Coupled Coincidence and Common Fixed Point Theorems for Set-valued and Single-valued Mappings in fuzzy Metric Space

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Abstract

In this paper, we define the tangential property and the generalized coincidence property for a pair of set-valued and single-valued mappings and use it to prove some coupled coincidence and common fixed point theorems for a hybrid pair of mappings without appeal to the completeness of the underlying space.

Keywords: w-compatible mappings; tangential property; weak tangent point; generalized coincidence property.

1 Introduction

Since the introduction of the fuzzy sets by Zadeh [1] in 1965, many authors have introduced the concept of fuzzy metric space in different ways [2, 3]. We know that in the setting of fuzzy metric space, the strict contractive condition do not insure the existence of a common fixed point unless the space is assumed complete or the strict conditions are replaced by strong conditions. The study of fixed points for multi-valued contraction mappings using the Hausdorff metric was initiated by Nadler [4] and Markin [5]. Later an interesting and rich fixed point theory for such maps was developed which has found applications in control theory, convex optimization, differential inclusion and economics. Klim and Wardowski [6] also obtained existence of fixed point for set-valued contraction in complete metric space. Bhashkar and Lakshmikantham [7] introduced the concept of a coupled fixed point of a mapping \( F : X \times X \rightarrow X \) (a nonempty set) and established

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some coupled fixed point theorems in partially ordered complete metric spaces. Later, Lakshmikantham and Ciric [8] proved coupled coincidence and coupled common fixed point results for nonlinear mappings \( F : X \times X \to X \) and \( g : X \to X \) satisfying certain contractive conditions in partially ordered complete metric spaces. Recently Abbas et al. [9] proved coupled common fixed point theorem for a hybrid pair of mappings satisfying w- compatibility in complete metric space.


In this paper we define the tangential property and generalized coincidence property for two mappings \( F : X \to CB(X) \) and \( g : X \to X \) in fuzzy metric space. Also we prove some common fixed point theorems using these properties without appeal to the completeness or closeness of the underlying space or the continuity of the mappings involved therein.

2 Preliminaries

Definition 2.1. [3] A triplet \((X, M, *)\) is said to be a fuzzy metric space if \( X \) is an arbitrary set, \( * \) is a continuous t-norm and \( M \) is a fuzzy set on \( X^2 \times [0, \infty) \) satisfying the following:

\[ (KM-1) \quad M(x, y, 0) = 0 \quad \text{for all} \quad x, y \in X, \]
\[ (KM-2) \quad M(x, y, t) = 1 \quad \text{for all} \quad t > 0 \quad \text{if and only if} \quad x = y, \]
\[ (KM-3) \quad M(x, y, t) = M(y, x, t) \quad \text{for all} \quad x, y \in X \quad \text{and} \quad t > 0, \]
\[ (KM-4) \quad M(x, y, t) \ast M(y, z, s) \leq M(x, z, t + s) \quad \text{for all} \quad x, y, z \in X \quad \text{and} \quad s, t > 0, \]

Note that \( M(x, y, t) \) can be thought of as the degree of nearness between \( x \) and \( y \) with respect to \( t \).

Let \((X, M, *)\) be a fuzzy metric space and we denote by \( CB(X) \), the class of all nonempty bounded closed subsets of \( X \). For \( A, B \in CB(X) \), we write
\[ M^\triangledown(A, b, t) = \max \{ M(a, b, t) : a \in A \} \]
And
\[ D_M(A, B, t) = \max \{ M(a, b, t) : a \in A, b \in B \} \]
And so we deduce that \( M^\triangledown(A, b, t) = 1 \Rightarrow b \in A \) also \( M(a, b, t) \leq M^\triangledown(A, b, t) \leq D_M(A, B, t) \).

Definition 2.2. Let \( X \) be a nonempty set, \( f : X \to X \) and \( A : X \times X \to CB(X) \). A point \((x, y) \in X \) is called (i) coupled coincidence point of the pair \( \{ f, A \} \) if \( f x \in A(x, y) \) and \( f y \in A(y, x) \) (ii) coupled common fixed point of the pair \( \{ f, A \} \) if \( x = f x \in A(x, y) \) and \( y = f y \in A(y, x) \).
We denote the set of coupled coincidence point of mappings \( f : X \rightarrow X \) and \( A : X \times X \rightarrow CB(X) \) by \( C(A, f) \). Note that if \( (x, y) \in C(A, f) \) then \( (y, x) \) is also in \( C(A, f) \).

**Definition 2.3.** Let \( f : X \rightarrow X \) and \( A : X \times X \rightarrow CB(X) \) then the hybrid pair \( \{ f, A \} \) is called \( w \)-compatible if \( f(A(x, y)) \subseteq A(f(x, y)) \) whenever \( (x, y) \in C(A, f) \).

Pathak and Shahzad [10] defined the concept of weak tangent point for a pair of single valued maps in metric space as follows:

**Definition 2.4.** Let \((X, d)\) be a metric space. Let \( f, g : X \rightarrow X \). Then \( z \in X \) is called weak tangent point to the pair \( \{ f, g \} \) if there exist two sequences \( \{ x_n \} \) and \( \{ y_n \} \) in \( X \) such that

\[
\lim_{n \to \infty} f x_n = \lim_{n \to \infty} g y_n = z \in X
\]

Sintunavarat and Kumam [11] defined the concept of tangential property for single-valued and multi-valued mapping in metric space as follows:

**Definition 2.5.** Let \((X, d)\) be a metric space. Let \( f : X \rightarrow X \) and \( A : X \rightarrow CB(X) \). The mapping \( f \) is called tangential w.r.t. the mapping \( A \) if there exist two sequences \( \{ x_n \} \) and \( \{ y_n \} \) in \( X \) such that

\[
\lim_{n \to \infty} f x_n = \lim_{n \to \infty} f y_n = z
\]

for some \( z \in X \), then

\[
z \in \lim_{n \to \infty} A x_n = \lim_{n \to \infty} A y_n = C \in CB(X)
\]

**Definition 2.6.** Let \((X, d)\) be a metric space. Let \( f, g : X \rightarrow X \) and \( A, B : X \rightarrow CB(X) \). The pair \( \{ f, g \} \) is called tangential w.r.t. the pair \( \{ A, B \} \) if there exist two sequences \( \{ x_n \} \) and \( \{ y_n \} \) in \( X \) such that

\[
\lim_{n \to \infty} f x_n = \lim_{n \to \infty} g y_n = z
\]

for some \( z \in X \), then

\[
z \in \lim_{n \to \infty} A x_n = \lim_{n \to \infty} B y_n = C \in CB(X)
\]

We define the concept of a coupled weak tangent point and tangential property for single-valued and multi-valued mapping in fuzzy metric space as follows:
**Definition 2.7.** Let $(X, M, *)$ be a fuzzy metric space. Let $f : X \to X$ and $A : X \times X \to CB(X)$. The mapping $f$ is called tangential w.r.t. the mapping $A$ if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in $X$ such that

\[
\lim_{n \to \infty} f x_n = z_1 \in \lim_{n \to \infty} A(x_n, y_n) = C_1 \in CB(X) \\
\lim_{n \to \infty} f y_n = z_2 \in \lim_{n \to \infty} A(y_n, x_n) = C_2 \in CB(X)
\]

for some $z_1, z_2 \in X$. And the pair $(z_1, z_2)$ is called a coupled weak tangent point to the mapping $f$.

Remark: If, in particular $z_1, z_2 \in f(X)$ then $(z_1, z_2)$ is called contained coupled weak tangent point to the mapping $f$.

Throughout this section $\mathbb{R}_+$ denotes the set of non-negative real numbers.

**Example 2.1.** Let $X = \mathbb{R}_+$. Then $(X, M, *)$ is a fuzzy metric space where $a * b = ab$ with $M(x, y, t) = t/(t+md(x, y))$ in which $m > 1$. Let $f : X \to X$ and $A : X \times X \to CB(X)$ are mappings defined by

$f x = x + 1$ and $A(x, y) = [x^2 + 1, y^2 + 2]$ for all $x, y \in \mathbb{R}_+$.

Since there exist two sequences $x_n = 1 + 1/n$ and $y_n = 2 + 1/n$ such that

\[
\lim_{n \to \infty} f x_n = 2 \in \lim_{n \to \infty} A(x_n, y_n) = [2, 6] \in CB(X) \\
\lim_{n \to \infty} f y_n = 3 \in \lim_{n \to \infty} A(y_n, x_n) = [3, 5] \in CB(X)
\]

Thus the mapping $f$ is tangential w.r.t. $A$.

Moreover, since $f1 = 2$ and $f2 = 3$ we have $2, 3 \in f(X)$ and so $(2, 3)$ is a contained weak tangent point to the mapping $f$.

In the above definition, if we replace $A : X \times X \to CB(X)$ by $A : X \times X \to X$ then we have the definition of tangential property for two single-valued mappings as follows:

**Definition 2.8.** Let $(X, M, *)$ be a fuzzy metric space. Let $f : X \to X$ and $A : X \times X \to X$. The mapping $f$ is called tangential w.r.t. the mapping $A$ if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in $X$ such that

\[
\lim_{n \to \infty} f x_n = \lim_{n \to \infty} A(x_n, y_n) = z_1 \\
\lim_{n \to \infty} f y_n = \lim_{n \to \infty} A(y_n, x_n) = z_2
\]

for some $z_1, z_2 \in X$. And the pair $(z_1, z_2)$ is called a coupled weak tangent point to the mapping $f$. 
Example 2.2. Let $X = R_+$. Then $(X, M, *)$ is a fuzzy metric space where $a * b = ab$ with $M(x, y, t) = t/(t + md(x, y))$ in which $m > 1$. Let $f : X \rightarrow X$ and $A : X \times X \rightarrow X$ are mappings defined by

$$fx = x + 3$$

and

$$A(x, y) = 2x + y$$

for all $x, y \in R_+$. Since there exist two sequences $x_n = 1 + 1/n$ and $y_n = 2 + 1/n$ such that

$$\lim_{n \to \infty} fx_n = 4 = \lim_{n \to \infty} A(x_n, y_n)$$

and

$$\lim_{n \to \infty} fy_n = 5 = \lim_{n \to \infty} A(y_n, x_n)$$

Thus the mapping $f$ is tangential w.r.t. $A$.

Moreover, since $f_1 = 4$ and $f_2 = 5$ we have $4, 5 \in f(X)$ and so $(4, 5)$ is a contained weak tangent point to the mapping $f$.

Now we define the generalized coincidence property for a pair of set-valued and single-valued mappings as follows:

Definition 2.9. Let $f : X \rightarrow X$ and $A : X \times X \rightarrow CB(X)$. The pair $\{f, A\}$ is said to satisfy the generalized coincidence property at $(x, y) \in X \times X$ if $fx_1 \in A(x_1, y_1)$ and $fy_1 \in A(y_1, x_1)$ for all $x_1 \in A(x, y)$ and $y_1 \in A(y, x)$.

Example 2.3. Let $X = R_+$. Then $(X, M, *)$ is a fuzzy metric space where $a * b = ab$ with $M(x, y, t) = t/(t + md(x, y))$ in which $m > 1$. Let $f : X \rightarrow X$ and $A : X \times X \rightarrow CB(X)$ are mappings defined by

$$fx = x + 1$$

and

$$A(x, y) = [x + 1, y^2]$$

for all $x, y \in R_+$. At $(1, 2) \in X \times X$ we have $A(1, 2) = [2, 4]$ and $A(2, 1) = [1, 3]$. Since $2 \in A(1, 2)$ and $5/2 \in A(2, 1)$ we have $f_2 = 3$ and $A(2, 5/2) = [3, 25/4]$ implying $f_2 \in A(2, 5/2)$ also $f(5/2) = 7/2$ and $A(5/2, 2) = [7/2, 4]$ implying $f(5/2) \in A(5/2, 2)$.

Similarly we can prove that $fx_1 \in A(x_1, y_1)$ and $fy_1 \in A(y_1, x_1)$ for all $x_1 \in A(1, 2)$ and $y_1 \in A(2, 1)$.

Hence the pair $\{f, A\}$ satisfies the generalized coincidence property at $(1, 2) \in X \times X$.

Similarly we can define the generalized coincidence property for a pair of single-valued mappings.

Definition 2.10. Let $f : X \rightarrow X$ and $A : X \times X \rightarrow X$. The pair $\{f, A\}$ is said to satisfy the generalized coincidence property at $(x, y) \in X \times X$ if $fx_1 = A(x_1, y_1)$ and $fy_1 = A(y_1, x_1)$ for all $x_1 = A(x, y)$ and $y_1 = A(y, x)$.

## 3 Coupled Coincidence point theorems

In this section, we give coupled coincidence point theorems for set-valued and single-valued mappings.

Theorem 3.1. Let $(X, M, *)$ be a fuzzy metric spaces. Let $g : X \rightarrow X$ and $A : X \times X \rightarrow CB(X)$ satisfying the condition

$$D_M(A(x, y), A(u, v), t) \leq \min \left\{ M(gx, gu, qt), M^\vee(A(x, y), gx, qt), M^\vee(A(u, v), gu, qt), M^\vee(A(x, y), gu, qt), M^\vee(A(u, v), gx, qt) \right\}$$

(3.1)
for all \( x, y, u, v \in X \) and \( q \in (0, 1) \). If there exist a contained coupled weak tangential point \((z_1, z_2)\) to the mapping \( g \) and the mapping \( g \) is tangential to the mapping \( A \) then the pairs \( \{g, A\} \) have a coupled coincident point.

**Proof.** Since \( z_1, z_2 \in g(X) \), there exist points \( w_1, w_2 \in X \) such that \( z_1 = gw_1 \) and \( z_2 = gw_2 \). Again since \( g \) is tangential w.r.t. the mapping \( A \) we have two sequences \( \{x_n\} \) and \( \{y_n\} \) in \( X \) such that

\[
\lim_{n \to \infty} gx_n = z_1 \in \lim_{n \to \infty} A(x_n, y_n) = C_1 \in CB(X)
\]

\[
\lim_{n \to \infty} gy_n = z_2 \in \lim_{n \to \infty} A(y_n, x_n) = C_2 \in CB(X)
\]

Now we shall prove that \( gw_1 \in A(w_1, w_2) \). If not, putting \( x = x_n, y = y_n, u = w_1 \) and \( v = w_2 \) in inequality (3.1), we get

\[
D_M(A(x_n, y_n), A(w_1, w_2), t) \leq \min \left\{ \begin{array}{l}
M(gx_n, gw_1, qt), M^\n(A(x_n, y_n), gx_n, qt), \\
M^\n(A(w_1, w_2), gw_1, qt), M^\n(A(x_n, y_n), gw_1, qt), \\
M^\n(A(w_1, w_2), gx_n, qt)
\end{array} \right\}
\]

Letting \( n \to \infty \)

\[
D_M(C_1, A(w_1, w_2), t) \leq \min \left\{ \begin{array}{l}
M(z_1, gw_1, qt), M^\n(C_1, z_1, qt), M^\n(A(w_1, w_2), gw_1, qt), \\
M^\n(C_1, gw_1, qt), M^\n(A(w_1, w_2), z_1, qt)
\end{array} \right\}
\]

Or

\[
D_M(C_1, A(w_1, w_2), t) \leq \min \left\{ \begin{array}{l}
M(z_1, z_1, qt), M^\n(C_1, z_1, qt), M^\n(A(w_1, w_2), z_1, qt), \\
M^\n(C_1, z_1, qt), M^\n(A(w_1, w_2), z_1, qt)
\end{array} \right\}
\]

which gives

\[
D_M(C_1, A(w_1, w_2), t) \leq \min \left\{ 1, 1, M^\n(A(w_1, w_2), z_1, qt), 1, M^\n(A(w_1, w_2), z_1, qt) \right\}
\]

Since \( z_1 \in C_1 \), we have

\[
M^\n(z_1, A(w_1, w_2), t) \leq D_M(C_1, A(w_1, w_2), t)
\]

and so

\[
M^\n(z_1, A(w_1, w_2), t) \leq M^\n(z_1, A(w_1, w_2), t)
\]

which is a contradiction. Hence \( z_1 = gw_1 \in A(w_1, w_2) \).

Similarly we can prove that \( z_2 = gw_2 \in A(w_2, w_1) \) by putting \( x = y_n, y = x_n, u = w_2 \) and \( v = w_1 \) in inequality (3.1).

Hence \((w_1, w_2)\) is a coupled coincidence point of the mapping \( A \) and \( g \).

\[\square\]

**Theorem 3.2.** Let \((X, M, \ast)\) be a fuzzy metric spaces. Let \( g : X \to X \) and \( A : X \times X \to CB(X) \) satisfying the condition

\[
D_M(A(x, y), A(u, v), t) \leq \varphi \left( \min \left\{ \begin{array}{l}
M(gx, gu, qt), M^\n(A(x, y), gx, qt), M^\n(A(u, v), gu, qt), \\
M^\n(A(x, y), gu, qt), M^\n(A(u, v), gx, qt)
\end{array} \right\} \right)
\]

(3.2)
for all \(x, y, u, v \in X\) and \(q \in (0, 1)\) where \(\varphi : [0, 1] \rightarrow [0, 1]\) is a continuous function s.t. \(\varphi(s) < s\) for each \(0 < s < 1\). If there exist a contained coupled weak tangential point \((z_1, z_2)\) to the mapping \(g\) and the mapping \(g\) is tangential to the mapping \(A\) then the pairs \(\{g, A\}\) have a coupled coincident point.

**Proof.** Proof directly follows from theorem (3.1) \(\square\)

**Corollary 3.1.** Let \((X, M, *)\) be a fuzzy metric spaces. Let \(g : X \rightarrow X\) and \(A : X \times X \rightarrow X\) satisfying the condition

\[
M(A(x, y), A(u, v), t) \leq \min \left\{ M(gx, gu, qt), M(A(x, y), gx, qt), M(A(u, v), gu, qt), M(A(x, y), gu, qt) \right\}
\]

for all \(x, y, u, v \in X\) and \(q \in (0, 1)\). If there exist a contained coupled weak tangent point \((z_1, z_2)\) to the mapping \(g\) and the mapping \(g\) is tangential to the mapping \(A\) then the pair \(\{g, A\}\) have a coupled coincident point.

**Proof.** In theorem (3.1), if we replace \(A : X \times X \rightarrow CB(X)\) by \(A : X \times X \rightarrow X\) and \(D_M\) and \(M^\nabla\) by \(M\) then the proof directly follows from the definition 2.8. \(\square\)

**Corollary 3.2.** Let \((X, M, *)\) be a fuzzy metric spaces. Let \(g : X \rightarrow X\) and \(A : X \times X \rightarrow X\) satisfying the condition

\[
M(A(x, y), A(u, v), t) \leq \varphi \left( \min \left\{ M(gx, gu, qt), M(A(x, y), gx, qt), M(A(u, v), gu, qt), M(A(x, y), gu, qt) \right\} \right)
\]

for all \(x, y, u, v \in X\) and \(q \in (0, 1)\) where \(\varphi : [0, 1] \rightarrow [0, 1]\) is a continuous function s.t. \(\varphi(s) < s\) for each \(0 < s < 1\). If there exist a contained coupled weak tangent point \((z_1, z_2)\) to the mapping \(g\) and the mapping \(g\) is tangential to the mapping \(A\) then the pair \(\{g, A\}\) have a coupled coincident point.

**Proof.** Proof directly follows from corollary (3.1). \(\square\)

## 4 Common Coupled fixed point theorems:

In this section, we give common fixed point theorems for set-valued and single valued mappings satisfying generalized coincidence property.

**Theorem 4.1.** Let \((X, M, *)\) be a fuzzy metric spaces. Let \(g : X \rightarrow X\) and \(A : X \times X \rightarrow CB(X)\) satisfying all the condition of theorem (3.1). Moreover, if the pairs \(\{g, A\}\) satisfy the generalized coincidence property at \((a, b) \in C(g, A)\) then \(g\) and \(A\) have a coupled common fixed point.

**Proof.** From theorem (3.1), there exist point \((w_1, w_2) \in C(g, A)\). Again since the pair \(\{g, A\}\) satisfy the generalized coincidence property at \((w_1, w_2) \in C(g, A)\) also \(z_1 \in A(w_1, w_2)\) and \(z_2 \in A(w_2, w_1)\), we have \(g z_1 \in A(z_1, z_2)\) and \(g z_2 \in A(z_2, z_1)\).

Now we shall prove that \(z_1 = g z_1\) and \(z_2 = g z_2\).

Again putting \(x = z_1, y = z_2, u = w_1\) and \(v = w_2\) in inequality 3.1 ,we get

\[
D_M(A(z_1, z_2), A(w_1, w_2), t) \leq \min \left\{ M(g z_1, gw_1, qt), M^\nabla(A(z_1, z_2), g z_1, qt), M^\nabla(A(w_1, w_2), gw_1, qt), M^\nabla(A(z_1, z_2), gw_1, qt), M^\nabla(A(w_1, w_2), g z_1, qt) \right\}
\]
Let \( D_M(A(z_1, z_2), A(w_1, w_2), t) \leq \min \left\{ M(gz_1, z_1, qt), M^\nabla(A(z_1, z_2), gz_1, qt), M^\nabla(A(w_1, w_2), z_1, qt), M^\nabla(A(z_1, z_2), z_1, qt), M^\nabla(A(w_1, w_2), gz_1, qt) \right\} \)

Since \( gz_1 \in A(z_1, z_2) \) and \( z_1 \in A(w_1, w_2) \), we have
\[
M(gz_1, z_1, t) \leq D_M(A(z_1, z_2), A(w_1, w_2), t)
\]
and so
\[
M(gz_1, z_1, t) \leq D_M(A(z_1, z_2), A(w_1, w_2), t)
\]

\[
\leq \min \left\{ M(gz_1, z_1, qt), 1, 1, M^\nabla(A(z_1, z_2), z_1, qt), M^\nabla(A(w_1, w_2), gz_1, qt) \right\}
\]
Again we have \( M(gz_1, z_1, qt) \leq M^\nabla(A(z_1, z_2), z_1, qt) \) and \( M(gz_1, z_1, qt) \leq M^\nabla(A(w_1, w_2), gz_1, qt) \) implying
\[
M(gz_1, z_1, t) \leq M(gz_1, z_1, qt)
\]
which is a contradiction. And so \( gz_1 = z_1 \).
Similarly on putting \( x = z_2, y = z_1, u = w_2 \) and \( v = w_1 \) in inequality 3.1, we get \( g z_2 = z_2 \).
Therefore we have \( z_1 = gz_1 \in A(z_1, z_2) \) and \( z_2 = gz_2 \in A(z_2, z_1) \).
Hence \((z_1, z_2)\) is a coupled common fixed point of \( g \) and \( A \).

**Theorem 4.2.** Let \((X, M, \ast)\) be a fuzzy metric spaces. Let \( g: X \rightarrow X \) and \( A: X \times X \rightarrow CB(X) \) satisfying all the condition of theorem (3.2). Moreover, if the pairs \( \{g, A\} \) satisfy the generalized coincidence property at \((a, b) \in C(g, A)\) then \( g \) and \( A \) have a coupled common fixed point.

*Proof.* Proof directly follows from theorem (3.2) and theorem (4.1).

**Corollary 4.1.** Let \((X, M, \ast)\) be a fuzzy metric spaces. Let \( g: X \rightarrow X \) and \( A: X \times X \rightarrow X \) satisfying all the condition of corollary (3.1). Moreover, if the pairs \( \{g, A\} \) satisfy the generalized coincidence property at \((a, b) \in C(g, A)\) then \( g \) and \( A \) have a coupled common fixed point.

*Proof.* In theorem (4.1), if we replace \( A: X \times X \rightarrow CB(X) \) by \( A: X \times X \rightarrow X \) and \( D_M \) and \( M^\nabla \) by \( M \), then the proof directly follows from the definition 2.10.

**Corollary 4.2.** Let \((X, M, \ast)\) be a fuzzy metric spaces. Let \( g: X \rightarrow X \) and \( A: X \times X \rightarrow X \) satisfying all the condition of corollary (3.2). Moreover, if the pairs \( \{g, A\} \) satisfy the generalized coincidence property at \((a, b) \in C(g, A)\) then \( g \) and \( A \) have a coupled common fixed point.

*Proof.* Proof directly follows from the corollary (4.1).

Now we establish coupled common fixed point theorems using \( w \)-compatibility.
Theorem 4.3. Let \((X, M, \ast)\) be a fuzzy metric spaces. Let \(g : X \rightarrow X\) and \(A : X \times X \rightarrow CB(X)\) satisfying all the condition of theorem (3.1). Moreover if \(gga = ga\) and \(ggb = gb\) for \((a, b) \in C(g, A)\) and the pair \(\{g, A\}\) is \(w\)- compatible then \(g\) and \(A\) have a coupled common fixed point.

Proof. From theorem (3.1), there exist point \((w_1, w_2) \in C(g, A)\). It follows that \(ggw_1 = gw_1\) and \(ggw_2 = gw_2\). Hence \(gz_1 = z_1\) and \(gz_2 = z_2\).

Since the pair \(\{g, A\}\) is \(w\)- compatible, \(g(A(w_1, w_2)) \subseteq A(gw_1, gw_2)\). Thus \(gz_1 \in g(A(w_1, w_2)) \subseteq A(gw_1, gw_2) = A(z_1, z_2)\). Also, since \((w_1, w_2) \in C(g, A)\) by the fact, \((w_2, w_1)\) will also be in \(C(g, A)\). And so by \(w\)- compatibility of the pair \(\{g, A\}\), we have \(g(A(w_2, w_1)) \subseteq A(gw_2, gw_1)\). Thus \(gz_2 \in g(A(w_2, w_1)) \subseteq A(gw_2, gw_1) = A(z_2, z_1)\).

Hence \((z_1, z_2)\) is a coupled common coupled fixed point of \(g\) and \(A\).

Theorem 4.4. Let \((X, M, \ast)\) be a fuzzy metric spaces. Let \(g : X \rightarrow X\) and \(A : X \times X \rightarrow CB(X)\) satisfying all the condition of theorem (3.2). Moreover if \(gga = ga\) and \(ggb = gb\) for \((a, b) \in C(g, A)\) and the pair \(\{g, A\}\) is \(w\)- compatible then \(g\) and \(A\) have a coupled common fixed point.

Proof. Proof directly follows from theorem (3.2) & theorem (4.3).

5 Conclusion

In this work we have defined some new properties for single-valued and set-valued mappings in fuzzy metric spaces. Moreover, we have proved the existence of coupled coincidence and coupled common fixed point for set-valued and single-valued mappings using these new properties in fuzzy metric spaces.

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References

http://dx.doi.org/10.1016/S0019-9958(65)90241-X

http://dx.doi.org/10.1016/0165-0114(84)90069-1


http://dx.doi.org/10.1090/S0002-9939-1973-0313897-4


