Ranking Exponential Trapezoidal Fuzzy Numbers by Median Value

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Abstract
In this paper, we want represented a method for ranking of two exponential trapezoidal fuzzy numbers. A median value is proposed for the ranking of exponential trapezoidal fuzzy numbers. For the validation the results of the proposed approach are compared with different existing approaches.

Keywords: Exponential Trapezoidal Fuzzy Numbers, Median Value, Ranking Method.

1 Introduction

In this paper, we want represented a method for ranking of two exponential trapezoidal fuzzy numbers. A median value is proposed for the ranking of exponential trapezoidal fuzzy numbers. For the validation the results of the proposed approach are compared with different existing approaches.

2 Preliminaries

Definition 2.1. Generally, a generalized fuzzy number \(A\) is described as any fuzzy subset of the real line \(R\), whose membership function \(\mu_A\) satisfies the following conditions,
We have all the above definitions apply to exponential trapezoidal fuzzy numbers.

Proposition 3.1. \[ f_A(x) = \begin{cases} \frac{we^{-(b-x)/(b-a)}}{w} & a \leq x \leq b, \\
\frac{we^{-(x-c)/(d-c)}}{d} & c \leq x \leq d, 
\end{cases} \quad (2.1) \]

\[ L(x) = we^{-(b-x)/(b-a)}, \quad R(x) = we^{-(x-c)/(d-c)} \quad (2.2) \]

3 Proposed Approach

In this section some important results, that are useful for the proposed approach, are proved.

Definition 3.1. Cardinality of a fuzzy number \( A \) is the value of the integral [3]

\[
\text{card } A = \int_a^b A(x) \, dx = \int_0^1 (b_\alpha - a_\alpha) \, d\alpha \quad (3.3)
\]

Definition 3.2. The median value of a fuzzy number \( A \) is the real number \( m_A \) from the support of \( A \) such that [3]

\[
\int_a^{m_A} A(x) \, dx = \int_{m_A}^d A(x) \, dx \quad (3.4)
\]

Proposition 3.1. If \( A = (a, b, c, d) \) is a fuzzy number with light tails then [3]

\[
m_A = \frac{a + b}{2} + 0.5 \left( \int_c^d A(x) \, dx - \int_a^b A(x) \, dx \right) \quad (3.5)
\]

We have all the above definitions apply to exponential trapezoidal fuzzy numbers.

Theorem 3.1. Cardinality of an exponential trapezoidal fuzzy number \( A \) characterized by (2.1) is the value of the integral

\[
\text{card } A = w(c - b) + \frac{w}{e} ((b - a)(e - 1) + (c - d)(1 - e)) \quad (3.6)
\]
Proof.

\[
\text{card } A = \int_a^b A(x) \, dx = \int_a^b we^{-[(b-x)/(b-a)]} \, dx + \int_b^c w \, dx + \int_c^d we^{-[(x-c)/(d-c)]]} \, dx
\]

\[
= w(b-a)(1 - \frac{1}{e}) + w(c-b) + w(c-d)(\frac{1}{e} - 1) = w(c-b) + \frac{w}{e}(b-a)(1 - e) + (c-d)(1 - e)
\]

Now the article will study location of the median value \(m_A\) in the support of \(A\). The article will also identify the fuzziness of \(m_A\) determined by its membership grade \(A(m_A)\).

**Theorem 3.2.** If \(A\) is a exponential trapezoidal fuzzy number with light tails then

\[
m_A = \frac{w(b+c)}{2} + \frac{w}{2e} [(c-d)(1 - e) - (b-a)(e - 1)]
\]

Proof.

\[
m_A = \frac{w(b+c)}{2} + \frac{1}{2} \int_c^d A(x) \, dx - \int_a^b A(x) \, dx
\]

\[
= \frac{w(b+c)}{2} + \frac{1}{2} \left[ \int_c^d we^{-[(x-c)/(d-c)]]} \, dx - \int_a^b we^{-[(b-x)/(b-a)]} \, dx \right]
\]

\[
= \frac{w(b+c)}{2} + \frac{1}{2} [w(c-d)(\frac{1}{e} - 1) - w(b-a)(1 - \frac{1}{e})] = \frac{w(b+c)}{2} + \frac{w}{2e} [(c-d)(1 - e) - (b-a)(e - 1)]
\]

So we can Define the ranking of median value in exponential trapezoidal fuzzy number.

**Theorem 3.3.** \(A = (a, b, c, d)\) is a exponential trapezoidal fuzzy number, and \(m_A\) the median value of them, So

(i) If \(m_A < m_B\) then \(A < B\).

(ii) If \(m_A > m_B\) then \(A > B\).

(iii) If \(m_A \sim m_B\) then \(A \sim B\).

4 Results

**Example 4.1.** Let \(A = (0.2, 0.4, 0.6, 0.8; 0.35)\) and \(B = (0.1, 0.2, 0.3, 0.4; 0.7)\) be two generalized trapezoidal fuzzy number, then

\[
m_A = \frac{w_A(b_A + c_A)}{2} + \frac{w_A}{2e} [(c_A - d_A)(1 - e) - (b_A - a_A)(e - 1)]
\]

\[
= \frac{0.35(0.4 + 0.6)}{2} + \frac{0.35}{2 \times 2.72} \left[(0.6 - 0.8)(1 - 2.72) - (0.4 - 0.2)(2.72 - 1) \right]
\]

\[
= 0.175 + 0.064[0.344 - 0.344] = 0.175
\]

and

\[
m_B = \frac{w_B(b_B + c_B)}{2} + \frac{w_B}{2e} [(c_B - d_B)(1 - e) - (b_B - a_B)(e - 1)]
\]

\[
= \frac{0.7(0.2 + 0.3)}{2} + \frac{0.7}{2 \times 2.72} \left[(0.3 - 0.4)(1 - 2.72) - (0.2 - 0.1)(2.72 - 1) \right]
\]

\[
= 0.175 + 0.13[0.172 - 0.172] = 0.175
\]

So with use of theorem 3.3, we have \(m_A \sim m_B\) then \(A \sim B\).
Example 4.2. Let $A = (0.1, 0.2, 0.4, 0.5; 1)$ and $B = (0.1, 0.3, 0.3, 0.5; 1)$ be two generalized trapezoidal fuzzy number, then

$$m_A = \frac{w_A(b_A + c_A)}{2} + \frac{w_A}{2e}[(c_A - d_A)(1 - e) - (b_A - a_A)(e - 1)]$$

$$= \frac{(0.2 + 0.4)}{2} + \frac{1}{2 \times 2.72}[(0.4 - 0.5)(1 - 2.72) - (0.2 - 0.1)(2.72 - 1)]$$

$$= 0.3 + 0.184[0.172 - 0.172] = 0.3$$

and

$$m_B = \frac{w_B(b_B + c_B)}{2} + \frac{w_B}{2e}[(c_B - d_B)(1 - e) - (b_B - a_B)(e - 1)]$$

$$= \frac{(0.3 + 0.3)}{2} + \frac{1}{2 \times 2.72}[(0.3 - 0.5)(1 - 2.72) - (0.3 - 0.1)(2.72 - 1)]$$

$$= 0.3 + 0.184[0.344 - 0.344] = 0.3$$

So with use of theorem 3.3, we have $m_A \sim m_B$ then $A \sim B$.

Example 4.3. Let $A = (0.1, 0.2, 0.4, 0.5; 1)$ and $B = (1, 1, 1, 1; 1)$ be two generalized trapezoidal fuzzy number, then

$$m_A = \frac{w_A(b_A + c_A)}{2} + \frac{w_A}{2e}[(c_A - d_A)(1 - e) - (b_A - a_A)(e - 1)]$$

$$= \frac{(0.2 + 0.4)}{2} + \frac{1}{2 \times 2.72}[(0.4 - 0.5)(1 - 2.72) - (0.2 - 0.1)(2.72 - 1)]$$

$$= 0.3 + 0.184[0.172 - 0.172] = 0.3$$

and

$$m_B = \frac{w_B(b_B + c_B)}{2} + \frac{w_B}{2e}[(c_B - d_B)(1 - e) - (b_B - a_B)(e - 1)]$$

$$= \frac{(1 + 1)}{2} + \frac{1}{2 \times 2.72}[(1 - 1)(1 - 2.72) - (1 - 1)(2.72 - 1)]$$

$$= 1 + 0.184[0 - 0] = 1$$

So with use of theorem 3.3, we have $m_A < m_B$ then $A < B$.

Example 4.4. Let $A = (-0.5, -0.3, -0.3, -0.1; 1)$ and $B = (0.1, 0.3, 0.3, 0.5; 1)$ be two generalized trapezoidal fuzzy number, then

$$m_A = \frac{w_A(b_A + c_A)}{2} + \frac{w_A}{2e}[(c_A - d_A)(1 - e) - (b_A - a_A)(e - 1)]$$

$$= \frac{(-0.3 - 0.3)}{2} + \frac{1}{2 \times 2.72}[-(-0.3 + 0.1)(1 - 2.72) - (-0.3 + 0.5)(2.72 - 1)]$$

$$= -0.3 + 0.184[0.344 - 0.344] = -0.3$$

and

$$m_B = \frac{w_B(b_B + c_B)}{2} + \frac{w_B}{2e}[(c_B - d_B)(1 - e) - (b_B - a_B)(e - 1)]$$

$$= \frac{(0.3 + 0.3)}{2} + \frac{1}{2 \times 2.72}[(0.3 - 0.5)(1 - 2.72) - (0.3 - 0.1)(2.72 - 1)]$$

$$= 0.3 + 0.184[0.344 - 0.344] = 0.3$$

So with use of theorem 3.3, we have $m_A < m_B$ then $A < B$.
Example 4.5. Let \( A = (0.3, 0.5, 0.5, 1; 1) \) and \( B = (0.1, 0.6, 0.6, 0.8; 1) \) be two generalized trapezoidal fuzzy number, then

\[
m_A = \frac{w_A}{2}(b_A + c_A) + \frac{w_A}{2e}[(c_A - d_A)(1 - e) - (b_A - a_A)(e - 1)]
\]

\[
= \left( \frac{0.5 + 0.5}{2} \right) + \frac{1}{2 \times 2.72} \left[ (0.5 - 1)(1 - 2.72) - (0.5 - 0.3)(2.72 - 1) \right]
\]

\[
= 0.5 + 0.184[0.86 - 0.344] = 0.5 + 0.095 = 0.595
\]

and

\[
m_B = \frac{w_B}{2}(b_B + c_B) + \frac{w_B}{2e}[(c_B - d_B)(1 - e) - (b_B - a_B)(e - 1)]
\]

\[
= \left( \frac{0.6 + 0.6}{2} \right) + \frac{1}{2 \times 2.72} \left[ (0.6 - 0.8)(1 - 2.72) - (0.6 - 0.1)(2.72 - 1) \right]
\]

\[
= 0.6 + 0.184[0.344 - 0.86] = 0.6 - 0.095 = 0.505
\]

So with use of theorem 3.3, we have \( m_A > m_B \) then \( A > B \).

Example 4.6. Let \( A = (0, 0.4, 0.6, 0.8; 1) \) and \( B = (0.2, 0.5, 0.5, 0.9; 1) \) and \( C = (0.1, 0.6, 0.7, 0.8; 1) \) be three generalized trapezoidal fuzzy number, then

\[
m_A = \frac{w_A}{2}(b_A + c_A) + \frac{w_A}{2e}[(c_A - d_A)(1 - e) - (b_A - a_A)(e - 1)]
\]

\[
= \left( \frac{0.4 + 0.6}{2} \right) + \frac{1}{2 \times 2.72} \left[ (0.6 - 0.8)(1 - 2.72) - (0.4 - 0.0)(2.72 - 1) \right]
\]

\[
= 0.5 + 0.184[0.344 - 0.688] = 0.5 - 0.06 = 0.44
\]

and

\[
m_B = \frac{w_B}{2}(b_B + c_B) + \frac{w_B}{2e}[(c_B - d_B)(1 - e) - (b_B - a_B)(e - 1)]
\]

\[
= \left( \frac{0.5 + 0.5}{2} \right) + \frac{1}{2 \times 2.72} \left[ (0.5 - 0.9)(1 - 2.72) - (0.5 - 0.2)(2.72 - 1) \right]
\]

\[
= 0.5 + 0.184[0.688 - 0.516] = 0.5 + 0.03 = 0.53
\]

and

\[
m_C = \frac{w_B}{2}(b_B + c_B) + \frac{w_B}{2e}[(c_B - d_B)(1 - e) - (b_B - a_B)(e - 1)]
\]

\[
= \left( \frac{0.6 + 0.7}{2} \right) + \frac{1}{2 \times 2.72} \left[ (0.7 - 0.8)(1 - 2.72) - (0.6 - 0.1)(2.72 - 1) \right]
\]

\[
= 0.65 + 0.184[0.172 - 0.86] = 0.65 - 0.13 = 0.52
\]

So with use of theorem 3.3, we have \( m_A < m_C < m_B \) then \( A < C < B \).

Example 4.7. Let \( A = (0.1, 0.2, 0.4, 0.5; 1) \) and \( B = (-2, 0, 0, 2; 1) \) be two generalized trapezoidal fuzzy number, then

\[
m_A = \frac{w_A}{2}(b_A + c_A) + \frac{w_A}{2e}[(c_A - d_A)(1 - e) - (b_A - a_A)(e - 1)]
\]

\[
= \left( \frac{0.2 + 0.4}{2} \right) + \frac{1}{2 \times 2.72} \left[ (0.4 - 0.5)(1 - 2.72) - (0.2 - 0.1)(2.72 - 1) \right]
\]

\[
= 0.3 + 0.184[0.172 - 0.172] = 0.3
\]

and

\[
m_B = \frac{w_B}{2}(b_B + c_B) + \frac{w_B}{2e}[(c_B - d_B)(1 - e) - (b_B - a_B)(e - 1)]
\]

\[
= \left( \frac{0 + 0}{2} \right) + \frac{1}{2 \times 2.72} \left[ (0 - 2)(1 - 2.72) - (0 + 2)(2.72 - 1) \right]
\]

\[
= 0 + 0.184[3.44 - 3.44] = 0
\]

So with use of theorem 3.3, we have \( m_A > m_B \) then \( A > B \).
Figure 1: Example 1, Example 2.

Figure 2: Example 3.

Figure 3: Example 4.
Table (1): A comparison of the ranking results for different approaches

<table>
<thead>
<tr>
<th>Approaches</th>
<th>Ex.1</th>
<th>Ex.2</th>
<th>Ex.3</th>
<th>Ex.4</th>
<th>Ex.5</th>
<th>Ex.6</th>
<th>Ex.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abbasbandy[1]</td>
<td>Error</td>
<td>$A \sim B$</td>
<td>$A &lt; B$</td>
<td>$A &lt; B$</td>
<td>$A &lt; B &lt; C$</td>
<td>$A &gt; B$</td>
<td></td>
</tr>
<tr>
<td>Proposed approach</td>
<td>$A \sim B$</td>
<td>$A \sim B$</td>
<td>$A &lt; B$</td>
<td>$A &lt; B$</td>
<td>$A &gt; B$</td>
<td>$A &lt; C &lt; B$</td>
<td>$A &gt; B$</td>
</tr>
</tbody>
</table>
5 Conclusion

It is clear from Table 1 that the results of the proposed approach are same as obtained by using the existing approach (Chen and Chen, 2009). The main advantage of the proposed approach is that the proposed approach provides the correct ordering of generalized and normal trapezoidal fuzzy numbers and also the proposed approach is very simple and easy to apply in the real life problems.

References

http://dx.doi.org/10.1016/j.camwa.2008.10.090

http://dx.doi.org/10.1016/0165-0114(85)90012-0

http://dx.doi.org/10.1016/j.ins.2004.07.018

http://dx.doi.org/10.1016/0165-0114(85)90050-8

http://dx.doi.org/10.1007/s10489-006-0003-5


http://dx.doi.org/10.1016/S0165-0114(96)00272-2

http://dx.doi.org/10.1016/S0898-1221(01)00277-2

http://dx.doi.org/10.1016/0165-0114(87)90028-5

http://dx.doi.org/10.1109/TSMC.1976.309421


http://dx.doi.org/10.1007/s12543-010-0036-7


