Representation theorem for finite intuitionistic fuzzy perfect distributive lattices

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Abstract
In this paper, we extend some results obtained by A. Amroune and B. Davvaz in [1]. More precisely, we will develop a representation theory of intuitionistic fuzzy perfect distributive lattices in the finite case. To that end, we introduce the notion of intuitionistic fuzzy perfect distributive lattices and the one of fuzzy perfect Priestley spaces. In this way, the results of A. Amroune and B. Davvaz are extended to the intuitionistic fuzzy perfect case and the equivalence between the category of finite intuitionistic fuzzy perfect Priestley spaces the dual of the category of finite intuitionistic fuzzy perfect distributive lattices is proved.

Keywords: Intuitionistic fuzzy perfect ordered relation, Intuitionistic fuzzy perfect ordered set, Intuitionistic fuzzy perfect lattice, Intuitionistic fuzzy perfect Priestley space, Homomorphism of intuitionistic fuzzy perfect lattices, Homomorphism of intuitionistic fuzzy perfect Priestley spaces.

1 Introduction

The representation theorems appeared in the thirties of the last century; M. Stone [19] proved that every Boolean algebra is isomorphic to a set of \( \{ I_a : a \in A \} \) (where \( I_a \) denotes the set of prime ideals of \( A \) not containing \( a \)). The representation theorem for distributive lattices proved by Birkhoff [3]; asserts that any finite distributive lattice \( L \) is isomorphic to the lattice of the ideals of the partial order of the join-irreducible elements of \( L \).

In series of papers, Priestley [15, 16], gave a theory of representation of distributive lattices. The duality is central in making the link between syntactical and semantic approaches to logic, also in theoretical computer science this link is central as the two sides correspond to specification languages and the space of computational states. This ability to translate faithfully between algebraic specification and spatial dynamics has often proved itself to be a powerful theoretical tool as well as a handle for making practical problems decidable.

Topological duality for Boolean algebras [18] and distributive lattices [19] is a useful tool for studying relational semantics for propositional logics. Canonical extensions [10, 11, 12, 13], provide a way of looking at these semantics algebraically.

Priestley’s duality for bounded distributive lattices has enjoyed growing attention and has been variously applied in the international literature since its inception in 1970. Since their first appearance in 1971, [22] fuzzy relations have known a vast development, as well many notions and

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In this paper, we extend some results of [1, 15, 16], more precisely, we give a representation theory of perfect intuitionistic fuzzy distributive lattices in the finite case. This paper is organized as follows: In the next section, basic definitions and notions are presented. In the third section, we give and prove the main result using a definition of intuitionistic fuzzy perfect ordering relation.

2 Preliminaries

In the following we recall some definitions of the intuitionistic fuzzy sets, intuitionistic fuzzy relations [2], [5].

Definition 2.1. [6] Let \( L^* \) = \( \{ (a_1, a_2) \in [0, 1]^2 : a_1 + a_2 \leq 1 \} \) and \( \leq_{L^*} \) be an order in \( L^* \) defined by \( \forall (a_1, a_2), (b_1, b_2) \in L^*: (a_1, a_2) \leq_{L^*} (b_1, b_2) \iff (a_1 \leq b_1 \text{ and } a_2 \geq b_2) \). (\( L^* \), \( \leq_{L^*} \)) is a complete lattice. \( 0_{L^*} = (0, 1) \) and \( 1_{L^*} = (1, 0) \) are the units of \( L^* \).

Let \( X \) be a given non-empty set. An intuitionistic fuzzy set in \( X \) is an expression \( A \) given by \( A = \{ (x, \mu_A(x), \nu_A(x)) \mid x \in X \} \) where \( \mu_A : X \rightarrow [0, 1] \), \( \nu_A : X \rightarrow [0, 1] \) with the condition \( \mu_A(x) + \nu_A(x) \leq 1 \), for all \( x \in X \). We will denote by \( A = (\mu_A, \nu_A) \). The numbers \( \mu_A(x) \) and \( \nu_A(x) \) denote respectively the degree of membership and the degree of non-membership of the element \( x \) in the set \( A \). We will denote by \( IFS_X(X) \) the set of all intuitionistic fuzzy sets on \( X \). In particular, \( 0 \) and \( 1 \) denote the intuitionistic fuzzy empty set and the intuitionistic fuzzy whole set in \( X \) defined by \( 0(x) = (0, 1) \) and \( 1(x) = (1, 0) \) for each \( x \in X \). Obviously, when \( \nu_A(x) = 1 - \mu_A(x) \) for every \( x \in X \), the set \( A \) is fuzzy set. We will denote the set of all \( IFS_X \) in \( X \) by \( IFS(X) \).

Definition 2.2. The following expressions are defined in [2] for all intuitionistic fuzzy sets \( A, B \) in \( X \):

1. \( A \subseteq B \) if and only if \( \mu_A \leq \mu_B \) and \( \nu_A \geq \nu_B \).
2. \( A = B \) if and only if \( A \subseteq B \) and \( B \subseteq A \).
3. \( A^c = (\nu_A, \mu_A) \).
4. \( A \cap B = (\mu_A \land \mu_B, \nu_A \lor \nu_B) \).
5. \( A \cup B = (\mu_A \lor \mu_B, \nu_A \land \nu_B) \).

An intuitionistic fuzzy relation (for short, \( IFR \)) \( R \) is an intuitionistic fuzzy subset of \( X \times Y \) given by the expression \( R = \{ (x, y, \mu_R(x, y), \nu_R(x, y)) \mid x \in X, y \in Y \} \), \( (R = (\mu_R, \nu_R) \) for short) where \( \mu_R : X \times Y \rightarrow [0, 1] \) and \( \nu_R : X \times Y \rightarrow [0, 1] \) satisfy the condition \( \mu_R(x, y) + \nu_R(x, y) \leq 1 \) for every \( (x, y) \in X \times Y \). In particular, if \( R \) is an intuitionistic fuzzy relation from \( X \) to itself, then \( R \) is called a binary intuitionistic fuzzy relation on \( X \), and we will denote the set of all intuitionistic fuzzy relations on \( X \) by: \( IFR(X) \).

Definition 2.3. [5] A fuzzy relation \( R = (\mu_R, \nu_R) \) on a nonempty set \( X \) is called:

1. reflexive if and only if for all \( x \in X \), \( \mu_R(x, x) = 1 \) and \( \nu_R(x, x) = 0 \),
2. perfect antisymmetrical intuitionistic, if for every \( (x, y) \in X \times X \) with \( x \neq y \) and \( \mu_R(x, y) > 0 \), or \( \mu_R(x, y) = 0 \) and \( \nu_R(x, y) < 1 \), then \( \mu_R(y, x) = 0 \) and \( \nu_R(y, x) = 1 \),
3. transitive if and only if for all \(x, y, z \in X\),
\[
\begin{aligned}
\mu_R(x, y) \land \mu_R(y, z) &\leq \mu_R(x, z), \\
\vartheta_R(x, y) \lor \vartheta_R(y, z) &\geq \vartheta_R(x, z).
\end{aligned}
\]

A reflexive, perfect antisymmetric intuitionistic and transitive intuitionistic fuzzy relation is called an intuitionistic fuzzy perfect partial ordering relation. An intuitionistic fuzzy perfect partial order relation \(R\) is an intuitionistic fuzzy perfect total order relation if and only if \((\mu_R(x, y) > 0 \text{ and } \vartheta_R(x, y) < 1)\) or \((\mu_R(y, x) > 0 \text{ and } \vartheta_R(y, x) < 1)\) for all \(x, y \in X\). A set equipped with an intuitionistic fuzzy perfect partial order relation is called an intuitionistic fuzzy perfect poset. The height of \(R\), denoted by \(h(R)\), is defined by: \(h(R) = \bigvee_{x \leq y} \{(x, y) \in X^2 | x \neq y\} R(x, y)\).

### 2.1 Intuitionistic fuzzy perfect lattices

In this section, we first extend the concept of fuzzy lattices studied in [7], to intuitionistic fuzzy case. Hence, we extend some results in this direction.

The following definition introduce the intuitionistic fuzzy lattice as a relational structure.

**Definition 2.4.** Let \((X, R)\) be an intuitionistic fuzzy perfect poset and let \(A\) be a subset of \(X\). An element \(u \in X\) is said to be an upper bound for \(A\) if and only if \(\mu_R(a, u) > 0\) and \(\vartheta_R(a, u) < 1\), for all \(a \in A\). An upper bound \(u_0\) for \(A\) is the least upper bound of \(A\) if and only if \(\mu_R(a, u_0) > 0\) and \(\vartheta_R(a, u_0) < 1\), for every upper bound \(u\) of \(A\). An element \(l \in X\) is said to be a lower bound for \(A\) if and only if \(\mu_R(l, a) > 0\) and \(\vartheta_R(l, a) < 1\), for all \(a \in A\). A lower bound \(l_0\) of \(A\) is the greatest lower bound of \(A\) if and only if \(\mu_R(l_0, a) > 0\) and \(\vartheta_R(l_0, a) < 1\), for every lower bound \(l\) of \(A\).

The least upper bound and the greatest lower bound of a set \(\{x, y\}\) are denoted by \(x \lor y\) and \(x \land y\) respectively.

**Definition 2.5.** Let \((X, R)\) be an intuitionistic fuzzy perfect poset. \((X, R)\) is an intuitionistic fuzzy perfect lattice if and only if \(x \lor y\) and \(x \land y\) exist for all \(x, y \in X\).

**Remark 2.1.** Since \(R\) is perfect antisymmetric intuitionistic, then the least upper (greatest lower) bound, if it exists, is unique.

**Proposition 2.1.** Let \((X, R)\) be an intuitionistic fuzzy perfect lattice and \(x, y, z \in X\). Then

1. \(\mu_R(x, x \lor y) > 0\), \(\vartheta_R(x, x \lor y) < 1\), \(\mu_R(x \land y, x) > 0\), \(\vartheta_R(x \land y, x) < 1\);
2. \(\mu_R(x, z) > 0\), \(\vartheta_R(x, z) < 1\) and \(\mu_R(y, z) > 0\), \(\vartheta_R(y, z) < 1\) implies \(\mu_R(x \lor y, z) > 0\) and \(\vartheta_R(x \lor y, z) < 1\);
3. \((\mu_R(z, x) > 0\) and \(\vartheta_R(z, x) < 1\) and \((\mu_R(z, y) > 0\) and \(\vartheta_R(z, y) < 1)\) implies \((\mu_R(z, x \land y) > 0\) and \(\vartheta_R(z, x \land y) < 1)\);
4. \((\mu_R(x, y) > 0\) and \(\vartheta_R(x, y) < 1)\) if and only if \(x \lor y = y\);
5. \((\mu_R(x, y) > 0\) and \(\vartheta_R(x, y) < 1)\) if and only if \(x \land y = x\);
6. If \(\mu_R(y, z) > 0\) and \(\vartheta_R(y, z) < 1\), then \((\mu_R(x \land y, x \land z) > 0\) and \(\vartheta_R(x \land y, x \land z) < 1)\) and \((\mu_R(x \lor y, x \lor z) > 0\) and \(\vartheta_R(x \lor y, x \lor z) < 1)\).

**Proof.** Straightforward.

**Proposition 2.2.** Let \((X, R)\) be an intuitionistic fuzzy perfect lattice and \(x, y, z \in X\). Then

a) \(x \lor x = x, x \land x = x\);

b) \(x \lor y = y \lor x, x \land y = y \land x\);

c) \((x \lor y) \lor z = x \lor (y \lor z), (x \land y) \land z = x \land (y \land z);\)

d) \((x \lor y) \land x = x, (x \land y) \lor x = x.\)

**Proof.** Straightforward.
The following definition and theorem give a characterizations of intuitionistic fuzzy perfect distributive lattices.

**Definition 2.6.** Let \((X, R)\) be an intuitionistic fuzzy perfect lattice. \((X, R)\) is distributive if and only if \(x \land (y \lor z) = (x \land y) \lor (x \land z)\) and \((x \lor y) \land (x \lor z) = x \lor (y \land z)\) for all \(x, y, z \in X\).

**Theorem 2.1.** Let \((X, R)\) be an intuitionistic fuzzy perfect totally ordered set. Then \((X, A)\) is an intuitionistic fuzzy perfect distributive lattice.

**Proof.** Straightforward.

**Definition 2.7.** Let \((X, R, \land, \lor)\) be an intuitionistic fuzzy perfect lattice, and \(F\) be a nonempty crisp subset of \(X\). \(F\) is called prime filter if \(F\) is proper \((F \neq X)\) and for all \(x, y \in X\), it holds that

1. If \(y \in F\) and \(\mu_y(x, y) > 0\), then \(x \in F\),
2. If \(x, y \in F\), then \(x \land y \in F\).

**Definition 2.8.** Let \((X, R, \land, \lor)\) be an intuitionistic fuzzy perfect lattice and \(F\) be a filter of \((X, R, \land, \lor)\). Then \(F\) is called prime filter if \(F\) is proper \((F \neq X)\) and for all \(x, y \in X\), \(x \lor y \in F\) imply \(x \in F\) or \(y \in F\).

2.1. Intuitionistic fuzzy lattices isomorphisms

Next, we extend the concept of fuzzy lattices isomorphism studied in [20], to intuitionistic fuzzy case.

**Definition 2.9.** Let \((L, r, \land, \lor)\), and \((M, R, \land, \lor)\) be two intuitionistic fuzzy perfect lattices.

1. A function \(f : L \to M\) is called monotone if for all \(x, y \in L\), \(r(x, y) \leq L R(f(x), f(y))\), i.e. \(\mu_r(x, y) \leq \mu_R(f(x), f(y))\) and \(\theta_r(x, y) \geq \theta_R(f(x), f(y))\).
2. Let \(f : L \to M\) be a monotone function between intuitionistic fuzzy perfect lattices. Then \(f\) is called a lattice homomorphism if for any \(x, y \in L\), \(f(x \land y) = f(x) \land f(y)\), and \(f(x \lor y) = f(x) \lor f(y)\). If \(f\) is a bijection, then \(f\) is said to be intuitionistic fuzzy lattices isomorphism.

This proposition is an intuitionistic fuzzy version of [4, Proposition 1.3.9].

**Proposition 2.3.** Let \((X, R, \land, \lor)\) be an intuitionistic fuzzy perfect lattice and \(F\) be a subset of \((X, R, \land, \lor)\). The following conditions are equivalents,

1. \(F\) is a prime filter;
2. There is a surjective lattice homomorphism \(f : X \to \{0, 1\}\) such that \(F = f^{-1}(\{1\})\).

**Proof.** Similar to [4, Proposition 1.3.9].

This corollary is an intuitionistic fuzzy version of [4, Corollary 1.3.13].

**Corollary 2.1.** Let \((X, R, \land, \lor)\) be an intuitionistic fuzzy perfect distributive lattice. If \(a, b \in X\) are such that \(R(a, b) = (0, 1)\) (i.e., \(\mu_R(x, y) = 0\) and \(\theta_R(x, y) = 1\)) there is a prime filter \(F\) such that \(a \in F\) and \(b \notin F\).

**Proof.** Similar to [4, Corollary 1.3.13].
2.2 Intuitionistic fuzzy perfect Priestley spaces

In this section we extend the notion of increasing (decreasing) subset introduced by [20] and some results obtained by A. Amroune and B. Davvaz in [1] in the intuitionistic fuzzy perfect case. Let \((X, R)\) be an intuitionistic fuzzy perfect ordered set. A subset \(E\) of \(X\) is called increasing if for all \(x\) belongs to \(E\) and \(\mu_R(x, y) > 0\) and \(\theta_R(x, y) < 1\) \((y\) is an upper bound of \(x)\), then \(y\) belongs to \(E\) \((a\) decreasing set is defined in a dually\).

An intuitionistic fuzzy ordered space is a triplet \((X, \tau, R)\), where \(X\) is a non empty set, \(\tau\) is a topology on \(X\) and \(R\) is an intuitionistic fuzzy perfect order on \(X\).

An intuitionistic fuzzy perfect ordered space \((X, \tau, R)\) is called perfect totally order disconnected if for \(x, y \in X\), \(\mu_R(x, y) = 0\) and \(\theta_R(x, y) = 1\), there exists an increasing \(\tau\)-clopen \(U\) and a decreasing \(\tau\)-clopen \(V\) such that \(U \cap V = \emptyset\) with \(x \in U\) and \(y \in V\). We recall that a \(\tau\)-clopen set in a topological space is a set which is both open and closed. An intuitionistic fuzzy perfect ordered space \((X, \tau, R)\) is called an intuitionistic fuzzy perfect Priestley space if it is compact and perfect totally order disconnected.

2.2.1 Intuitionistic fuzzy perfect Priestley spaces isomorphisms

**Definition 2.10.** Let \((X, \tau, r)\) and \((X', \tau', r')\) be two intuitionistic fuzzy perfect Priestley spaces.

1. A function \(f : X \to X'\) is called monotone if for all \(x, y \in X\), \(r(x, y) \leq r'(f(x), f(y))\), i.e. \(\mu_r(x, y) \leq \mu_{r'}(f(x), f(y))\) and \(\theta_r(x, y) \geq \theta_{r'}(f(x), f(y))\).

2. Let \(f : X \to X'\) be a function between intuitionistic fuzzy perfect Priestley spaces. Then \(f\) is called a Priestley spaces homomorphism if \(f\) is monotone and continuous. If \(f\) is a bijection, then \(f\) is said to be intuitionistic fuzzy perfect Priestley space isomorphism.

3 Duality for intuitionistic fuzzy perfect distributive lattices

In this section, we extend the concept of Priestley duality for distributive lattices studied in [1, 15, 16], to intuitionistic fuzzy case.

Throughout this section, all fuzzy intuitionistic lattices are perfect distributive lattices and homomorphisms preserve first (0) and last (1) elements. If \((A, \wedge, \vee, R)\) is an intuitionistic fuzzy perfect distributive lattice, then its dual space is defined by: \(T(A) = (X, \tau, R_1)\), where \(X\) is the set of 0−1 homomorphisms from \(A\) onto \(\{0, 1\}\), and \(\tau\) be the topology induced on \(X\) by the product topology on \(\{0, 1\}^A\) and \(R_1\) is an intuitionistic fuzzy perfect order adequately chosen on \(X\). Indeed, \(R_1\) is defined by \(R\), see Lemma 3.1.

If \(\delta = (X, \tau, r)\) is an intuitionistic fuzzy perfect Priestley space, then its dual is defined by: \((L(\delta), \vee, \wedge, r_1)\), where \(L(\delta) = \{Y \subseteq X\mid Y\) is increasing and \(\tau\)-clopen\} and \(r_1\) is an intuitionistic fuzzy perfect order adequately chosen.

**Lemma 3.1.** If \((A, \wedge, \vee, R)\) is an intuitionistic fuzzy finite perfect distributive lattice, then there exists two intuitionistic fuzzy orders \(R_1, R_2\) such that:

1. \(T(A) = (X, \tau, R_1)\) is an intuitionistic fuzzy perfect Priestley space,
2. \((L(T(A)), \wedge, \vee, R_2)\) is an intuitionistic fuzzy perfect distributive lattice.

**Proof.** (1) Let \(R_1 = (\mu_{R_1}, \theta_{R_1})\) be the relation defined by:

\[
\mu_{R_1}(f, g) = \begin{cases} 
\mu_R \left( \wedge g^{-1}(1), \wedge f^{-1}(1) \right) & \text{if } f^{-1}(1) \subseteq g^{-1}(1), \\
0 & \text{otherwise.}
\end{cases}
\]

\[
\theta_{R_1}(f, g) = \begin{cases} 
\theta_R \left( \wedge g^{-1}(1), \wedge f^{-1}(1) \right) & \text{if } f^{-1}(1) \subseteq g^{-1}(1), \\
1 & \text{otherwise.}
\end{cases}
\]

where the symbol \(\wedge\) stands for an infimum with respect to the intuitionistic fuzzy relation \(R\). We show that \(R_1\) is an intuitionistic fuzzy perfect order. We have \(\mu_{R_1}(f, f) = \mu_R \left( \wedge f^{-1}(1), \wedge f^{-1}(1) \right) = 1\) and \(\theta_{R_1}(f, f) = \theta_R \left( \wedge f^{-1}(1), \wedge f^{-1}(1) \right) = 0\) for all \(f \in X\), then \(R_1\) is reflexive.

On the other hand, for all \(f, g \in X\) such that \(f \neq g\), if \(\mu_{R_1}(f, g) > 0\), then \(f^{-1}(1) \subseteq g^{-1}(1)\), which imply \(g^{-1}(1) \nsubseteq f^{-1}(1)\), it follows that \(\mu_{R_1}(g, f) = 0\) and \(\theta_{R_1}(g, f) = 1\), the case \(\mu_{R}(f, g) = 0\) and \(\theta_{R}(f, g) < 1\), is impossible case.
Hence, \( R_1 \) is perfect antisymmetric relation.

In order to verify the transitivity of \( R_1 \), let \( f, g, h \in X \), we show that \( \mu_{R_1}(f, g) \wedge \mu_{R_1}(g, h) \leq \mu_{R_1}(f, h) \) and \( \vartheta_{R_1}(f, g) \vee \vartheta_{R_1}(g, h) \geq \vartheta_{R_1}(f, h) \). We use the following truth table, where the proposition \((P)\) is \( \mu_{R_1}(f, g) \wedge \mu_{R_1}(g, h) \leq \mu_{R_1}(f, h) \) and the proposition \((P')\) is \( \vartheta_{R_1}(f, g) \vee \vartheta_{R_1}(g, h) \geq \vartheta_{R_1}(f, h) \).

### Table 1: \((P)\).

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The only case for investigating is \( f^{-1}(1) \subseteq g^{-1}(1) \) and \( g^{-1}(1) \subseteq h^{-1}(1) \). By the transitivity of \( R \), for every \( a, b, c \) in \( A \), we have \( \mu_{R}(a, b) \wedge \mu_{R}(b, c) \leq \mu_{R}(a, c) \) and \( \vartheta_{R}(a, b) \vee \vartheta_{R}(b, c) \geq \vartheta_{R}(a, c) \). This yields

\[
\mu_{R} \left( \wedge g^{-1}(1), \wedge f^{-1}(1) \right) \wedge \mu_{R} \left( \wedge h^{-1}(1), \wedge g^{-1}(1) \right) \leq \mu_{R} \left( \wedge h^{-1}(1), \wedge f^{-1}(1) \right)
\]

and

\[
\vartheta_{R} \left( \wedge g^{-1}(1), \wedge f^{-1}(1) \right) \vee \vartheta_{R} \left( \wedge h^{-1}(1), \wedge g^{-1}(1) \right) \geq \vartheta_{R} \left( \wedge h^{-1}(1), \wedge f^{-1}(1) \right).
\]

Then for all \( f, g, h \in X \), \( \mu_{R_1}(f, g) \wedge \mu_{R_1}(g, h) \leq \mu_{R_1}(f, h) \) and \( \vartheta_{R_1}(f, g) \vee \vartheta_{R_1}(g, h) \geq \vartheta_{R_1}(f, h) \) are hold, i.e. \( R_1 \) is transitive. Hence, \( R_1 \) is an intuitionistic fuzzy perfect order and by [15], [16] \( T(A) = (X, \tau, R_1) \) is an intuitionistic fuzzy perfect Priestley space.

(2) Let \( M = (M_0, M_1) \) such that

\[
M = \bigwedge_{x' \in x} \{ R(x, y) : x, y \in X, x \neq y \ \text{and} \ R(x, y) \neq (0, 1) \}.
\]

We define \( R_2 = (\mu_{R_2}, \vartheta_{R_2}) \) by:

\[
\mu_{R_2}(H, D) = \begin{cases} 
1 & \text{if } H = D, \\
\mu_{R} \left( \wedge f \in H f^{-1}(1), \wedge g \in D g^{-1}(1) \right) & \text{if } H \subseteq D \text{ and } H \neq \emptyset, \\
M_0 & \text{if } H = \emptyset, \\
0 & \text{otherwise}.
\end{cases}
\]

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In order to verify the transitivity, we use the following truth table, where the proposition

\[ \vartheta_R(H,D) = \begin{cases} 0 & \text{if } H = D, \\ \vartheta_R \left( \bigwedge_{f \in H} f^{-1}(1), \bigwedge_{g \in D} g^{-1}(1) \right) & \text{if } H \subseteq D \text{ and } H \neq \phi, \\ M_1 & \text{if } H = \phi, \\ 1 & \text{otherwise.} \end{cases} \]

for all \( H, D \in L(T(A)) \),

where the symbol \( \bigwedge \) stands for an infimum with respect to the fuzzy relation \( R \).

First, we show that \( R_2 \) is an intuitionistic fuzzy perfect order. Since, \( \mu_{R_2}(H,D) + \vartheta_{R_2}(H,D) \leq 1 \) for all \( H, D \in L(T(A)) \), \( R_2 \) is an intuitionistic fuzzy relation. To show the reflexivity we have \( \mu_{R_2}(H,H) = 1 \) and \( \vartheta_{R_2}(H,H) = 0 \), then \( R_2(H,H) = (1,0) \). To prove the perfect antisymmetrical of \( R_2 \) let \( H, D \in L(T(A)) \) such that \( H \neq D \), if \( \mu_{R_2}(H,D) > 0 \) then \( \mu_{R_2}(D,H) = 0 \) and \( \vartheta_{R_2}(D,H) = 1 \), if \( \mu_{R_2}(H,D) = 0 \) and \( \vartheta_{R_2}(H,D) < 1 \), then \( \mu_{R_2}(D,H) = 0 \) and \( \vartheta_{R_2}(D,H) = 1 \).

In order to verify the transitivity, we use the following truth table, where the proposition \( (P) \) is \( \mu_{R_2}(H,D) \land \mu_{R_2}(D,E) \leq \mu_{R_2}(H,E) \) and the proposition \( (P') \) is \( \vartheta_{R_2}(H,D) \lor \vartheta_{R_2}(D,E) \geq \vartheta_{R_2}(H,E) \).

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<th>( H \subseteq D )</th>
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</tr>
</tbody>
</table>

Table 3: \( (P) \).

<table>
<thead>
<tr>
<th>( H \subseteq D )</th>
<th>( D \subseteq E )</th>
<th>( H \subseteq E )</th>
<th>( (P') )</th>
</tr>
</thead>
<tbody>
<tr>
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<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>Impossible case</td>
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<tr>
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<td>0</td>
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</table>

Table 4: \( (P') \).

First, if one of the three elements \( H, D, E \) is empty, then the transitivity is a trivial fact.

If \( H \neq \phi \) and \( D \neq \phi \) and \( E \neq \phi \), the only case that need investigation is when \( H \subseteq D \) and \( D \subseteq E \). Setting \( \alpha = \bigwedge_{f \in H} f^{-1}(1), \beta = \bigwedge_{g \in D} g^{-1}(1) \) and \( \gamma = \bigwedge_{h \in E} h^{-1}(1) \), since \( \mu_R(\alpha,\beta) \land \mu_R(\beta,\gamma) \leq \mu_R(\alpha,\gamma) \) and \( \vartheta_R(\alpha,\beta) \lor \vartheta_R(\beta,\gamma) \geq \vartheta_R(\alpha,\gamma) \), we have \( \mu_{R_2}(H,D) \land \mu_{R_2}(D,E) \leq \mu_{R_2}(H,E) \) and \( \vartheta_{R_2}(H,D) \lor \vartheta_{R_2}(D,E) \geq \vartheta_{R_2}(H,E) \).

Hence, \( R_2 \) is transitive.

Finally, the least upper and greatest lower bounds of \( H \) and \( D \) (with respect of the intuitionistic fuzzy perfect ordering relation \( R_2 \)) are denoted by \( H \lor_{R_2} D \) and \( H \land_{R_2} D \), respectively, to show that \( H \lor_{R_2} D = H \lor D \) and \( H \land_{R_2} D = H \land D \).
have that $H \cup D$ is an upper bound of $\{H, D\}$ because $\mu_{R_2}(H, H \cup D) > 0$, $\vartheta_{R_2}(H, H \cup D) < 1$ and $\mu_{R_2}(D, H \cup D) > 0$, $\vartheta_{R_2}(D, H \cup D) < 1$, if $C$ is the least upper bound of $\{H, D\}$ we have four cases:

1) if $H = \phi$ and $D = \phi$, it follows that $\mu_{R_2}(C, H \cup D) = \mu(C, \phi)$ and $\vartheta_{R_2}(C, H \cup D) = \vartheta(C, \phi)$, which imply that $(\mu_{R_2}(C, H \cup D), \vartheta_{R_2}(C, H \cup D))$ different to $(0, 1)$ if and only if $C = \phi$, hence $C = H \cup D$.

2) if $H = \phi$, $D \neq \phi$ and $D \subseteq C$ (because $\mu_{R}(D, C) > 0$ and $\vartheta_{R}(D, C) < 1$), it follows that $\mu_{R_2}(C, H \cup D) = \mu_{R}(C, D)$ and $\vartheta_{R_2}(C, H \cup D) = \vartheta_{R}(C, D)$, which imply that $(\mu_{R_2}(C, H \cup D), \vartheta_{R_2}(C, H \cup D))$ different from $(0, 1)$ if and only if $C \subseteq D$, hence $C = H \cup D$.

3) if $H \neq \phi$, $D = \phi$ and $H \subseteq C$ (because $\mu_{R}(H, C) > 0$ and $\vartheta_{R}(H, C) < 1$), it follows that $\mu_{R_2}(C, H \cup D) = \mu_{R}(C, H)$ and $\vartheta_{R_2}(C, H \cup D) = \vartheta_{R}(C, H)$, which imply that $(\mu_{R_2}(C, H \cup D), \vartheta_{R_2}(C, H \cup D))$ different to $(0, 1)$ if and only if $C \subseteq H$, hence $C = H \cup D$.

4) if $H \neq \phi$, $D \neq \phi$, $H \subseteq C$ and $D \subseteq C$, which imply $H \cup D \subseteq C$, it follows that $(\mu_{R_2}(C, H \cup D), \vartheta_{R_2}(C, H \cup D))$ different to $(0, 1)$ if and only if $C \subseteq H \cup D$, hence $C = H \cup D$.

Similarly, we prove that $H \wedge R_2 D = H \cap D$, its known that $H \cup D$ and $H \cap D$ are increasing and $\tau$–clopen. This shows that $(L(T(A)), \vee, \wedge, R_2)$ is a fuzzy distributive lattice.

Lemma 3.2. If $\mathbf{d} = (X, \tau, r)$ is an intuitionistic fuzzy finite perfect Priestley space, then there exist two intuitionistic fuzzy perfect orders $r_1$ and $r_2$ such that:

1. $(L(\mathbf{d})), \vee, \wedge, r_1)$ is an intuitionistic fuzzy perfect distributive lattice,

2. $(T(L(\mathbf{d})), \tau, r_2)$ is an intuitionistic fuzzy perfect Priestley space.

Proof. (1) (i) If $h(r) = (0, 1)$, then $X$ is an antichain and we can write $r_1$ as follows:

\[
\begin{array}{ll}
\mu_{r_1}(A, B) = & \begin{cases} 
1 & \text{if } A = B, \\
1 - \frac{\text{card}A}{\text{card}B} & \text{if } A \subseteq B, \\
0 & \text{otherwise}. 
\end{cases}
\end{array}
\]

and

\[
\begin{array}{ll}
\vartheta_{r_1}(A, B) = & \begin{cases} 
0 & \text{if } A = B, \\
\delta \frac{\text{card}A}{\text{card}B} & \text{if } A \subseteq B, \text{ where } \delta \in [0, 1[ , \\
1 & \text{otherwise}. 
\end{cases}
\end{array}
\]

$r_1$ is an intuitionistic fuzzy relation. It is easy to show that $r_1$ is an intuitionistic fuzzy perfect order, and $A \vee_{r_1} B = A \lor B$, $A \wedge_{r_1} B = A \land B$ exists for every $A$ and $B$ from $L(\mathbf{d})$, and $A \lor B, A \land B$ are increasing and $\tau$-clopen sets of $L(\mathbf{d})$, where $(L(\mathbf{d})), \vee, \wedge, r_1$ is an intuitionistic fuzzy perfect distributive lattice.

If $h(r) \neq 0$, then $X$ is not an antichain, setting $M = (M_0, M_1)$ such that $M = \wedge_{r_1} \{r(x, y)/x, y \in X, x \neq y \text{ and } r(x, y) \neq (0, 1)\}$. Then, $M \neq (0, 1)$ and we can take $r_1 = (\mu_{r_1}, \vartheta_{r_1})$ such that for every $A$ and $B$ from $L(\mathbf{d})$

\[
\begin{array}{ll}
\mu_{r_1}(A, B) = & \begin{cases} 
1 & \text{if } A = B, \\
\text{Max} \left(M_0, \vee_{A \subseteq B, B \subseteq A} \mu_{r_1}(a, b) \right) & \text{if } A \subseteq B \text{ and } A \neq \phi, \\
M_0 & \text{if } A = \phi, \\
0 & \text{otherwise}. 
\end{cases}
\end{array}
\]

and

\[
\begin{array}{ll}
\vartheta_{r_1}(A, B) = & \begin{cases} 
0 & \text{if } A = B, \\
\text{Min} \left(M_1, \wedge_{A \subseteq B, B \subseteq A} \vartheta_{r_1}(a, b) \right) & \text{if } A \subseteq B \text{ and } A \neq \phi, \\
M_1 & \text{if } A = \phi, \\
1 & \text{otherwise}. 
\end{cases}
\end{array}
\]

intuitionistic fuzzy perfect order and we can assume that $A \vee_{r_1} B = A \lor B$ and $A \wedge_{r_1} B = A \land B$, for every $A$ and $B$ from $L(\mathbf{d})$, where $(L(\mathbf{d})), \vee, \wedge, r_1)$ is an intuitionistic fuzzy perfect distributive lattice.

(2) To prove the second assertion, let $r_2 = (\mu_{r_2}, \vartheta_{r_2})$, such that
\( \mu_{r_2}(f, g) = \begin{cases} 
1 & \text{if } f = g, \\
\mu_r \left( \bigwedge_{A \in f^{-1}(1)} A, \bigwedge_{B \in g^{-1}(1)} B \right) & \text{if } f^{-1}(1) \subseteq g^{-1}(1), \\
0 & \text{otherwise.} 
\end{cases} \)
and
\( \vartheta_{r_2}(f, g) = \begin{cases} 
0 & \text{if } f = g, \\
\vartheta_r \left( \bigwedge_{A \in f^{-1}(1)} A, \bigwedge_{B \in g^{-1}(1)} B \right) & \text{if } f^{-1}(1) \subseteq g^{-1}(1), \\
1 & \text{otherwise.} 
\end{cases} \)

where the first infimum \( \bigwedge \) is in the sense of the intuitionistic fuzzy perfect ordering relation \( r \) and the second infimum \( \bigwedge \) is in the sense of the intuitionistic fuzzy relation \( r_1 \). Note that \( r_2 \) is well defined: \( A_1 = \bigwedge_{A \in f^{-1}(1)} A \), where the symbol \( \bigwedge \) stands for an infimum with respect to the intuitionistic fuzzy perfect ordering relation \( r_1 \), it exists because \( L(\delta) \) is a lattice and \( a = \bigwedge A_1 \), where the symbol \( \bigwedge \) stands for an infimum with respect to the intuitionistic fuzzy relation \( r_1 \). Then, \( a \) exists because \( A_1 \) is a finite increasing \( \tau \)-clopen, if \( A_1 \) has two minimal elements \( x, y \), then \( r(x, y) = (0, 1) \), there exists an increasing \( \tau \)-clopen \( U \) and a decreasing \( \tau \)-clopen \( V \) such that \( U \cap V = \emptyset \) with \( x \in U \) and \( y \in V \). It is easy to see that \( A_1 \subseteq U \), then \( U \cap V = \emptyset \) contradiction. By definition \( r_2 \) is an intuitionistic fuzzy relation. It is easy to show that \( r_2 \) is an intuitionistic fuzzy perfect ordering relation. Furthermore, By [15],[16] \((T(L(\delta)), \tau, r_2)\) is an intuitionistic fuzzy perfect Priestley space.

The following, shows that the category of finite intuitionistic fuzzy perfect Priestley spaces is equivalent to the dual of the category of finite intuitionistic fuzzy perfect distributive lattices.

**Lemma 3.3.** Let \( A \) be an intuitionistic fuzzy perfect distributive lattice. The map \( F_A : A \mapsto L(T(A)) \) defined by \( F_A(a) = \{ f \in X / f(a) = 1 \} \) is an intuitionistic fuzzy perfect lattice isomorphism.

**Proof.** It is easy to see that for all \( a, b \in A \) we have \( F_A(a \land b) = F_A(a) \land F_A(b) \) and \( F_A(a \lor b) = F_A(a) \lor F_A(b) \). If \( f, g \in F_A(a) \), then \( f(a) = 1 \) and \( g(a) = 1 \), it follows that \( (f \land g)(a) = 1 \). Hence \( f \land g \in F_A(a) \). On the other hand if \( f \in F_A(a) \) and \( d \leq g \), we have \( f \in F_A(a) \).

If \( f \lor g \in F_A(a) \), then \( f(a) \lor g(a) = 1 \), it follows that \( f(a) = 1 \) or \( g(a) = 1 \), hence \( f \in F_A(a) \) or \( g \in F_A(a) \). Therefore \( F_A(a) \) is prime filter.

Suppose that \( a \neq b \), it follows \( R(a, b) = (0, 1) \) or \( R(b, a) = (0, 1) \).

If \( R(a, b) = (0, 1) \) then, there exist a prime filter \( F \) such that \( a \in F \) and \( b \notin F \), it follows that there exists a surjection \( f : A \rightarrow \{0, 1\} \) such that \( a \in f^{-1} \{1\} \) and \( b \notin f^{-1} \{1\} \), hence \( f(a) = 1 \) and \( f(b) = 0 \). Setting \( R_2(F_A(a), F_A(b)) = (0, 1) \).

Similarly if \( R(b, a) = (0, 1) \) we have \( R_2(F_A(b), F_A(a)) = (0, 1) \). Hence, \( a \neq b \) imply \( F_A(a) \neq F_A(b) \) i.e., \( F_A \) is injective.

To prove that \( F_A \) is surjective let \( U \subseteq L(T(A)) \), for all \( f \in U \) and \( g \in L(T(A)) \) we have \( g < f \) , it follows that \( \exists a_{fg} \in A \) such that \( f(a_{fg}) = 1 \) and \( g(a_{fg}) = 0 \). Then, \( f \in F_A(a_{fg}) \) and \( g \in L(T(A)) \). For fixed \( f \in U \) we have \( g \in L(T(A)) \) \( \subseteq \bigcup_{i=1}^{n} (L(T(A)) - F_A(a_{fg})) = L(T(A)) - F_A(a_{fg}) \). Setting \( \bigwedge_{i=1}^{n} a_{fg} = a_f \), it follows \( F_A(a_f) = F_A(a) \subseteq U \), on the other hand \( f(a_f) = 1 \) then \( f \in F_A(a_f) \). Setting \( U = \bigcup_{f \in U} F_A(a_f) \), it follows \( U = \bigcup_{f \in U} F_A(a_f) = F_A(a) \subseteq L(T(A)) \), hence \( \exists a \in \bigcup_{f \in U} F_A(a) \) such that \( U = F_A(a) \) i.e., \( F_A \) is surjective.

Let us show that the map \( F_A(a) = \{ f \in X / f(a) = 1 \} \) is increasing.

We show that: \( R(x, y) \leq L \) \( R_2(F_A(x), F_A(y)) \), i.e. \( \mu_R(x, y) \leq \mu_{R_2}(F_A(x), F_A(y)) \) and \( \vartheta_R(x, y) \leq \vartheta_{R_2}(F_A(x), F_A(y)) \) for all \( x, y \in A \), where
\[
\mu_{R_2}(F_A(x), F_A(y)) = \begin{cases} 
1 & \text{if } F_A(x) \subseteq F_A(y), \\
\mu_R \left( \bigwedge \left( f \in F_A(x) \right) f^{-1}(1), \bigwedge \left( g \in F_A(y) \right) g^{-1}(1) \right) & \text{otherwise.} 
\end{cases}
\]
and the symbol \( \bigwedge \) stands for an infimum with respect to the fuzzy relation \( R \).

Note that \( F_A(x) \neq \emptyset \) for all \( x \in A \).
If $x = y$, it follows that $R(x, y) = R_2(F_A(x), F_A(y))$.
If $x \neq y$, we consider two cases:

1. If $R(x, y) = (0, 1)$, then we have $R(x, y) \leq L \cdot R_2(F_A(x), F_A(y))$.

2. If $R(x, y) > (0, 1)$, it follows that $F_A(x) \subset F_A(y)$, which implies that $R(x, y) = R_2(F_A(x), F_A(y))$.

Since, $\land \cap_{f \in F_A(x)} f^{-1}(1) = x$ and $\land \cap_{g \in F_A(y)} g^{-1}(1) = y$, because if $\land \cap_{f \in F_A(x)} f^{-1}(1) \neq x$, it follows that $\cap_{f \in F_A(x)} f^{-1}(1)$ has minimal element $z \neq x$, then $r(x, z) = (0, 1)$, there exists an increasing $\tau$–clopen $U$ and a decreasing $\tau$–clopen $V$ such that $U \cap V = \emptyset$ with $x \in U$ and $z \in V$. It is easy to see that $\cap_{f \in F_A(x)} f^{-1}(1) \subset U$, then $U \cap V = \emptyset$, contradiction. It follows that the map $F_A$ is an intuitionistic fuzzy perfect lattice isomorphism.

Lemma 3.4. If $f : A_1 \mapsto A_2$ is an intuitionistic fuzzy perfect lattice homomorphism, then the map $T(f) : T(A_1) \mapsto T(A_2)$ defined by $T(f)(g) = g \circ f$ is a homomorphism of intuitionistic fuzzy perfect Priestley space, i.e., a continuous and increasing map.

Proof. For all $g_1, g_2 \in T(A_1)$, $g_1 \leq g_2 \Rightarrow g_1 \circ f \leq g_2 \circ f$, hence $T(f)$ is increasing.

The continuity of $T(f)$ follows from the fact that for every $a \in A_1$,

$$T(f)^{-1}(F_{A_1}(a)) = \{ g \in T(A_2) : T(f)(g) \in F_{A_1}(a) \} = \{ g \in T(A_2) : g \circ f(a) = 1 \} = F_{A_2}(f(a)).$$

Hence, $T(f)$ is continuous.

Lemma 3.5. If $\delta = (X, \tau, r)$ is a finite intuitionistic fuzzy perfect Priestley space, then the map $G_\delta : \delta \mapsto T(L(\delta))$ defined by $G_\delta(x)(y) = \begin{cases} 1 & \text{if } x \in Y, \\ 0 & \text{if } x \notin Y. \end{cases}$ for all $y \in L(\delta)$ is an isomorphism of intuitionistic fuzzy perfect Priestley space, i.e., a bijection, continuous and increasing map.

Proof. Let $G_\delta : \delta \mapsto T(L(\delta))$ defined by $G_\delta(x)(y) = \begin{cases} 1 & \text{if } x \in Y, \\ 0 & \text{if } x \notin Y. \end{cases}$

To prove the surjection of $G_\delta$, let $f \in T(L(\delta))$ and setting $U = \{ Y \in L(\delta) : f(Y) = 1 \}$, $V = \{ Z \in L(\delta) : f(Z) = 0 \}$, $A = \cap_{Y \in U} Y$ and $B = \cup_{Z \in V} Z$. To show that $A \cap B \neq \emptyset$, suppose that $A \cap B = \emptyset$, it follows that $(\cap_{Y \in U} Y) \cap (\cup_{Z \in V} Z^c) = \emptyset$, then $(\cap_{Y \in U} Y) \cap (\cup_{Z \in V} Z^c) = \emptyset$, since $X$ is compact we have $(\cap_{j=1}^n Y_j) \cap (\cup_{j=1}^m Z_j) = \emptyset$, it follows that $\cap_{j=1}^n Y_j \subseteq \cup_{j=1}^m Z_j$, hence $f(\cup_{j=1}^m Z_j) = 1$.

Conduction since $f(\cup_{j=1}^m Z_j) = \bigvee_{j=1}^m f(Z_j) = 0$, hence $A \cap B \neq \emptyset$. Then, there exist $x \in A \cap B$ such that $G_\delta(x) = f$ and therefore

$$G_\delta(x)(Y) = 1 \Leftrightarrow x \in Y \Leftrightarrow y \in U \Leftrightarrow f(Y) = 1$$

hence $G_\delta$ is surjective.

To prove the injectivity, let $x_1, x_2 \in \delta$,

$x_1 \neq x_2 \Rightarrow r(x_1, x_2) = (0, 1)$ or $r(x_2, x_1) = (0, 1)$.

If $r(x_1, x_2) = (0, 1)$, then since $L(\delta)$ is totally disconnected, there exist $Y_0 \in L(\delta)$ such that $x_1 \in Y_0$ and $x_2 \notin Y_0$, hence $G_\delta(x_1)(Y_0) \neq G_\delta(x_2)(Y_0)$.

If $r(x_2, x_1) = (0, 1)$ then, there exist $Y_1 \in L(\delta)$ such that $x_2 \in Y_1$ and $x_1 \notin Y_1$, hence $G_\delta(x_2)(Y_1) \neq G_\delta(x_1)(Y_1)$. It follows that $x_1 \neq x_2 \Rightarrow G_\delta(x_1)(Y_1) \neq G_\delta(x_2)(Y_1)$, hence $G_\delta$ is injective.

To prove that $G_\delta$ is continuous, let $Z$ a $\tau$–clopen of $T(L(\delta))$. Then, there exist $y \in L(\delta)$ such that $Y = F_{L(\delta)}(y)$.  

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Case 2: if $G_\delta$ is increasing it suffices to show that $r(x, y) \leq L \cdot r_2(G_\delta(x), G_\delta(y))$. Then

$$
G_\delta^{-1}(Y) = G_\delta^{-1}(F_{\delta}(\delta)) = \{x \in X : G_\delta(x) \in F_{\delta}(\delta)\} = \{x \in X : G_\delta(x)(y) = 1\} = \{x \in X : x \in y\} = X \cap y = y (\tau - clopen)
$$

Hence, $G_\delta$ is continuous.

To prove that $G_\delta$ is increasing it suffices to show that $r(x, y) \leq L \cdot r_2(G_\delta(x), G_\delta(y))$. Then

$$
\mu_2(G_\delta(x), G_\delta(y)) = \begin{cases} 
1 & \text{if } G_\delta(x)^{-1} = G_\delta(y)^{-1}, \\
\mu_r(\land_{A \in G_\delta(x)^{-1}} A, \land_{B \in G_\delta(y)^{-1}} B) & \text{if } G_\delta(x)^{-1} (1) \subset G_\delta(y)^{-1} (1), \\
0 & \text{otherwise.}
\end{cases}
$$

and

$$
\vartheta_2(G_\delta(x), G_\delta(y)) = \begin{cases} 
0 & \text{if } G_\delta(x)^{-1} = G_\delta(y)^{-1}, \\
\vartheta_r(\land_{A \in G_\delta(x)^{-1}} A, \land_{B \in G_\delta(y)^{-1}} B) & \text{if } G_\delta(x)^{-1} (1) \subset G_\delta(y)^{-1} (1), \\
1 & \text{otherwise.}
\end{cases}
$$

If $x = y$, then so $r(x, y) = r_2(G_\delta(x), G_\delta(y))$. If $x \neq y$, then there are two cases as follows:

Case 1: if $r(x, y) = (0, 1)$, then we have $r(x, y) \leq L \cdot r_2(G_\delta(x), G_\delta(y))$.

Case 2: if $r(x, y) > (0, 1)$, then $y$ belongs to each $\tau$-clopen which contains $x$,

so,

$$
G_\delta^{-1}(x) \subset G_\delta^{-1}(y), \text{ it follows that } \land_{A \in G_\delta(x)^{-1}} A = x \text{ and } \land_{B \in G_\delta(y)^{-1}} B = y.
$$

Then, $\mu_2(G_\delta(x), G_\delta(y)) = \mu_r(\land_{A \in G_\delta(x)^{-1}} A, \land_{B \in G_\delta(y)^{-1}} B) = \mu_r(x, y)$

and

$$
\vartheta_2(G_\delta(x), G_\delta(y)) = \vartheta_r(\land_{A \in G_\delta(x)^{-1}} A, \land_{B \in G_\delta(y)^{-1}} B) = \vartheta_r(x, y), \text{ hence } r_2(G_\delta(x), G_\delta(y)) = r(x, y). \quad \square
$$

**Lemma 3.6.** If $h : G_1 \to G_2$ is a homomorphism of intuitionistic fuzzy perfect Priestley space, then the map $L(h) : L(G_2) \to L(G_1)$ defined by $L(h)(y) = h^{-1}(y)$ for every $y \in L(G_2)$ is a fuzzy perfect lattices homomorphism.

**Proof.** For all $y \in L(G_2)$ we have $L(h)(y) \in L(G_1)$.

For all $y, z \in L(G_2)$ since $f^{-1}$ commutes with set-theoretical operations we have,

$$
L(h)(y \cup z) = h^{-1}(y \cup z) = h^{-1}(y) \cup h^{-1}(z) = L(h)(y) \cup L(h)(z).
$$

and

$$
L(h)(y \cap z) = h^{-1}(y \cap z) = h^{-1}(y) \cap h^{-1}(z) = L(h)(y) \cap L(h)(z).
$$

and for all $y, z \in L(G_2)$

$$
y \subseteq z \Rightarrow h^{-1}(y) \subseteq h^{-1}(z) \Rightarrow L(h)(y) \subseteq L(h)(z).
$$

Hence, $L(h)$ is fuzzy perfect lattices homomorphism. \quad \square

**Theorem 3.1.** If $f : A_1 \to A_2$ is an intuitionistic fuzzy perfect lattice homomorphism, then $L(T(f)) \circ F_{A_1} = F_{A_2} \circ f$. 

---

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\[ A_1 \xrightarrow{f} A_2 \]
\[ F_{A_1} \xrightarrow{\downarrow} F_{A_2} \]
\[ L(T(A_1)) \xrightarrow{\downarrow} L(T(A_2)) \]

**Proof.** For all \( a \in A_1 \),
\[ (L(T(f)) \circ F_{A_1})(a) = L(T(f))(F_{A_1}(a)) \]
\[ = T^{-1}(f)(F_{A_1}(a)) \]
\[ = \{ g \in T(A_2) : T(f)(g) \in F_{A_1}(a) \} \]
\[ = \{ g \in T(A_2) : g \circ f \in F_{A_1}(a) \} \]
\[ = \{ g \in T(A_2) : g(f(a)) = 1 \} \]
\[ = F_{A_2}(f(a)) \]
\[ = F_{A_2} \circ f(a). \]

**Theorem 3.3.** If \( h : \delta_1 \rightarrow \delta_2 \) is a homomorphism of intuitionistic fuzzy perfect Priestley space, then
\[ T(L(h)) \circ G_{\delta_1} = G_{\delta_2} \circ h \]

**Proof.** For all \( f \in \delta_1 \),
\[ (T(L(h)) \circ G_{\delta_1})(f) \]
\[ = T(L(h))(G_{\delta_1}(f)) \]
\[ = G_{\delta_2}(f) \circ L(h)(\text{since } T(f)(g) = g \circ f) \]

due to \( T(L(h)) \circ G_{\delta_1}(f) \) being a homomorphism.

**Theorem 3.3.** The dual of the category of intuitionistic fuzzy distributive perfect lattices is equivalent to the category of intuitionistic fuzzy perfect Priestley spaces.

**Proof.** Lemma 3.3, Lemma 3.5, Theorem 3.1 and Theorem 3.2 establish the functorial isomorphisms.

**Example 3.1.** Let \( (A, \vee, \wedge, R) \) be an intuitionistic fuzzy perfect distributive lattice, where \( A = \{ a, b, c, d, e, f \} \) and \( R \) is an intuitionistic fuzzy perfect relation defined by:
Table 5: $R$.

<table>
<thead>
<tr>
<th>$\mu_R$</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$d$</th>
<th>$e$</th>
<th>$f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>1</td>
<td>0.1</td>
<td>0.2</td>
<td>0.2</td>
<td>0.3</td>
<td>0.4</td>
</tr>
<tr>
<td>$b$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0.3</td>
<td>0.4</td>
<td>0.5</td>
</tr>
<tr>
<td>$c$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.3</td>
<td>0.4</td>
<td>0.5</td>
</tr>
<tr>
<td>$d$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>$e$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$f$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\varphi_R$</th>
<th>$a$</th>
<th>$b$</th>
<th>$c$</th>
<th>$d$</th>
<th>$e$</th>
<th>$f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>0</td>
<td>0.5</td>
<td>0.4</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>$b$</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>$c$</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>$d$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>$e$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>$f$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Then its dual is: $T(A) = \{\text{homomorphisms from } A \text{ onto } \{0, 1\}\} = \{f_1, f_2, f_3, f_4\}$

Table 6

<table>
<thead>
<tr>
<th>$A$</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
<th>$f_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$b$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$c$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$d$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$e$</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$f$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

and its bidual is: $L(T(A)) = \{\phi, \{f_3\}, \{f_4\}, \{f_3, f_4\}, \{f_2, f_3, f_4\}, X\}$, where $R_2$ is given by:

Table 7: $\mu_{R_2}$.

<table>
<thead>
<tr>
<th>$\mu_{R_2}$</th>
<th>$\phi$</th>
<th>${f_3}$</th>
<th>${f_4}$</th>
<th>${f_3, f_4}$</th>
<th>${f_2, f_3, f_4}$</th>
<th>$X$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>${f_3}$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0.3</td>
<td>0.4</td>
<td>0.5</td>
</tr>
<tr>
<td>${f_4}$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.3</td>
<td>0.4</td>
<td>0.5</td>
</tr>
<tr>
<td>${f_3, f_4}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>${f_2, f_3, f_4}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.3</td>
</tr>
<tr>
<td>$X$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

and
Table 8: $\vartheta_{R_2}$.

<table>
<thead>
<tr>
<th>$\vartheta_{R_2}$</th>
<th>$\phi$</th>
<th>${f_3}$</th>
<th>${f_4}$</th>
<th>${f_3, f_4}$</th>
<th>${f_2, f_3, f_4}$</th>
<th>$X$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>0</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>${f_3}$</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>${f_4}$</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>${f_3, f_4}$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>${f_2, f_3, f_4}$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0.1</td>
</tr>
<tr>
<td>$X$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Finally, $F_A : A \mapsto L(T(A))$ is given by:

Table 9: $F_A$.

<table>
<thead>
<tr>
<th>$A$</th>
<th>$F_A(a_i)/i = 1$ to $6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>$\phi$</td>
</tr>
<tr>
<td>$b$</td>
<td>${f_3}$</td>
</tr>
<tr>
<td>$c$</td>
<td>${f_4}$</td>
</tr>
<tr>
<td>$d$</td>
<td>${f_3, f_4}$</td>
</tr>
<tr>
<td>$e$</td>
<td>${f_2, f_3, f_4}$</td>
</tr>
<tr>
<td>$f$</td>
<td>$X$</td>
</tr>
</tbody>
</table>

Example 3.2. Let $(X, \tau, r)$ be a Priestley space, where $X = \{x, y, z\}$ and $r$ is given by:

Table 10: $r$.

<table>
<thead>
<tr>
<th>$\mu_r$</th>
<th>$x$</th>
<th>$y$</th>
<th>$z$</th>
<th>$\vartheta_r$</th>
<th>$x$</th>
<th>$y$</th>
<th>$z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>$x$</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$y$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>$y$</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$z$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>$z$</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Then $L(X) = \{\phi, \{x\}, \{y\}, \{z\}, \{x, y\}, \{x, z\}, \{y, z\}, X\}$ and $r_1$ is given by:

$r_1 = (\mu_{r_1}, \vartheta_{r_1})$ such that

$\mu_{r_1}(A, B) = \begin{cases} 
1 - \frac{\text{card} A}{\text{card} B} & \text{if } A = B, \\
0 & \text{if } A \subset B \\
\frac{\text{card} A}{\text{card} B} & \text{otherwise}
\end{cases}$

and $\vartheta_{r_1}(A, B) = \begin{cases} 
0 & \text{if } A = B, \\
\delta & \text{if } A \subset B \text{ where } \delta \in [0, 1], \\
1 & \text{otherwise}
\end{cases}$

Then $r_1$ will be given by: $\left(\delta = \frac{1}{2}\right)$.
Table 11: $\mu_{r_1}$.

<table>
<thead>
<tr>
<th>$\mu_{r_1}$</th>
<th>$\emptyset$</th>
<th>${x}$</th>
<th>${y}$</th>
<th>${z}$</th>
<th>${x,y}$</th>
<th>${x,z}$</th>
<th>${y,z}$</th>
<th>$X$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\emptyset$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>${x}$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>$\frac{1}{4}$</td>
<td>0</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>${y}$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>$\frac{1}{4}$</td>
<td>0</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>${z}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>$\frac{1}{4}$</td>
<td>0</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>${x,y}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>${x,z}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>${y,z}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>$X$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

and

Table 12: $\vartheta_{r_1}$.

<table>
<thead>
<tr>
<th>$\vartheta_{r_1}$</th>
<th>$\emptyset$</th>
<th>${x}$</th>
<th>${y}$</th>
<th>${z}$</th>
<th>${x,y}$</th>
<th>${x,z}$</th>
<th>${y,z}$</th>
<th>$X$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\emptyset$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>${x}$</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>$\frac{1}{4}$</td>
<td>1</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>${y}$</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>$\frac{1}{4}$</td>
<td>1</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>${z}$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>$\frac{1}{4}$</td>
<td>1</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>${x,y}$</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>${y,z}$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>$\frac{1}{4}$</td>
</tr>
<tr>
<td>$X$</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

and the set of $0-1$ homomorphisms from $L(X)$ onto $\{0,1\}$, i.e., $T(L(X))$ is equal to $\{f_1, f_2, f_3\}$. 
Table 13: $L(X)$.

<table>
<thead>
<tr>
<th>$L(X)$</th>
<th>$f_1(X_i)$</th>
<th>$f_2(X_i)$</th>
<th>$f_3(X_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\emptyset$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>${x}$</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>${y}$</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>${z}$</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>${x,y}$</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>${x,z}$</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>${y,z}$</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$X$</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

And $r_2$ will be given by:

Table 14: $r_2$.

<table>
<thead>
<tr>
<th>$\mu_{r_2}$</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
<th>$\theta_{r_2}$</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$</td>
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<td>0</td>
<td>0</td>
<td>$f_1$</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$f_2$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>$f_2$</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$f_3$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>$f_3$</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

and the isomorphism $G_X$ is defined by: $G_X : X \rightarrow T(L(X))$, where

Table 15: $G_X$.

<table>
<thead>
<tr>
<th>$X$</th>
<th>$G_X(X_i)$ $/X_i \in X$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$f_1$</td>
</tr>
<tr>
<td>$y$</td>
<td>$f_2$</td>
</tr>
<tr>
<td>$z$</td>
<td>$f_3$</td>
</tr>
</tbody>
</table>

Example 3.3. Let $(X, \tau, r)$ be a Priestley space, where $X = \{x, y, z, t\}$ and $r$ is given by:

Table 16: $r$.

<table>
<thead>
<tr>
<th>$\mu_r$</th>
<th>$x$</th>
<th>$y$</th>
<th>$z$</th>
<th>$t$</th>
<th>$\theta_r$</th>
<th>$x$</th>
<th>$y$</th>
<th>$z$</th>
<th>$t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>1</td>
<td>0.2</td>
<td>0.3</td>
<td>0</td>
<td>$x$</td>
<td>0</td>
<td>0.5</td>
<td>0.4</td>
<td>1</td>
</tr>
<tr>
<td>$y$</td>
<td>0</td>
<td>1</td>
<td>0.4</td>
<td>0</td>
<td>$y$</td>
<td>1</td>
<td>0</td>
<td>0.5</td>
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<tr>
<td>$z$</td>
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<td>0</td>
<td>1</td>
<td>0</td>
<td>$z$</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$t$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>$t$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

and $L(X) = \{\emptyset, \{t\}, \{z\},\{y,z\},\{z,t\},\{x,y,z\},\{y,z,t\},X\}$, where $r_1$ is given by:
Table 17: $\mu_{r_1}$.

<table>
<thead>
<tr>
<th>$\mu_{r_1}$</th>
<th>$\phi$</th>
<th>${t}$</th>
<th>${z}$</th>
<th>${t,z}$</th>
<th>${y,z}$</th>
<th>${y,z,t}$</th>
<th>${x,y,z}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>1</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>${t}$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0.2</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>${z}$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.2</td>
<td>0.2</td>
<td>02</td>
<td>0</td>
</tr>
<tr>
<td>${t,z}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>${y,z}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0.4</td>
</tr>
<tr>
<td>${y,z,t}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0.4</td>
</tr>
<tr>
<td>${x,y,z}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0.4</td>
</tr>
<tr>
<td>$X$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 18: $\vartheta_r$.

<table>
<thead>
<tr>
<th>$\vartheta_r$</th>
<th>$\phi$</th>
<th>${t}$</th>
<th>${z}$</th>
<th>${t,z}$</th>
<th>${y,z}$</th>
<th>${y,z,t}$</th>
<th>${x,y,z}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>0</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>${t}$</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0.5</td>
<td>1</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>${z}$</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>${t,z}$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>${y,z}$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>${y,z,t}$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>${x,y,z}$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>$X$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 19: $L(X)$.

<table>
<thead>
<tr>
<th>$L(X)$</th>
<th>$f_1(X_i)$</th>
<th>$f_2(X_i)$</th>
<th>$f_3(X_i)$</th>
<th>$f_4(X_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>${t}$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>${z}$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>${t,z}$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>${y,z}$</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>${y,z,t}$</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>${x,y,z}$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$X$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

The isomorphism $G_X$ is defined as follows: $G_X : X \mapsto T(L(X))$

Table 20: $G_X$.

<table>
<thead>
<tr>
<th>$X$</th>
<th>$G_X(X_i)X_i \in X$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>$f_1$</td>
</tr>
<tr>
<td>$y$</td>
<td>$f_2$</td>
</tr>
<tr>
<td>$z$</td>
<td>$f_3$</td>
</tr>
<tr>
<td>$t$</td>
<td>$f_4$</td>
</tr>
</tbody>
</table>
4 Conclusion

In this paper, we have proposed a way to represent finite intuitionistic fuzzy perfect distributive lattices. This, by constructing adequate intuitionistic fuzzy perfect orders. In this context, a theory of representation of finite intuitionistic fuzzy perfect distributive lattices in the finite case is presented. The main result extends the one obtained in [1] and shows that the category of finite intuitionistic fuzzy perfect Priestley spaces is equivalent to the dual of the category of finite intuitionistic fuzzy perfect distributive lattices.

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