Fuzzy number Intuitionistic fuzzy soft sets and its properties

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Abstract
In this paper, we have defined the notion of the fuzzy number intuitionistic fuzzy soft sets. For it, different operations such as union, intersection, complement, max, min, AND and OR have been introduced on fuzzy number intuitionistic fuzzy soft sets environment. Some examples of these operations are given and a few properties are also studied.

Keywords: Fuzzy soft set; Fuzzy number; Fuzzy number intuitionistic fuzzy soft sets; Intuitionistic fuzzy sets.

1 Introduction
The concept of an intuitionistic fuzzy set (IFS) (Atanassov, 1986 [5]) can be viewed as an alternative approach to define a fuzzy set (FS) (Zadeh, 1965) [25] in cases where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set. But, Molodtsov (1999) [17] has been analyzed that these IFS and FS theories have incompatibility with the parameterization tools in dealing with the uncertainties. To handle this, Molodtsov (1999) [17] introduced the concept of a soft set as a new mathematical tool which is free from the parameterization inadequacy measures of IFS and FS. After their pioneering work, the various authors have gained their interest in it and derived various types of operators like equality, subset, superset, binary operations (Maji et al. 2003) [13]. After that, many extensions of fuzzy soft sets have been proposed in different areas (Jiang et al. 2010 [10]; Maji et al. 2001 [15]; Maji et al. 2004 [14]; Majumdar and Samanta, 2010 [16]; Neog and Sut, 2011 [19]; Zou and Xiao, 2008 [27], Alkhazaleh et al. 2011a,b,c [2-4]). Now-a-days soft sets theory has been applied in variety of disciplines such as such as smoothness of functions, game theory, operation research, measurement theory, medical diagnosis, decision making, algebra etc. (Acar, 2010 [1]; Molodtsov, 1999 [17]; Molodtsov, 2004 [18]; Roy and Maji, 2007 [20]; Celik and Yamak, 2013 [6], Majumdar and Samanta, 2010 [16], Ma et al., 2014 [12]). Yang et al. (2009) [23] presented the concept of interval-valued fuzzy soft set by combining the interval-valued fuzzy set (Gorzalczyzny, 1987 [8]; Zadeh 1975 [24]) and soft set (Maji et al. 2001 [15]) models. Yang et al. (2009) [23] proposed interval-valued fuzzy soft sets

Thus, motivated by the theory of intuitionistic fuzzy set and fuzzy soft set, the present paper addressed the new hybrid structure of the set theories called fuzzy number Intuitionistic fuzzy soft set. This theory combines the soft set with the fuzzy number Intuitionistic fuzzy set theory. In it, fuzzy soft set theory has been generalized by assigning a degree with the parameterization of intuitionistic fuzzy sets. Various operations, such as Union, Intersection, Complement, Max-min, etc. have been defined and their corresponding properties in the fuzzy number Intuitionistic fuzzy soft sets environment. In order to do so, the rest of the paper has been summarized as follows. Some basic definition related to the intuitionistic fuzzy set theory and Soft set theory are given in the next section. In section 3 the notion of fuzzy number intuitionistic fuzzy soft sets is proposed. Some set theoretic operations like union, intersection, complement etc are also defined here. In Section 4 based on set theoretic operations, a number of properties of fuzzy number intuitionistic soft sets are stated and proved. In section 5 some operators such as max, min defined with their properties. The concrete conclusion about the work has been summarized in Section 6.

2 Preliminaries

In this section, basic concepts of intuitionistic fuzzy set and soft set have been discussed. Let $X$ be a universe of objects and $E$ be the set of parameters with the connection to the objects in $X$.

2.1. Intuitionistic Fuzzy Set [5]

An intuitionistic fuzzy set $A$ defined in $X$ is given by $A = \{x, \mu_A(x), \nu_A(x) \mid x \in X\}$, where, $\mu_A(x) : X \to [0,1]$ and $\nu_A(x) : X \to [0,1]$, with the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1$. The numbers $\mu_A(x)$ and $\nu_A(x)$ respectively, denote the degree of membership and degree of non-membership of $x$ in $A$. For each $x \in X$, $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called the intuitionistic index or hesitancy degree of $x$ in $A$.

2.2. Fuzzy Soft Set [15]:

Let $FS(X)$ be the set of all fuzzy sets in $X$. A pair $\langle F, A \rangle$ is called a fuzzy soft set over $X$ if and only if $F$ is a mapping from $A$ to $FS(X)$ i.e. $F : A \to FS(X)$. Every set $F(e), e \in A$ from this family may be considered as the set $e$ approximate elements of the fuzzy soft set $\langle F, A \rangle$. $F(e)$ can be written as a fuzzy set such that $F(e) = \{\langle x, \mu_{F(e)}(x) \rangle : x \in X\}$. If for any parameter $e \in A$, $F(e)$ is a crisp subset of $X$ then $\langle F, A \rangle$ degenerates the standard soft set. But if we apply the membership degree $\mu_{F(e)}(x)$, then $\langle F, A \rangle$ is simply called fuzzy soft set.

2.3. Intuitionistic Fuzzy Soft Set [14]:

Let $IFS(X)$ be the set of all intuitionistic fuzzy set in $X$. A pair $\langle F, A \rangle$ is called an intuitionistic fuzzy soft set over $X$ if and only if $F$ is a mapping from $A$ to $IFS(X)$ i.e. $F : A \to IFS(X)$. Every set $F(e), e \in A$ from this family may be considered as the set $e$ approximate elements of the intuitionistic
fuzzy soft set $\langle F, A \rangle$ and $F(e)$ can be written as $F(e) = \{ (x, \mu_A(x), \gamma_A(x)) : x \in X \}$, where $\mu_A : E \to [0,1]$ and $\gamma_A : E \to [0,1]$ define the membership and the non-membership functions respectively and if $\gamma_{F(e)}(x) = 1 - \mu_{F(e)}(x), \forall x \in X$, $\langle F, A \rangle$ will be treated as traditional fuzzy soft set.

3 Fuzzy Number Intuitionistic Fuzzy Soft Sets

In defining the fuzzy number intuitionistic fuzzy soft set, let $X$ be a universal set and $E$ be a set of parameters such that $A \subseteq E$.

3.1. Fuzzy Number Intuitionistic Fuzzy Soft Sets ($FNIFSS$)

Let $FNIFSS(X)$ be the set of all fuzzy number intuitionistic fuzzy sets in $X$. A pair $\langle F, A \rangle$ is called a fuzzy number intuitionistic fuzzy soft set over $X$ if and only if $F$ is a mapping from $A$ to $FNIFSS(X)$ i.e. $F : A \to FNIFSS(X)$. In other words, the fuzzy number intuitionistic fuzzy soft set is a parameterized family of all fuzzy subsets of the set $X$. Every set $F(e), e \in A$ from this family may be considered as the set of approximate elements of the fuzzy number intuitionistic fuzzy soft set $\langle F, A \rangle$ and $F(e)$ can be written as $F(e) = \{ (x, t_A(x), f_A(x)) : x \in X \}$, where $t_A : X \to F[0,1]$ and $f_A : X \to F[0,1]$ are the fuzzy numbers such that $t_A(x) = (t_A^1(x), t_A^2(x), t_A^3(x))$ and $f_A(x) = (f_A^1(x), f_A^2(x), f_A^3(x))$ respectively denote the degree of membership and non-membership of $x$ to the set $A$, and for every $x \in X$, $0 \leq t_A^i(x) + f_A^i(x) \leq 1$.

Especially, if $t_A^1(x) = t_A^2(x) = t_A^3(x)$ and $f_A^1(x) = f_A^2(x) = f_A^3(x)$, then the fuzzy number intuitionistic fuzzy soft sets set will be converted into intuitionistic fuzzy soft set and if any two values are equal in membership as well as in non-membership function, then this theory will be converted into “interval-valued intuitionistic fuzzy soft set theory. For instance,

**Example 3.1.** Consider a fuzzy number intuitionistic fuzzy soft set $\langle F, A \rangle$, where $X$ is a set of six houses under the consideration of a person to purchase denoted by $X = \{ h_1, h_2, h_3, h_4, h_5, h_6 \}$ and $A$ is a set of parameters such that $A = \{ e_1, e_2, e_3, e_4, e_5 \} = \{ \text{expensive, beautiful, wooden, in good repair, in good surroundings} \}$. Then the fuzzy number intuitionistic fuzzy soft set $\langle F, A \rangle$ describes the “attractiveness of the houses” to the person is represented of each house for a particular parameter in the form of triangular intuitionistic fuzzy number is as follows:

$$
F(e_1) = \left\{ \begin{array}{l}
\langle h_1, [0.30, 0.50, 0.60], [0.25, 0.30, 0.35] \rangle, \langle h_2, [0.30, 0.50, 0.60], [0.25, 0.30, 0.35] \rangle, \\
\langle h_3, [0.30, 0.50, 0.60], [0.25, 0.30, 0.35] \rangle, \langle h_4, [0.30, 0.50, 0.60], [0.25, 0.30, 0.35] \rangle, \\
\langle h_5, [0.30, 0.50, 0.60], [0.25, 0.30, 0.35] \rangle, \langle h_6, [0.30, 0.50, 0.60], [0.25, 0.30, 0.35] \rangle,
\end{array} \right.
\right.$$

$$
F(e_2) = \left\{ \begin{array}{l}
\langle h_1, [0.60, 0.70, 0.80], [0.10, 0.15, 0.20] \rangle, \langle h_2, [0.80, 0.85, 0.90], [0.05, 0.07, 0.10] \rangle, \\
\langle h_3, [0.60, 0.65, 0.70], [0.10, 0.20, 0.25] \rangle, \langle h_4, [0.50, 0.60, 0.80], [0.10, 0.15, 0.20] \rangle, \\
\langle h_5, [0.65, 0.68, 0.78], [0.15, 0.19, 0.21] \rangle, \langle h_6, [0.68, 0.70, 0.82], [0.11, 0.16, 0.18] \rangle.
\end{array} \right.
\right.$$

Finally, the fuzzy number intuitionistic fuzzy soft set (FNIFSS) \( \langle F, A \rangle \), a parameterized family 

\[ \{ F(e_i), i = 1, 2, 3, 4, 5 \} \]

of fuzzy number intuitionistic fuzzy sets (FNIFSS) in \( X \) can be written as:

\[
\begin{align*}
\text{expensive} &= \left\{ \langle h_1, [0.30, 0.50, 0.60], [0.25, 0.30, 0.35] \rangle, \langle h_2, [0.30, 0.50, 0.60], [0.25, 0.30, 0.35] \rangle, \langle h_3, [0.30, 0.50, 0.60], [0.25, 0.30, 0.35] \rangle, \langle h_4, [0.30, 0.50, 0.60], [0.25, 0.30, 0.35] \rangle, \langle h_5, [0.30, 0.50, 0.60], [0.25, 0.30, 0.35] \rangle \right\}, \\
\text{beautiful} &= \left\{ \langle h_1, [0.60, 0.70, 0.80], [0.10, 0.15, 0.20] \rangle, \langle h_2, [0.80, 0.85, 0.90], [0.05, 0.07, 0.10] \rangle, \langle h_3, [0.65, 0.68, 0.78], [0.15, 0.19, 0.21] \rangle, \langle h_4, [0.50, 0.60, 0.80], [0.10, 0.15, 0.20] \rangle, \langle h_5, [0.68, 0.70, 0.82], [0.11, 0.16, 0.18] \rangle \right\}, \\
\text{wooden} &= \left\{ \langle h_1, [0.75, 0.80, 0.85], [0.10, 0.12, 0.15] \rangle, \langle h_2, [0.50, 0.55, 0.60], [0.10, 0.20, 0.30] \rangle, \langle h_3, [0.60, 0.70, 0.80], [0.10, 0.15, 0.20] \rangle, \langle h_4, [0.40, 0.50, 0.60], [0.05, 0.10, 0.30] \rangle, \langle h_5, [0.70, 0.75, 0.80], [0.10, 0.16, 0.18] \rangle \right\}, \\
\text{In good repair} &= \left\{ \langle h_1, [0.80, 0.85, 0.90], [0.02, 0.05, 0.10] \rangle, \langle h_2, [0.65, 0.70, 0.75], [0.10, 0.20, 0.24] \rangle, \langle h_3, [0.30, 0.50, 0.60], [0.25, 0.30, 0.35] \rangle, \langle h_4, [0.56, 0.61, 0.68], [0.10, 0.15, 0.20] \rangle, \langle h_5, [0.72, 0.76, 0.80], [0.05, 0.10, 0.20] \rangle \right\}, \\
\text{Green Surroundings} &= \left\{ \langle h_1, [0.77, 0.80, 0.88], [0.02, 0.06, 0.10] \rangle, \langle h_2, [0.30, 0.50, 0.70], [0.10, 0.15, 0.20] \rangle, \langle h_3, [0.60, 0.70, 0.73], [0.10, 0.20, 0.25] \rangle, \langle h_4, [0.63, 0.68, 0.76], [0.06, 0.10, 0.20] \rangle, \langle h_5, [0.70, 0.75, 0.80], [0.07, 0.10, 0.15] \rangle \right\}.
\end{align*}
\]

3.2. Null Fuzzy Number Intuitionistic Fuzzy Soft Set:

A fuzzy number intuitionistic fuzzy soft set \( \langle F, A \rangle \) over \( X \) is said to be a null fuzzy number intuitionistic fuzzy soft set denoted by \( \phi \), if \( \forall e \in A, t_{F(e)}(x) = [0, 0, 0] \) and \( f_{F(e)}(x) = [1, 1, 1] \), \( x \in X \).

3.3. Absolute Fuzzy Number Intuitionistic Fuzzy Soft Set:
A fuzzy number intuitionistic fuzzy soft set \( \{ F, A \} \) over \( X \) is said to be an absolute fuzzy number intuitionistic fuzzy soft set denoted by \( \sum \), if 
\[ \forall e \in A, \ t_{F(e)}(x) = [1,1,1] \quad \text{and} \quad f_{F(e)}(x) = [0,0,0], \ x \in X. \]

### 3.4. Not Set

The not set of a parameter set \( E = \{e_1, e_2, \ldots, e_n\} \) is denoted by \( \neg E \) and defined by 
\[ \neg E = \{-e_1, -e_2, \ldots, -e_n\} \text{ where } -e_i = \text{not } e_i. \]

### 3.5. Set operations on FNIFSSs

Based on the definition of fuzzy number intuitionistic fuzzy soft set theory, some basic operations like union, intersection complement etc., are define as below.

#### 3.5.1. Union of FNIFSSs

Let \( \{ F, A \} \) and \( \{ G, B \} \) be the two FNIFSSs on universal set \( X \) and \( A, B \subseteq E \), then union of two FNIFSSs denoted by \( \{ F, A \} \cup \{ G, B \} = \{ H, C \} \) where \( C = A \cup B \), \( \forall e \in C \) is also FNIFSSs and its degree of membership and non-membership are defined as

- \( t_{H(e)}(x) = \begin{cases} t_{F(e)}(x), & \text{if } e \in A - B, \ x \in X \\ t_{G(e)}(x), & \text{if } e \in B - A, \ x \in X \\ [\sup(t_{F(e)}(x), t_{G(e)}(x)), \sup(t_{F(e)}^2(x), t_{G(e)}^2(x)), \sup(t_{F(e)}^3(x), t_{G(e)}^3(x))], & \text{if } e \in A \cap B, \ x \in X \end{cases} \)
- \( f_{H(e)}(x) = \begin{cases} f_{F(e)}(x), & \text{if } e \in A - B, \ x \in X \\ f_{G(e)}(x), & \text{if } e \in B - A, \ x \in X \\ [\inf(f_{F(e)}(x), f_{G(e)}(x)), \inf(f_{F(e)}^2(x), f_{G(e)}^2(x)), \inf(f_{F(e)}^3(x), f_{G(e)}^3(x))], & \text{if } e \in A \cap B, \ x \in X \end{cases} \)

**Example 3.2.** Let \( \{ F, A \} \) and \( \{ G, B \} \) be the two fuzzy number intuitionistic fuzzy soft sets defined on universal set \( X = \{h_1, h_2, h_3, h_4, h_5, h_6\} \) containing the houses and \( E = \{e_1, e_2, e_3, e_4, e_5\} = \{\text{expensive, beautiful, wooden, in good repair, in good surroundings}\} \) be the set of parameters such that \( A, B \subseteq E \). Take \( A = \{e_1, e_2\} \) and \( B = \{e_1, e_2, e_3\} \) then the rating values of each house for a particular parameter is represented in the form of intuitionistic fuzzy numbers as:

\[
F(e_1) = \begin{cases} \{h_1,[0.30,0.50,0.60],[0.25,0.30,0.35]\}, \{h_2,[0.30,0.50,0.60],[0.25,0.30,0.35]\}, \\
\{h_3,[0.30,0.50,0.60],[0.25,0.30,0.35]\}, \{h_4,[0.30,0.50,0.60],[0.25,0.30,0.35]\}, \\
\{h_5,[0.30,0.50,0.60],[0.25,0.30,0.35]\}, \{h_6,[0.30,0.50,0.60],[0.25,0.30,0.35]\} \end{cases};
\]
\[
F(e_2) = \begin{cases} \{h_1,[0.60,0.70,0.80],[0.10,0.15,0.20]\}, \{h_2,[0.80,0.85,0.90],[0.05,0.07,0.07]\}, \\
\{h_3,[0.60,0.65,0.70],[0.10,0.20,0.25]\}, \{h_4,[0.50,0.60,0.80],[0.10,0.15,0.20]\}, \\
\{h_5,[0.65,0.68,0.78],[0.15,0.19,0.21]\}, \{h_6,[0.68,0.70,0.82],[0.11,0.16,0.18]\} \end{cases};
\]

and
Then the union is also a fuzzy number intuionistic fuzzy soft set denoted as $\langle H, C \rangle$ where $C = A \cup B$ and $\forall e \in C$ and defined as

$$G(e_1) = \begin{cases} \{h_1, [0.20,0.30,0.40], [0.20,0.23,0.30]\}, \{h_2, [0.50,0.55,0.60], [0.10,0.20,0.30]\} \\ \{h_3, [0.60,0.70,0.80], [0.10,0.15,0.20]\}, \{h_4, [0.40,0.50,0.60], [0.05,0.10,0.30]\} \\ \{h_5, [0.70,0.75,0.80], [0.10,0.16,0.18]\}, \{h_6, [0.40,0.45,0.50], [0.10,0.20,0.30]\} \end{cases}$$

$$G(e_2) = \begin{cases} \{h_1, [0.80,0.85,0.90], [0.02,0.05,0.10]\}, \{h_2, [0.80,0.86,0.92], [0.03,0.05,0.07]\} \\ \{h_3, [0.70,0.80,0.90], [0.05,0.08,0.10]\}, \{h_4, [0.50,0.58,0.68], [0.10,0.15,0.20]\} \\ \{h_5, [0.72,0.76,0.80], [0.05,0.10,0.20]\}, \{h_6, [0.70,0.80,0.90], [0.02,0.03,0.05]\} \end{cases}$$

$$G(e_3) = \begin{cases} \{h_1, [0.77,0.80,0.88], [0.02,0.06,0.10]\}, \{h_2, [0.30,0.50,0.70], [0.10,0.15,0.20]\} \\ \{h_3, [0.60,0.70,0.73], [0.10,0.20,0.25]\}, \{h_4, [0.63,0.68,0.76], [0.06,0.10,0.20]\} \\ \{h_5, [0.70,0.80,0.86], [0.04,0.10,0.12]\}, \{h_6, [0.70,0.75,0.80], [0.07,0.10,0.15]\} \end{cases}$$

### 3.5.2. Intersection of FNIFSSs

Let $\langle F, A \rangle$ and $\langle G, B \rangle$ be the two FNIFSSs on universal set $X$ and $A, B \subseteq E$, then intersection of two FNIFSSs denoted by $\langle F, A \rangle \cap \langle G, B \rangle = \langle K, C \rangle$ where $C = A \cap B$, $\forall e \in C$ is also FNIFSSs and its degree of membership and non-membership are defined as

$$t_{K(e)}(x) = \begin{cases} t_{F(e)}(x), & \text{if } e \in A - B, x \in X \\ t_{G(e)}(x), & \text{if } e \in B - A, x \in X \end{cases}$$

$$f_{K(e)}(x) = \begin{cases} f_{F(e)}(x), & \text{if } e \in A - B, x \in X \\ f_{G(e)}(x), & \text{if } e \in B - A, x \in X \end{cases}$$

and

$$[\inf(t_{F(e)}^1(x), t_{G(e)}^1(x)), \inf(t_{F(e)}^2(x), t_{G(e)}^2(x)), \inf(t_{F(e)}^3(x), t_{G(e)}^3(x))], \text{ if } e \in A \cap B, x \in X,$$

$\forall e \in C$ is given by

**Example 3.3.** Consider the data as presented in Example 3.2, then the intersection between FNIFSSs, denoted by $\langle F, A \rangle \cap \langle G, B \rangle = \langle K, C \rangle$ where $C = \{e_1, e_3\}$ is given by
3.5.3. Complement of FNIFSS

The complement of $\langle F, A \rangle$ is denoted by $\langle F, A \rangle^C$ and is defined by $\langle F, A \rangle^C = \langle F^C, A \rangle$, where $F^C : \rightarrow FNIFSS(X)$ is a mapping given by $F^C(\varepsilon) = (x, f_{F(\varepsilon)}(x), t_{F(\varepsilon)}(x))$.

Example 3.4. In order to illustrate the complement of FNIFSS, an example as described in Example 3.1. has been considered here. Then based on the rating values of each house for describing the “attractiveness of the house”, denoted by $\langle F, A \rangle$, the complement of it is given as:

$$\langle F, A \rangle^C = \begin{cases} \langle h_1, [0.25,0.30,0.35], [0.30,0.50,0.60], [0.30,0.50,0.60] \rangle, \\
\langle h_2, [0.20,0.32,0.35], [0.30,0.50,0.60], [0.30,0.50,0.60] \rangle, \\
\langle h_3, [0.21,0.30,0.3], [0.30,0.50,0.60], [0.30,0.50,0.60] \rangle, \\
\langle h_4, [0.25,0.30,0.35], [0.30,0.50,0.70], [0.30,0.50,0.60] \rangle, \\
\langle h_5, [0.10,0.15,0.20], [0.60,0.70,0.80], [0.80,0.85,0.90] \rangle, \\
\langle h_6, [0.10,0.15,0.20], [0.50,0.60,0.80], [0.60,0.65,0.70] \rangle, \\
\langle h_7, [0.15,0.19,0.21], [0.65,0.68,0.78], [0.68,0.70,0.82] \rangle, \\
\langle h_8, [0.11,0.16,0.18], [0.68,0.70,0.82], [0.60,0.55,0.60] \rangle, \\
\langle h_9, [0.10,0.12,0.15], [0.75,0.80,0.85], [0.50,0.55,0.60] \rangle, \\
\langle h_{10}, [0.10,0.20,0.30], [0.50,0.55,0.60], [0.60,0.70,0.80] \rangle, \\
\langle h_{11}, [0.10,0.15,0.20], [0.50,0.55,0.60], [0.40,0.50,0.60] \rangle, \\
\langle h_{12}, [0.10,0.16,0.18], [0.70,0.75,0.80], [0.70,0.75,0.80] \rangle, \\
\langle h_{13}, [0.10,0.20,0.40], [0.60,0.70,0.80], [0.60,0.70,0.80] \rangle, \\
\langle h_{14}, [0.02,0.05,0.10], [0.80,0.85,0.90], [0.65,0.70,0.75] \rangle, \\
\langle h_{15}, [0.02,0.05,0.10], [0.80,0.85,0.90], [0.65,0.70,0.75] \rangle, \\
\langle h_{16}, [0.05,0.10,0.20], [0.72,0.76,0.80], [0.70,0.80,0.90] \rangle, \\
\langle h_{17}, [0.05,0.10,0.20], [0.72,0.76,0.80], [0.70,0.80,0.90] \rangle, \\
\langle h_{18}, [0.77,0.80,0.88], [0.02,0.06,0.10], [0.10,0.15,0.20] \rangle, \\
\langle h_{19}, [0.77,0.80,0.88], [0.02,0.06,0.10], [0.10,0.15,0.20] \rangle, \\
\langle h_{20}, [0.04,0.10,0.12], [0.70,0.80,0.86], [0.70,0.75,0.80] \rangle, \\
\langle h_{21}, [0.04,0.10,0.12], [0.70,0.80,0.86], [0.70,0.75,0.80] \rangle \end{cases}$$

3.6. Inclusion

Let $X$ be the universe of discourse and $E$ be the set of parameters. Suppose $A, B \subseteq E$, $\langle F, A \rangle$ and $\langle G, B \rangle$ be the two fuzzy number intuitionistic fuzzy soft sets, then $\langle F, A \rangle$ is said to be the subset of $\langle G, B \rangle$ if and only if

a) $A \subseteq B$;
b) \( \forall e \in A, F(e) \) is a fuzzy number intuitionistic fuzzy subset of \( G(e) \) i.e. for all \( x \in X \) and \( e \in A \), \( t_A(x) \leq t_B(x), f_A(x) \geq f_B(x) \) and denoted by \( \langle F, A \rangle \in \langle G, B \rangle \), where, \( t_A(x) = (t^1_A(x), t^2_A(x), t^3_A(x)) \), \( f_A(x) = (f^1_A(x), f^2_A(x), f^3_A(x)) \).

Similarly, \( \langle F, A \rangle \) is said to be the superset of \( \langle G, B \rangle \) if and only if
a) \( A \supseteq B \);

b) \( \forall e \in A, F(e) \) is a fuzzy number intuitionistic fuzzy superset of \( G(e) \) i.e. for all \( x \in X \) and \( e \in A \), \( t_A(x) \geq t_B(x), f_A(x) \leq f_B(x) \) and denoted by \( \langle F, A \rangle \supseteq \langle G, B \rangle \), where, \( t_B(x) = (t^1_B(x), t^2_B(x), t^3_B(x)) \) and \( f_B(x) = (f^1_B(x), f^2_B(x), f^3_B(x)) \).

### 3.7. Equal set

Two fuzzy number intuitionistic fuzzy softs \( \langle F, A \rangle \) and \( \langle G, B \rangle \) are said to be equal sets denoted by \( \langle F, A \rangle = \langle G, B \rangle \) if and only if
a) \( \langle F, A \rangle \) is a fuzzy number intuitionistic fuzzy soft subset of \( \langle G, B \rangle \).

b) \( \langle G, B \rangle \) is a fuzzy number intuitionistic fuzzy soft subset of \( \langle F, A \rangle \).

### 4 Properties on set operations

Here, we derive some properties on operations defined in the above section. Let \( \langle F, A \rangle \) and \( \langle G, B \rangle \) be the two fuzzy number intuitionistic fuzzy soft sets on universal set \( X \), then we have the following.

**Theorem 4.1. Demorgan’s law**

I. \( \langle \langle F, A \rangle \cup \langle G, B \rangle \rangle^C = \langle F, A \rangle^C \cap \langle G, B \rangle^C \)

II. \( \langle \langle F, A \rangle \cap \langle G, B \rangle \rangle^C = \langle F, A \rangle^C \cup \langle G, B \rangle^C \)

**Proof.** I. Let \( \langle F, A \rangle \cup \langle G, B \rangle = \langle H, C \rangle \), where \( C = A \cup B \), \( \forall e \in C \), then

\[
t_{H(e)}(x) = \begin{cases} t_{F(e)}(x), & \text{if } e \in A - B, \ x \in X \\
t_{G(e)}(x), & \text{if } e \in B - A, \ x \in X \\
[\sup(t^1_{F(e)}(x), t^1_{G(e)}(x)), \sup(t^2_{F(e)}(x), t^2_{G(e)}(x)), \sup(t^3_{F(e)}(x), t^3_{G(e)}(x))] & \text{if } e \in A \cap B, \ x \in X, \end{cases}
\]

and

\[
f_{H(e)}(x) = \begin{cases} f_{F(e)}(x), & \text{if } e \in A - B, \ x \in X \\
f_{G(e)}(x), & \text{if } e \in B - A, \ x \in X \\
[\inf(f^1_{F(e)}(x), f^1_{G(e)}(x)), \inf(f^2_{F(e)}(x), f^2_{G(e)}(x)), \inf(f^3_{F(e)}(x), f^3_{G(e)}(x))] & \text{if } e \in A \cap B, \ x \in X. \end{cases}
\]

Since \( \langle F, A \rangle \cup \langle G, B \rangle = \langle H, C \rangle \), we have \( \langle \langle F, A \rangle \cup \langle G, B \rangle \rangle^C = \langle H, C \rangle^C = \langle H^C, -C \rangle \), where \( H^C(-e) = \{x, f_{H(-e)}(x), t_{H(-e)}(x)\} \forall x \in X \) and \( -e \in -C = -(A \cup B) = -A \cap -B. \) Hence,
\[
\begin{align*}
t_{H^{(e)}}(x) &= \begin{cases} 
  f_{F(e)}(x), & \text{if } e \in A - B, \ x \in X \\
  f_{G(e)}(x), & \text{if } e \in B - A, \ x \in X
\end{cases}, \\
\{\inf(f_{F(e)}^1(x), f_{G(e)}^1(x)), \inf(f_{F(e)}^2(x), f_{G(e)}^2(x)), \inf(f_{F(e)}^3(x), f_{G(e)}^3(x))\}, & \text{if } e \in A \cap B, \ x \in X,
\end{align*}
\]
and
\[
\begin{align*}
t_{H^{(e)}}(x) &= \begin{cases} 
  t_{F(e)}(x), & \text{if } e \in A - B, \ x \in X \\
  t_{G(e)}(x), & \text{if } e \in B - A, \ x \in X
\end{cases}, \\
\{\sup(t_{F(e)}^1(x), t_{G(e)}^1(x)), \sup(t_{F(e)}^2(x), t_{G(e)}^2(x)), \sup(t_{F(e)}^3(x), t_{G(e)}^3(x))\}, & \text{if } e \in A \cap B, \ x \in X,
\end{align*}
\]

Since \( \{F, A\}^C = \{F^C, -A\} \) and \( \{G, B\}^C = \{G^C, -B\} \), we have
\[
\{F, A\}^C \cap \{G, B\}^C = \{F^C, -A\} \cap \{G^C, -B\}.
\]
Suppose that \( \{F^C, -A\} \cap \{G^C, -B\} = \{O, D\} \) and
\( \neg e \in D \), where \( D = \neg C = \neg A \cap -B \); then
\[
\begin{align*}
t_{F(e)}(x) &= \begin{cases} 
  f_{F(e)}(x), & \text{if } -e \in \neg A - B, \ x \in X \\
  f_{G(e)}(x), & \text{if } -e \in -B - A, \ x \in X
\end{cases}, \\
\{\inf(f_{F(e)}^1(x), f_{G(e)}^1(x)), \inf(f_{F(e)}^2(x), f_{G(e)}^2(x)), \inf(f_{F(e)}^3(x), f_{G(e)}^3(x))\}, & \text{if } -e \in \neg A \cap -B, \ x \in X,
\end{align*}
\]
and
\[
\begin{align*}
t_{G(e)}(x) &= \begin{cases} 
  f_{F(e)}(x), & \text{if } e \in A - B, \ x \in X \\
  f_{G(e)}(x), & \text{if } e \in B - A, \ x \in X
\end{cases}, \\
\{\inf(f_{F(e)}^1(x), f_{G(e)}^1(x)), \inf(f_{F(e)}^2(x), f_{G(e)}^2(x)), \inf(f_{F(e)}^3(x), f_{G(e)}^3(x))\}, & \text{if } e \in A \cap B, \ x \in X,
\end{align*}
\]

Hence, \( \{F, A\} \cup \{G, B\}\)^C = \(\{F, A\}^C \cap \{G, B\}^C\)

Proof. II. can be proved analogously.

**Theorem 4.2. Associativity**

For three FNIFSSs \( \{F, A\}, \{G, B\} \) and \( \{H, C\} \), we have

I. \( \{F, A\} \cap \{G, B\} \cap \{H, C\} = \{F, A\} \cap \{G, B\} \cap \{H, C\} \);

II. \( \{F, A\} \cup \{G, B\} \cup \{H, C\} = \{F, A\} \cup \{G, B\} \cup \{H, C\} \).

**Proof.** I. Let \( \{G, B\} \cap \{H, C\} = \{I, S\} \), where \( S = B \cap C \) and \( \forall e \in S \), then membership and non-membership degree of \( \{I, S\} \) is defined as
Let $t_{i(e)}(x) = \begin{cases} t_{G(e)}(x), & \text{if } e \in B - C, \ x \in X \\ t_{H(e)}(x), & \text{if } e \in C - B, \ x \in X \end{cases}$ for $i \in I \cup J \cup K$. Then membership and non-membership degrees of $e I$, $e J$, and $e K$ are given as

$$
\begin{align*}
\inf(t_{G(e)}(x), t_{H(e)}(x)), \ inf(t_{G(e)}(x), t_{H(e)}(x)), \ inf(t_{G(e)}(x), t_{H(e)}(x)), \ inf(t_{G(e)}(x), t_{H(e)}(x)), & \text{ if } e \in B \cap C, \ x \in X, \\
\sup(f_{G(e)}(x), f_{H(e)}(x)), \ sup(f_{G(e)}(x), f_{H(e)}(x)), \ sup(f_{G(e)}(x), f_{H(e)}(x)), \ sup(f_{G(e)}(x), f_{H(e)}(x)), & \text{ if } e \in B \cap C, \ x \in X,
\end{align*}
$$

So we have the degrees of membership and non-membership as follows:

$$
\begin{align*}
t_{G(e)}(x), & \text{ if } e \in B - C - A, \ x \in X \\
t_{H(e)}(x), & \text{ if } e \in C - B - A, \ x \in X \\
t_{F(e)}(x), & \text{ if } e \in A - B - C, \ x \in X.
\end{align*}
$$

On the other hand, let $\langle F, A \rangle \cap \langle G, B \rangle = \langle K, V \rangle$, where $V = A \cap B$ and $\forall e \in V$, then membership and non-membership degrees of $\langle K, V \rangle$ is given as

$$
\begin{align*}
t_{K(e)}(x) = \begin{cases} t_{F(e)}(x), & \text{if } e \in A - B, \ x \in X \\ t_{G(e)}(x), & \text{if } e \in B - A, \ x \in X \end{cases},
\end{align*}
$$

and

$$
\begin{align*}
f_{K(e)}(x) = \begin{cases} f_{F(e)}(x), & \text{if } e \in A - B, \ x \in X \\ f_{G(e)}(x), & \text{if } e \in B - A, \ x \in X \end{cases}.
\end{align*}
$$
Since \( \langle F, A \rangle \cap \langle G, B \rangle \cap \langle H, C \rangle = \langle K, V \rangle \cap \langle H, C \rangle = \langle L, W \rangle \), where, \( W = V \cap C = A \cap B \cap C \), we have the following

\[
\begin{align*}
t_{L(e)}(x) &= \left\{ \begin{array}{ll}
t_{G(e)}(x), & \text{if } e \in B - C - A, \ x \in X \\
t_{H(e)}(x), & \text{if } e \in C - B - A, \ x \in X \\
t_{F(e)}(x), & \text{if } e \in A - B - C, \ x \in X \\
\{ \inf(t_{G(e)}^1(x), t_{H(e)}^1(x)), \inf(t_{G(e)}^2(x), t_{H(e)}^2(x)), \inf(t_{G(e)}^3(x), t_{H(e)}^3(x)), \} & \text{if } e \in B \cap C - A, \ x \in X, \\
\{ \inf(t_{F(e)}^1(x), t_{H(e)}^1(x)), \inf(t_{F(e)}^2(x), t_{H(e)}^2(x)), \inf(t_{F(e)}^3(x), t_{H(e)}^3(x)), \} & \text{if } e \in A \cap C - B, \ x \in X, \\
\{ \inf(t_{F(e)}^1(x), t_{G(e)}^1(x)), \inf(t_{F(e)}^2(x), t_{G(e)}^2(x)), \inf(t_{F(e)}^3(x), t_{G(e)}^3(x)), \} & \text{if } e \in A \cap B - C, \ x \in X, \\
\{ \inf(t_{F(e)}^1(x), t_{H(e)}^1(x)), \inf(t_{F(e)}^2(x), t_{H(e)}^2(x)), \inf(t_{F(e)}^3(x), t_{H(e)}^3(x)), \} & \text{if } e \in A \cap B \cap C, \ x \in X \\
\end{array} \right. \\
f_{L(e)}(x) &= \left\{ \begin{array}{ll}
f_{G(e)}(x), & \text{if } e \in B - C - A, \ x \in X \\
f_{H(e)}(x), & \text{if } e \in C - B - A, \ x \in X \\
f_{F(e)}(x), & \text{if } e \in A - B - C, \ x \in X \\
\{ \sup(f_{G(e)}^1(x), f_{H(e)}^1(x)), \sup(f_{G(e)}^2(x), f_{H(e)}^2(x)), \sup(f_{G(e)}^3(x), f_{H(e)}^3(x)), \} & \text{if } e \in B \cap C - A, \ x \in X, \\
\{ \sup(f_{F(e)}^1(x), f_{H(e)}^1(x)), \sup(f_{F(e)}^2(x), f_{H(e)}^2(x)), \sup(f_{F(e)}^3(x), f_{H(e)}^3(x)), \} & \text{if } e \in A \cap C - B, \ x \in X, \\
\{ \sup(f_{F(e)}^1(x), f_{G(e)}^1(x)), \sup(f_{F(e)}^2(x), f_{G(e)}^2(x)), \sup(f_{F(e)}^3(x), f_{G(e)}^3(x)), \} & \text{if } e \in A \cap B - C, \ x \in X, \\
\{ \sup(f_{F(e)}^1(x), f_{H(e)}^1(x)), \sup(f_{F(e)}^2(x), f_{H(e)}^2(x)), \sup(f_{F(e)}^3(x), f_{H(e)}^3(x)), \} & \text{if } e \in A \cap B \cap C, \ x \in X \\
\end{array} \right. 
\end{align*}
\]

Clearly, \( t_{J(e)}(x) = t_{L(e)}(x) \) and \( f_{J(e)}(x) = f_{L(e)}(x) \) \( \forall e \in A \cup B \cup C, x \in X \). i.e. \( J \) and \( L \) are the equivalent sets.

Hence, \( \langle F, A \rangle \cap \langle G, B \rangle \cap \langle H, C \rangle = \langle (F, A) \cap \langle G, B \rangle \cap \langle H, C \rangle \); \( \langle (F, A) \cap \langle G, B \rangle \cap \langle H, C \rangle \); \( \langle (F, A) \cap \langle G, B \rangle \cap \langle H, C \rangle \);

**Proof.** II. can be proved analogously.

5 Max-Min Operators

In this section, we propose two operators named as max, min operators and derive some properties on fuzzy number intuitionistic fuzzy soft set theory.

If \( \langle F, A \rangle \) and \( \langle G, B \rangle \) be the two fuzzy number intuitionistic fuzzy soft sets defined on universal set \( X \), then

\( \langle F, A \rangle \) and \( \langle G, B \rangle \) denoted by \( \langle F, A \rangle \cap \langle G, B \rangle \), called as min operator, is also a fuzzy number intuitionistic fuzzy soft set and is defined by \( \langle F, A \rangle \cap \langle G, B \rangle = (H, A \times B) \), where \( H(\alpha, \beta) = F(\alpha) \cap G(\beta), \forall (\alpha, \beta) \in A \times B \), that is,

\[
H(\alpha, \beta)(x) = \left\{ x, \{ \inf(f_{F(\alpha)}^1(x), f_{G(\beta)}^1(x)), \inf(f_{F(\alpha)}^2(x), f_{G(\beta)}^2(x)), \inf(f_{F(\alpha)}^3(x), f_{G(\beta)}^3(x)), \} \right\} 
\]

\[
\{ \sup(f_{F(\alpha)}^1(x), f_{H(\beta)}^1(x)), \sup(f_{F(\alpha)}^2(x), f_{H(\beta)}^2(x)), \sup(f_{F(\alpha)}^3(x), f_{H(\beta)}^3(x)), \} \}
\]
Theorem 5.1. Demorgan’s law

I. \( (\langle F, A \rangle \land \langle G, B \rangle)^c = \langle F, A \rangle^c \lor \langle G, B \rangle^c \).

II. \( (\langle F, A \rangle \lor \langle G, B \rangle)^c = \langle F, A \rangle^c \land \langle G, B \rangle^c \).

Proof. I. Suppose that \( \langle F, A \rangle \land \langle G, B \rangle = \langle H, A \times B \rangle \) and
\[
\left\langle \langle F, A \rangle \land \langle G, B \rangle \right\rangle^c = \langle H, A \times B \rangle^c = \langle H^c, -(A \times B) \rangle.
\]
We have,
\[
H^c(-\alpha, -\beta)(x) = \left\{ x, \left\{ \sup(f_{F(-\alpha)}(x), f_{G(-\beta)}(x)), \sup(f_{F(-\alpha)}^2(x), f_{G(-\beta)}^2(x)), \sup(f_{F(-\alpha)}^3(x), f_{G(-\beta)}^3(x)) \right\}, \left\{ \inf(t_{F(-\alpha)}(x), t_{G(-\beta)}(x)), \inf(t_{F(-\alpha)}^2(x), t_{G(-\beta)}^2(x)), \inf(t_{F(-\alpha)}^3(x), t_{G(-\beta)}^3(x)) \right\} \right\}
\]
Since \( \langle F, A \rangle^c = \langle F^c, -A \rangle \) and \( \langle G, B \rangle^c = \langle G^c, -B \rangle \) implies
\[
F^c(-\alpha) = \left\{ x, f_{F(-\alpha)}(x), t_{F(-\alpha)}(x) \right\} \quad \text{and} \quad G^c(-\beta) = \left\{ x, f_{G(-\beta)}(x), t_{G(-\beta)}(x) \right\},
\]
then by definition the join of these two will be denoted as \( \langle F, A \rangle^c \lor \langle G, B \rangle^c = \langle F^c, -A \rangle \lor \langle G^c, -B \rangle = \langle O, -(A \times B) \rangle \). Hence
\[
O(-\alpha, -\beta)(x) = \left\{ x, \left\{ \sup(f_{F(-\alpha)}^1(x), f_{G(-\beta)}^1(x)), \sup(f_{F(-\alpha)}^2(x), f_{G(-\beta)}^2(x)), \sup(f_{F(-\alpha)}^3(x), f_{G(-\beta)}^3(x)) \right\}, \left\{ \inf(t_{F(-\alpha)}^1(x), t_{G(-\beta)}^1(x)), \inf(t_{F(-\alpha)}^2(x), t_{G(-\beta)}^2(x)), \inf(t_{F(-\alpha)}^3(x), t_{G(-\beta)}^3(x)) \right\} \right\}
\]
Clearly, \( \langle H^c, -(A \times B) \rangle = \langle O, -(A \times B) \rangle \). Consequently, \( H^c \) and \( O \) are the equivalent operators.

Hence, \( \left\langle \langle F, A \rangle \land \langle G, B \rangle \right\rangle^c = \langle F, A \rangle^c \lor \langle G, B \rangle^c \)

Proof. II. Similarly, the result \( \left\langle \langle F, A \rangle \lor \langle G, B \rangle \right\rangle^c = \langle F, A \rangle^c \land \langle G, B \rangle^c \) holds.

Theorem 5.2. Associatively:

I. \( \langle F, A \rangle \land \left( \langle G, B \rangle \land \langle H, C \rangle \right) = \langle F, A \rangle \land \langle G, B \rangle \land \langle H, C \rangle \).

II. \( \langle F, A \rangle \lor \left( \langle G, B \rangle \lor \langle H, C \rangle \right) = \langle F, A \rangle \lor \langle G, B \rangle \lor \langle H, C \rangle \).

Proof. I. Let \( \langle G, B \rangle \land \langle H, C \rangle = \langle I, B \times C \rangle \), where, \( I(\beta, \delta) = G(\beta) \cap H(\delta), \forall (\beta, \delta) \in B \times C \), then
\[ I(\beta, \delta)(x) = \begin{cases} x, & \{\inf(t^1_{G(\beta)}(x), t^1_{H(\delta)}(x)), \inf(t^2_{G(\beta)}(x), t^2_{H(\delta)}(x)), \inf(t^3_{G(\beta)}(x), t^3_{H(\delta)}(x))\}, \\ \{\sup(f^1_{\bar{G}(\beta)}(x), f^1_{H(\delta)}(x)), \sup(f^2_{\bar{G}(\beta)}(x), f^2_{H(\delta)}(x)), \sup(f^3_{\bar{G}(\beta)}(x), f^3_{H(\delta)}(x))\}, \end{cases} \]

Assume \( F(A) \land (G, B) \land (H, C) = F(A) \land (I, B \times C) = (J, A \times (B \times C)) \), where \( J(\alpha, \beta, \delta) = F(\alpha) \cap I(\beta, \delta) \) and \((\alpha, \beta, \delta) \in A \times (B \times C) \). Hence

\[ J(\alpha, \beta, \delta)(x) = \begin{cases} x, & \{\inf(t^1_{F(\alpha)}(x), t^1_{I(\beta, \delta)}(x)), \inf(t^2_{F(\alpha)}(x), t^2_{I(\beta, \delta)}(x)), \inf(t^3_{F(\alpha)}(x), t^3_{I(\beta, \delta)}(x))\}, \\ \{\sup(f^1_{F(\alpha)}(x), f^1_{I(\beta, \delta)}(x)), \sup(f^2_{F(\alpha)}(x), f^2_{I(\beta, \delta)}(x)), \sup(f^3_{F(\alpha)}(x), f^3_{I(\beta, \delta)}(x))\}, \end{cases} \]

Again considering \( F(A) \land (G, B) = (K, A \times B) \), where \( K(\alpha, \beta) = F(\alpha) \cap G(\beta), \forall (\alpha, \beta) \in A \times B \), then

\[ J(\alpha, \beta)(x) = \begin{cases} x, & \{\inf(t^1_{K(\alpha, \beta)}(x), t^1_{G(\beta)}(x)), \inf(t^2_{K(\alpha, \beta)}(x), t^2_{G(\beta)}(x)), \inf(t^3_{K(\alpha, \beta)}(x), t^3_{G(\beta)}(x))\}, \\ \{\sup(f^1_{K(\alpha, \beta)}(x), f^1_{G(\beta)}(x)), \sup(f^2_{K(\alpha, \beta)}(x), f^2_{G(\beta)}(x)), \sup(f^3_{K(\alpha, \beta)}(x), f^3_{G(\beta)}(x))\}, \end{cases} \]

and \( (F, A) \land (G, B) \land (H, C) = (K, A \times B) \land (H, C) = (L, (A \times B) \times C) \), where \( L(\alpha, \beta, \delta) = K(\alpha, \beta) \cap (H(\delta) \land (\alpha, \beta, \delta) \in (A \times B) \times C = A \times B \times C \).

Hence,

\[ L(\alpha, \beta, \delta)(x) = \begin{cases} x, & \{\inf(t^1_{K(\alpha, \beta)}(x), t^1_{H(\delta)}(x)), \inf(t^2_{K(\alpha, \beta)}(x), t^2_{H(\delta)}(x)), \inf(t^3_{K(\alpha, \beta)}(x), t^3_{H(\delta)}(x))\}, \\ \{\sup(f^1_{K(\alpha, \beta)}(x), f^1_{H(\delta)}(x)), \sup(f^2_{K(\alpha, \beta)}(x), f^2_{H(\delta)}(x)), \sup(f^3_{K(\alpha, \beta)}(x), f^3_{H(\delta)}(x))\}, \end{cases} \]

Therefore, \( F(A) \land (G, B) \land (H, C) = (F, A) \land (G, B) \land (H, C) \) holds.

Similarly, we can prove the other part.

**Theorem 5.3. Idempotent Laws**

I. \( F(A) \cup (F, A) = (F, A) \);

II. \( F(A) \cap (F, A) = (F, A) \).

**Theorem 5.4. Identity Laws:**

I. \( F(A) \cup \emptyset = (F, A) \);

II. \( F(A) \cap \emptyset = \emptyset \);

III. \( (F, A) \cup \Sigma = \Sigma \);

IV. \( (F, A) \cap \Sigma = (F, A) \).

**Proof.** By using the definitions 3.2 and 3.3 we can easily obtain Theorem 5.3 and 5.4.
6 Illustrative example

Let \( \langle F, A \rangle \) and \( \langle G, B \rangle \) be the two fuzzy number intuitionistic fuzzy soft sets defined on universal set \( X = \{ h_1, h_2, h_3, h_4, h_5, h_6 \} \) containing the houses and \( E = \{ e_1, e_2, e_3, e_4, e_5 \} = \{ \text{expensive, beautiful, wooden, in good repair, in good surroundings} \} \) be the set of parameters such that \( A, B \subseteq E \) where, \( A = \{ e_1, e_2 \} \) and \( B = \{ e_1, e_2, e_3 \} \). Then, the FNIFSS for individual parameters are given as:

\[
\langle F, A \rangle = \begin{cases} 
\{ (h_1, [0.20,0.30,0.40],[0.25,0.35,0.45]), (h_2, [0.10,0.20,0.30],[0.35,0.45,0.55]) \} \\
\{ (h_3, [0.30,0.40,0.50],[0.10,0.20,0.30]), (h_4, [0.15,0.20,0.25],[0.25,0.35,0.45]) \} \\
\{ (h_5, [0.05,0.10,0.15],[0.20,0.30,0.40]), (h_6, [0.20,0.30,0.45],[0.30,0.40,0.50]) \}
\end{cases}
\]

\[
e_1 = \begin{cases} 
\{ (h_1, [0.05,0.10,0.20],[0.20,0.30,0.40]), (h_2, [0.10,0.15,0.25],[0.25,0.35,0.45]) \} \\
\{ (h_3, [0.20,0.30,0.40],[0.10,0.15,0.25]), (h_4, [0.30,0.40,0.50],[0.10,0.20,0.30]) \} \\
\{ (h_5, [0.15,0.25,0.40],[0.10,0.20,0.35]), (h_6, [0.05,0.15,0.25],[0.30,0.40,0.50]) \}
\end{cases}
\]

\[
\langle G, B \rangle = \begin{cases} 
\{ (h_1, [0.10,0.20,0.30],[0.30,0.40,0.50]), (h_2, [0.15,0.20,0.25],[0.30,0.40,0.45]) \} \\
\{ (h_3, [0.05,0.10,0.20],[0.20,0.30,0.40]), (h_4, [0.10,0.15,0.25],[0.25,0.35,0.45]) \} \\
\{ (h_5, [0.15,0.20,0.25],[0.40,0.50,0.60]), (h_6, [0.20,0.30,0.40],[0.30,0.40,0.50]) \}
\end{cases}
\]

\[
e_2 = \begin{cases} 
\{ (h_1, [0.20,0.30,0.40],[0.30,0.40,0.50]), (h_2, [0.10,0.20,0.30],[0.40,0.50,0.60]) \} \\
\{ (h_3, [0.10,0.15,0.25],[0.20,0.25,0.35]), (h_4, [0.20,0.25,0.30],[0.30,0.40,0.50]) \} \\
\{ (h_5, [0.05,0.10,0.15],[0.30,0.40,0.50]), (h_6, [0.10,0.20,0.25],[0.40,0.50,0.60]) \}
\end{cases}
\]

\[
e_3 = \begin{cases} 
\{ (h_1, [0.20,0.25,0.30],[0.30,0.40,0.50]), (h_2, [0.15,0.20,0.25],[0.20,0.30,0.40]) \} \\
\{ (h_3, [0.10,0.20,0.30],[0.40,0.50,0.60]), (h_4, [0.10,0.20,0.30],[0.15,0.25,0.35]) \}
\end{cases}
\]

Then the union \( \langle F, A \rangle \cup \langle G, B \rangle \), denoted by \( \langle H, C \rangle \) where \( C = A \cup B \) and \( \forall e \in C \) and defined as

\[
\langle H, C \rangle = \begin{cases} 
\{ (h_1, [0.20,0.30,0.40],[0.25,0.35,0.45]), (h_2, [0.15,0.20,0.30],[0.30,0.40,0.45]) \} \\
\{ (h_3, [0.30,0.40,0.50],[0.10,0.20,0.30]), (h_4, [0.15,0.20,0.25],[0.25,0.35,0.45]) \} \\
\{ (h_5, [0.15,0.20,0.25],[0.20,0.30,0.40]), (h_6, [0.20,0.30,0.45],[0.30,0.40,0.50]) \}
\end{cases}
\]

\[
e_1 = \begin{cases} 
\{ (h_1, [0.20,0.30,0.40],[0.20,0.30,0.40]), (h_2, [0.10,0.20,0.30],[0.25,0.35,0.45]) \} \\
\{ (h_3, [0.20,0.30,0.40],[0.10,0.15,0.25]), (h_4, [0.30,0.40,0.50],[0.10,0.20,0.30]) \} \\
\{ (h_5, [0.15,0.25,0.40],[0.10,0.20,0.25]), (h_6, [0.10,0.20,0.25],[0.30,0.40,0.50]) \}
\end{cases}
\]

\[
e_2 = \begin{cases} 
\{ (h_1, [0.20,0.30,0.40],[0.20,0.30,0.40]), (h_2, [0.10,0.20,0.30],[0.25,0.35,0.45]) \} \\
\{ (h_3, [0.20,0.30,0.40],[0.10,0.15,0.25]), (h_4, [0.30,0.40,0.50],[0.10,0.20,0.30]) \} \\
\{ (h_5, [0.15,0.25,0.35],[0.20,0.30,0.40]), (h_6, [0.25,0.30,0.35],[0.30,0.40,0.50]) \}
\end{cases}
\]

\[
e_3 = \begin{cases} 
\{ (h_1, [0.20,0.25,0.30],[0.30,0.40,0.50]), (h_2, [0.15,0.20,0.25],[0.20,0.30,0.40]) \} \\
\{ (h_3, [0.10,0.20,0.30],[0.40,0.50,0.60]), (h_4, [0.10,0.20,0.30],[0.15,0.25,0.35]) \}
\end{cases}
\]

And hence \( \langle \langle F, A \rangle \cup \langle G, B \rangle \rangle \) = \( \langle H^C, \neg C \rangle \) and is given by
Now, denote $\langle F, A \rangle^C \cap \langle G, B \rangle^C$ to be $\langle I, D \rangle$ where $D = \neg A \cap \neg B$ and is defined as
\[
\begin{align*}
-\varepsilon_1 &= \{h_1, [0.25, 0.35, 0.45], [0.20, 0.30, 0.40], h_2, [0.30, 0.40, 0.45], [0.15, 0.20, 0.30]\}, \\
-\varepsilon_2 &= \{h_1, [0.20, 0.30, 0.40], [0.20, 0.30, 0.40], h_2, [0.25, 0.35, 0.45], [0.10, 0.20, 0.30]\}, \\
-\varepsilon_3 &= \{h_1, [0.20, 0.30, 0.40], [0.15, 0.25, 0.35], h_2, [0.30, 0.40, 0.50], [0.25, 0.30, 0.35]\}, \\
-\varepsilon_4 &= \{h_1, [0.30, 0.40, 0.50], [0.20, 0.25, 0.30], h_2, [0.20, 0.30, 0.40], [0.15, 0.20, 0.25]\}, \\
-\varepsilon_5 &= \{h_1, [0.40, 0.50, 0.60], [0.10, 0.20, 0.30], h_2, [0.15, 0.25, 0.35], [0.10, 0.20, 0.30]\},
\end{align*}
\]

Therefore, \(\{F, A\} \cup \{G, B\}^c = \{F, A\}^c \cap \{G, B\}^c\)

7 Conclusion

In this paper, the notion of fuzzy number intuitionistic fuzzy soft sets (FNIFSSs) has been proposed as a combination of soft set theory with fuzzy number intuitionistic fuzzy set. The presented theory generalizes of many theories like the fuzzy soft set theory, interval-valued intuitionistic fuzzy set theory, intuitionistic fuzzy soft set theory and interval-valued intuitionistic fuzzy soft set theory, etc. Various operations like union, intersection, compliments, max, min etc. are presented in FNIFSS environment and their corresponding properties. Numerical examples are shown to elaborate the operations and its properties. In future, we are engaged to extend this theory to some other domains.

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