not true in general.

Theorem 4.7. A SGTS \((F_E, \mu)\) is soft \(\mu\)-compact if and only if every collection of soft \(\mu\)-closed subsets of \(F_E\) which satisfies the FIP has, itself, a non-soft empty soft intersection.

Proof. Suppose \(F_E\) is soft \(\mu\)-compact. \(\mathcal{S} = \{F_i\}_{i \in J}\) be a collection of soft \(\mu\)-closed subsets of \(F_E\) which satisfies the FIP. Then \(\cap_{i=1}^n F_i \neq F_\emptyset\). Since \(F_E\) is soft \(\mu\)-compact, by theorem 4.6, \(\cap_{i \in J} F_i = F_\emptyset \Rightarrow \cap_{i \in J} F_i \neq F_\emptyset\). Therefore \(\cap_{i=1}^n F_i \neq F_\emptyset \Rightarrow \cap_{i \in J} F_i \neq F_\emptyset\).

Conversely, suppose that \(\{F_i\}_{i \in J}\) is a collection of soft \(\mu\)-closed subsets of \(F_E\) which satisfies the FIP has, itself, a non-soft empty soft intersection. That is \(\cap_{i=1}^n F_i \neq F_\emptyset \Rightarrow \cup_{i \in J} F_i = F_\emptyset\). Then \(\cap_{i \in J} F_i = F_\emptyset \Rightarrow \cap_{i=1}^n F_i = F_\emptyset\). Then again by theorem 4.6, \(F_E\) is soft \(\mu\)-compact. \(\square\)

Theorem 4.8. Let \(F_B\) be a soft subset of a SGTS \((F_A, \mu)\). Then the following are equivalent:

1. \(F_B\) is soft \(\mu\)-compact with respect to \(\mu\).

2. \(F_B\) is soft \(\mu_{F_B}\)-compact with respect to the SSGT \(\mu_{F_B}\) on \(F_B\).

Proof. (1) \(\Rightarrow\) (2) Suppose \(F_B\) is soft \(\mu\)-compact w.r.t. \(\mu\). Let \(\mathcal{S} = \{F_i\}_{i \in J}\) be a soft \(\mu_{F_B}\)-cover of \(F_B\). Then for each \(i, F_i\) is of the form \(F_i = F_{\hat{B}} \cap F_B\) for some \(F_{\hat{B}} \in \mu\). Now fix for each \(F_i\) of \(\mathcal{S}\) an element \(F_{\hat{B}} \in \mu\) such that \(F_{\hat{B}} = F_B \cap F_{\hat{B}}\). Then the collection \(\{F_{\hat{B}} : F_i = F_{\hat{B}} \cap F_B, F_{\hat{B}} \in \mathcal{S}\}\) is a soft \(\mu\)-open soft cover of \(F_B\) using soft \(\mu\)-open subsets of \(F_A\). Since \(F_B\) is soft \(\mu\)-compact w.r.t. \(\mu\), there is a finite sub collection of soft \(\mu\)-open sets say, \(\{F_{\hat{B}_1}, F_{\hat{B}_2}, \ldots, F_{\hat{B}_n}\}\) soft covering \(F_B\). Then the collection \(\{F_{\hat{B}_1} \cap F_B, F_{\hat{B}_2} \cap F_B, \ldots, F_{\hat{B}_n} \cap F_B\}\) is a finite sub collection of \(\mathcal{S}\) soft covering \(F_B\). Hence \(F_B\) is soft \(\mu_{F_B}\)-compact.

(2) \(\Rightarrow\) (1) Suppose \(F_B\) is soft \(\mu_{F_B}\)-compact w.r.t. the SSGT \(\mu_{F_B}\) on \(F_B\). Let \(\mathcal{S} = \{F_i\}_{i \in J}\) be a soft \(\mu\)-open soft cover of \(F_B\). Then the collection \(\{F_i \setminus F_B\}_{i \in J}\) is a soft \(\mu_{F_B}\)-covering of \(F_B\). Since \(F_B\) is soft \(\mu_{F_B}\)-compact w.r.t. the SSGT \(\mu_{F_B}\) on \(F_B\), there is a finite sub collection say, \(\{F_{\hat{B}} \cap F_B, F_{\hat{B}} \cap F_B, \ldots, F_{\hat{B}_n} \cap F_B\}\) soft covering \(F_B\). Clearly the collection \(\{F_{\hat{B}_1}, F_{\hat{B}_2}, \ldots, F_{\hat{B}_n}\}\) is a finite sub collection of \(\mathcal{S}\) soft covering \(F_B\). Thus \(F_B\) is soft \(\mu_{F_B}\)-compact. \(\square\)

Theorem 4.9. Let \((F_A, \mu)\) be a SGTS and \(F_C \subseteq F_B \subseteq F_A\). Let \((F_B, \mu_{F_B})\) be a SGTSS. Then \(F_C\) is soft \(\mu\)-compact if and only if \(F_C\) is soft \(\mu_{F_B}\)-compact.

Proof. Let \(\mu_{F_C}\) and \((\mu_{F_B})_{F_C}\) be the SSGTs on \(F_C\). Then by theorem 4.8, we have the following results:

1. \(F_C\) is soft \(\mu\)-compact if and only if \(F_C\) is soft \(\mu_{F_C}\)-compact
2. \( F_C \) is soft \( \mu_{F_b} \)-compact if and only if \( F_C \) is soft \( (\mu_{F_b})_{F_C} \)-compact.

But \((\mu_{F_b})_{F_C} = \mu_{F_C} \). Hence the proof. □

**Theorem 4.10.** Let \((F_E, \mu)\) be a SGTS and \(F_B \subseteq F_E\). If \(F_E\) is soft \( \mu \)-compact and \(F_B\) is soft \( \mu \)-closed in \(F_E\), then \(F_B\) in its SS\( \mu \) is also soft \( \mu_{F_b} \)-compact.

**Proof.** Let \( \mathcal{S} = \{F_G : F_G \subseteq \mu_{F_b}, i \in J\} \) be a soft \( \mu_{F_b} \)-cover of \(F_B\). Then for each \(F_G \subseteq \mathcal{S}\) is of the form \(F_G = F_{H_i} \cap F_B\) for some \(F_{H_i} \subseteq \mu\). Now the collection \( \mathcal{G} = \{F_H : F_H = F_{H_i} \cap F_B, F_G \in \mathcal{S}\} \) is \( \mu \)-open soft covering of \(F_E\). Since \(F_E\) is soft \( \mu \)-compact, there is a finite sub collection of \( \mathcal{G} \) of soft \( \mu \)-open sets soft covering \(F_E\) which can be either:

1. \( \{F_{H_1}, F_{H_2}, \ldots, F_{H_n}\} \) or
2. \( \{F_{H_1}, F_{H_2}, \ldots, F_{H_n}, F_B \} \)

Consider (i). Since \( \cup_{i=1}^{n} F_{H_i} = F_E \) and \( F_B \subseteq F_E \), \( F_B \subseteq \cup_{i=1}^{n} F_{H_i} \). Then \( \{F_{H_1}, F_{H_2}, \ldots, F_{H_n}\} \) is a finite sub collection of soft \( \mu \)-open sets and then the collection \( \{F_{H_1} \cap F_B, F_{H_2} \cap F_B, \ldots, F_{H_n} \cap F_B\} = \{F_{G_1}, F_{G_2}, \ldots, F_{G_n}\} \) is a finite sub collection of \( \mathcal{S} \) soft covering \(F_B\). Hence \(F_B\) is soft \( \mu_{F_b} \)-compact.

Consider (ii). Since \( \cup_{i=1}^{n} (F_{H_i} \cup (F_E \setminus F_B)) = F_E \), \( F_B \subseteq \cup_{i=1}^{n} F_{H_i} \). If \( (F_E \setminus F_B) \) \( \beta \in \cup_{i=1}^{n} (F_{H_i} \cup (F_E \setminus F_B)) \Rightarrow \beta \in \cup_{i=1}^{n} F_{H_i} \) or \( \beta \in (F_E \setminus F_B) \Rightarrow \beta \in \cup_{i=1}^{n} F_{H_i} \), since \( \beta \in F_E \Rightarrow \beta \notin (F_E \setminus F_B) \). For, if \( (e_i, f_B(e_i)) \in F_B \Rightarrow \beta \in F_E \setminus F_B \), then \( (F_{H_1}, F_{H_2}, \ldots, F_{H_n}) \) is a finite sub collection of soft \( \mu \)-open sets and then the collection \( \{F_{H_1} \cap F_B, F_{H_2} \cap F_B, \ldots, F_{H_n} \cap F_B\} = \{F_{G_1}, F_{G_2}, \ldots, F_{G_n}\} \) is a finite sub collection of \( \mathcal{S} \) soft covering \(F_B\). Hence \(F_B\) is soft \( \mu_{F_b} \)-compact.

**Theorem 4.11.** If \((F_A, \mu)\) and \((F_B, \eta)\) be two SGTSs and suppose \( \varphi : (F_A, \mu) \rightarrow (F_B, \eta) \) is a soft continuous surjective function. If \(F_A, \mu\) is soft \( \mu \)-compact, then \(F_B, \eta\) is soft \( \eta \)-compact.

**Proof.** Assume that \((F_A, \mu)\) is soft \( \mu \)-compact and \( \varphi : (F_A, \mu) \rightarrow (F_B, \eta) \) is a soft continuous surjective function. Let \( \mathcal{S}\) be any soft \( \eta \)-open soft cover of \(F_B\). Let \( \varphi = \{\varphi^{-1}_F : F_C \in \mathcal{S}\} \). Clearly \( \varphi \) is a soft cover of \(F_A\). Since \( \varphi \) is soft continuous, \( \varphi \) is a soft \( \mu \)-open soft cover of \(F_A\). Since \(F_A\) is soft \( \mu \)-compact there exists a finitely many members of \( \varphi \), say, \( \varphi^{-1}_F(C_1), \varphi^{-1}_F(C_2), \ldots, \varphi^{-1}_F(C_n) \), \(F_C \in \mathcal{S}\) \( \forall i \) soft cover \(F_A\). But then \( \{F_{C_1}, F_{C_2}, \ldots, F_{C_n}\} \) is a finite sub collection of \( \mathcal{S} \) which soft cover \(F_B\). Hence \(F_B\) is soft \( \eta \)-compact. □

**Theorem 4.12.** Let \((F_E, \mu)\) be a soft \( \mu \)-compact SGTS. Then the space \((U, \mu_U)\) is a \( \mu_U \)-compact generalized topological space.

**Proof.** Let \( \mathcal{S} = \{f_{B_i}(e)\}_{i \in J} \) be a collection of \( \mu_U \)-open sets covering \(U\). Then by the definition of \( \mu_U \) there exists soft sets \( F_{B_i} \subseteq F_A \) such that \( (e, f_{B_i}(e)) \in F_{B_i}, \forall i \in J \). But then \( \{f_{B_i}(e)\}_{i \in J} \) is a collection of soft \( \mu \)-open sets soft covering \(F_E\). Since \(F_E\) is soft \( \mu \)-compact, there is a finite sub collection of soft \( \mu \)-open sets say, \( \{f_{B_1}(e), f_{B_2}(e), \ldots, f_{B_n}(e)\} \) of \(F_E\). But then \( \{f_{B_1}(e), f_{B_2}(e), \ldots, f_{B_n}(e)\} \) is a finite sub collection of \( \mathcal{S} \) of \( \mu_U \)-open sets covering \(U\). Hence \(U\) is \( \mu_U \)-compact. □

The above theorem shows that corresponding to each parameter \( e \in E \), we have a \( \mu_U \)-compact generalized topological space. Thus a SGTS on \(F_E\) gives a parameterized family of \( \mu_U \)-compact generalized topological spaces.

**Acknowledgements**

The authors would like to thank the referees for their valuable comments.
References

   http://dx.doi.org/10.1016/j.asoc.2011.01.003

   http://dx.doi.org/10.1016/j.camwa.2008.11.009

   http://dx.doi.org/10.1016/j.camwa.2010.07.014

   http://dx.doi.org/10.1016/j.ejor.2010.05.004

   http://dx.doi.org/10.1016/0022-247X(68)90057-7

   http://dx.doi.org/10.1023/A:1019713018007


   http://dx.doi.org/10.1142/S1793005711002025

   555-562.
   http://dx.doi.org/10.1016/S0898-1221(03)00016-6

    http://dx.doi.org/10.1016/S0898-1221(02)00216-X

    http://dx.doi.org/10.1016/S0898-1221(99)00056-5