Assessing the Efficiency of commercial Tunisian Banks using Fuzzy Data Envelopment Analysis

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Abstract
The banking sector is of great importance to Tunisian’s economy. Major commercial banks continue to spend high proportion of their budgets on new technologies and innovation in order to satisfy their customers and enhance their competitiveness. Consequently, performance analysis has become part of their management practices. This paper aims to evaluate the efficiency of commercial Tunisian banks in terms of several crisp and imprecise data. Two approaches of fuzzy data envelopment analysis (FDEA), the possibility approach and the approach based on relations between fuzzy numbers (BRONF), are used to obtain the efficiency score of each bank. The results show that, in a competitive environment, no-financial inputs and outputs should be taken into account in order to obtain credible and realistic efficiency scores.

Keywords: Tunisian commercial banks, Efficiency, FDEA, BRONF approach, Possibility approach.

1 Introduction

Tunisian commercial banks continue to spend high proportion of their budgets on new technologies and innovation in order to enhance their competitiveness. Consequently, performance analysis has become part of their management practices.
The decision makers of banks want to identify and reduce the underlying causes of inefficiencies, thus helping their firms to gain competitive advantage. Traditionally, banks have focused on various profitability ratios to estimate their efficiency. However, ratio analysis provides relatively insignificant amount of information when considering the effects of economies of scale, the identification of benchmarking policies, and the estimation of overall performance measures of firms. Another important method used in the evaluation of bank performances is Data Envelopment Analysis (DEA). DEA is a non-parametric method based on linear programming. It provides a relative evaluation of technical efficiency of different firms. This technique can deal with the case of multiple inputs and outputs. Applications of DEA in banking industry are numerous [1-9]. In these studies, only crisp financial data are used. However, it is not always sufficient to evaluate bank efficiency by taking only financial inputs and outputs as a basis. Nowadays, we see that non-financial performance criteria show up as an emerging asset especially in performance measurement. In general terms, non-financial data are defined as the criteria which cannot be measured with a precise manner and generally given in the form of linguistic terms. These data can be categorized as follows: customer satisfaction, market share, quality-process relation, personnel turnover, quality and flexibility, innovation, supply resources. Fuzzy data envelopment analysis (FDEA) represents an interesting method to deal with this kind of data. This method was applied in some studies to measure the efficiency of banks. BO [10] proposed a fuzzy super-efficiency slack-based measure DEA to analyze the performance of 24 commercial banks facing problems on loan and investment parameters with vague characteristics. Kao and Liu [11] used fuzzy CCR (FCCR) model to predict the performance of 24 commercial banks in Taiwan based on their financial forecasts. Wu et al. [12] used fuzzy BCC (FBCC) model to deal with environmental variables in order to assess the efficiency of bank branches from different regions in Canada. Yalcin et al. [13] developed a multi-criteria decision model to evaluate the performances of Turkish banks. Pramodh et al. [14] proposed a measurement technique that combines DEA method and Fuzzy Multi Attribute Decision Making technique to measure the productivity levels of Indian banks. Wang et al. [15] investigated the association between the performance of bank holding companies and their intellectual capital and applied fuzzy multiple objective programming approaches to calculate efficiency scores. Puri and Yadav [16] evaluated the fuzzy input mix-efficiency using the α-level based approach for the State Bank of Patiala in the Punjab state of India. Puri and Yadav [17] proposed another fuzzy DEA model with undesirable fuzzy outputs to calculate the efficiency scores. Chen et al. [18] applied the Fuzzy Slack-Based Measurement model in the Taiwan banking sector under market risk. Wanke et al [19] used FDEA and Bootstrap truncated regressions to evaluate the efficiency of Mozambican banks. Nazila [20] proposed a technique based on the α-level to evaluate the efficiency of all branches of the National Bank of Iran across Ardabil Province, Iran. Kordrostami et al [21] applied FDEA method to measure the efficiency scores of 25 branches of the Iranian commercial bank while undesirable and fuzzy factors are present.

This paper analyses the performance of Tunisian commercial banks in the presence of crisp and imprecise inputs and outputs. In order to obtain credible and realistic efficiency scores, we take into account non-financial data, the customer’s satisfaction as output and the innovation level as input. Further more, we propose a methodology to deal with these data. We used the possibility approach [22] and the BRONF approach [23] to obtain the efficiency scores and we compared the results provided by the two approaches.

The remainder of the paper is organised as follow: section 2 gives a brief summary for the evolution of the structure of Tunisian banking sector. Section 3 presents CCR and FCCR models. Section 4 gives the methodology used in this paper. Section 5 gives the empirical results and further discussion. Finally, our conclusions are given in section 6.

2 The banking system: structure and reforms

Tunisian financial system is dominated by the commercial banks. They hold most of the loans deposits, but also are the major of holders of the other financial institutions (insurance companies, leasing
companies...). The Tunisian banking sector is composed by 14 commercial banks, 5 development banks, 10 leasing companies, 8 offshore banks and 2 merchant banks. Commercial banks dominates the industry in terms of deposits and loans, they account for 97% of total assets.

In this paper we focus on commercial banks due to their great importance in this industry. The share of these banks in the credit distribution in 2008 was 85% compared to 6.7% for development banks. Commercial banks provide multi product services to satisfy the needs of the Tunisian economy, they are also allowed to collect deposits of any forms and makes short and medium term loans.

Since 1986, there was a limited movement of new entries the exception was the BH’s (Banque de l’Habitat, BH) entry which became the main bank in 1989 specialized in housing credit. This bank represents the third commercial bank in terms of total assets. Two other small banks were created: the city bank and the bank of solidarity which remained with limited activity. The absence of new entries are modest over the period 1986 to 2003, however, deposit banks continued to develop by the extension of their networks, the number of bank branches has increased over the period 1986 -2005 (456 in 1986 compared to 939 in 2005). During the last years the competitive environment has improved, foreign institutions participation in the domestic bank capital structure has increased, and new banking laws has been introduced to reinforce competition. The Tunisian authorities adopted in 2001 the universal bank which gave the development banks approval to compete commercial banks in their activities. This legislation provided a more liberal environment for the exercise of the banking activities.

The new banking law of 2006 consists to encourage the good governance and recover the credit traditions. This law has some aspects. First, it reinforces banks’ governance through internal audit, credit committees and compliance control. Second, it clarifies the conditions of banking activity. Third, it simplifies relationship between customers and their banks. In addition, the central bank of Tunisia (CBT) has illicit, since 2004, the distribution of dividends by banks that are inadequately provisioned.

The liberalization program of the Tunisian banking sector aims at the emergence of a new banking landscape. First, deregulation targeted interest rates, allowing banks to make freely credit decisions. The money market average rate encouraged banks to fix their rates. In 1994, the equity market was restructured and aims to improve its liquidity. In 1996, the rediscount refinancing technique was replaced by the market open technique. In addition, government financing moved gradually to be market-based and banks were less and less concerned by holding treasury bills.

3 DEA and FDEA models

The basic DEA models are the CCR model proposed by Charnes et al. [24] and the model of Banker et al. [25] called the BCC model. The two models differ in the way they treat returns to scale. The CCR model assumes constant return to scale. The BCC model is more flexible and allows variable returns to scale. Other DEA models exist and all are extensions of the CCR model. Consider $N$ decision making units ($DMU_j$), each uses $m$ inputs $(x_{1j},...,x_{mj})$ to produce $s$ different outputs $(y_{1j},...,y_{sj})$. The programming statement for the CCR primal model (input oriented) and its dual is given as follow:

$$\min \theta_0$$

$$s.t \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta_0 x_{i0}, \quad i = 1,\ldots,m,$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{r0}, \quad r = 1,\ldots,s,$$

$$\lambda_j \geq 0, \quad j = 1,\ldots,N.$$
In this model, $u_i$ represents the weight of the $i$th input and $v_r$ is the weight of the $r$th output. The target decision maker unit (DMU) is technically efficient if and only if the value of $\theta$ at the optimality is equal to 1 and so it’s not possible to make improvement without worsening any other input or output. The dual form of model (3.1) is given by

$$\max \sum_{r=1}^{s} v_r y_{r0}, \quad r = 1, \ldots, s,$$

$$s.t. \sum_{i=1}^{m} u_i x_{i0} = 1, \quad i = 1, \ldots, m,$$

$$\sum_{i=1}^{m} u_i x_{ij} \geq \sum_{r=1}^{s} v_r y_{rj}, \quad j = 1, \ldots, N,$$

$$u_i, v_r \geq 0.$$

When fuzzy inputs and fuzzy outputs exist in the performance evaluation process, model (3.2) becomes:

$$\max \sum_{r=1}^{s} v_r \tilde{y}_{r0}, \quad r = 1, \ldots, s,$$

$$s.t. \sum_{i=1}^{m} u_i \bar{x}_{i0} = 1, \quad i = 1, \ldots, m,$$

$$\sum_{i=1}^{m} u_i \bar{x}_{ij} \geq \sum_{r=1}^{s} v_r \tilde{y}_{rj}, \quad j = 1, \ldots, n,$$

$$u_i, v_r \geq 0, \quad \bar{x}_{ij} \text{ and } \tilde{y}_{rj} \text{ are respectively the } i\text{th fuzzy input and the } r\text{th fuzzy output}.$$

The FCCR model (3.3) takes the form of fuzzy linear programming problems. Fuzzy set theory is used as alternative to treat the imprecision and the vagueness in DEA models. The interpretation of constraints of FCCR model is similar to the crisp CCR model. The difference between the two models resides on the manner of resolution. The crisp CCR model can be simply solved by a standard LP solver. For the FCCR model, the resolution is more difficult and requires some ranking methods of fuzzy sets. These methods are usually categorised into four approaches.

The first is the tolerance approach developed by Sengupta [26] and further improved by Kahraman and Tolga [27]. The main idea of this approach consists to incorporate uncertainty into DEA models by specifying tolerance levels on constraint violations. This approach fuzzifies the inequality or equality signs but it does not treat fuzzy coefficients directly. The limitation of the tolerance approach appears in the case of DEA models with a fuzzy objective function and fuzzy constraints which may or may not be satisfied [28]. Although in most production processes fuzziness is present both in terms of not meeting specific objectives and in terms of the imprecision of the data, the tolerance approach provides flexibility by relaxing the DEA relationships while the input and output coefficients are treated as crisp.

The $\alpha$ -level approach is frequently used approach to treat the imprecision in FDEA models. The main idea consists to convert the fuzzy DEA model into a pair of parametric programs in order to find the lower and upper bounds at an $\alpha$ -level of the membership functions of the efficiency scores. Kao and Liu [29] used this approach to transform the fuzzy DEA model to a family of conventional crisp DEA models and developed a solution procedure to measure the efficiencies of the DMUs with fuzzy observations in the BCC model. Their method found approximately the membership functions of the fuzzy efficiency obtained by the application of the $\alpha$ -level approach and Zadeh’s extension principle [30], [31]. Saati et al. [32] suggested a fuzzy CCR model as a possibilistic programming problem and transformed it into an interval programming problem using $\alpha$ -level based approach. The resulting interval programming problem could be solved as a
crisp LP model for a given $\alpha$ with some variable substitutions. Saati and Memariani [33] proposed a technique for finding a common set of weights in fuzzy DEA based on the $\alpha$-level approach with triangular fuzzy data. Liu [34] suggested a fuzzy DEA procedure to obtain the efficiency measures embedded with assurance region (AR) concept when some observations were triangular fuzzy numbers. He combined the $\alpha$-level approach and the extension principle to transform the fuzzy DEA/AR model into a pair of parametric mathematical programs and worked out the lower and upper bounds of the efficiency scores of the DMUs. Wang et al. [35] developed a fuzzy DEA–Neural approach with a self-organizing map for classification in their neural network. The fuzzy ranking approach is also another popular technique that has attracted a great deal of attention in the fuzzy DEA literature. In this approach the main idea is to obtain the fuzzy efficiency scores of the DMUs using fuzzy linear programs which require ranking fuzzy sets. The fuzzy ranking approach was initially developed by Guo and Tanaka [36]. Tlig and Rebai [23] proposed an approach based on the ordering relations between LR-fuzzy numbers to solve the primal and the dual of FCCR. They suggested a procedure based on the resolution of a goal programming problem to transform the fuzzy normalisation equality in the primal of FCCR. Marbini et al. [37] suggested a multi-objective linear programming (MOLP) problem to solve FDEA models. Their approach classifies DMUs into three distinct categories based on their fuzzy efficiencies. Damghani et al. [38] proposed a FDEA model with the presence of desirable input and undesirable output data. A preference ratio method was used to rank the interval efficiency scores.

Other works related to this approach can be found in [39-44]. Another popular approach is the possibility approach. Guo et al. [45] initially built fuzzy DEA models based on possibility and necessity measures and then Lertworasirikul [46] and Lertworasirikul et al. [22] have proposed two methods for solving the ranking problem in fuzzy DEA models called the “possibility models” and the “credibility models.” They used the possibility approach from both optimistic and pessimistic view points by considering the uncertainty in fuzzy objectives and fuzzy constraints with possibility measures. In their credibility approach, fuzzy DEA model was transformed into a credibility programming-DEA model and fuzzy variables were replaced by ”expected credits,” which were obtained by using credibility measures. The mathematical details of the credibility model can be found in [47]. Azadah and kokabi [48], proposed a Z-number version of the FCCR (named after Charnes, Cooper, and Rhodes) and the FBCC (named after Banker, Charnes and Coopers) DEA models. They used the possibility theory to obtain crisp DEA models.

4 Methodology

In this paper, our study aims to evaluate the efficiency scores of 14 commercial Tunisian banks in terms of crisp and imprecise data during the period 2011-2013. We focus on the intermediary approach.

Used data

Three crisp inputs (deposits, labour and fixed assets) and two crisp outputs (loans and portfolio investment) have been used. The source of these data is the PATB (Professional association of Tunisian Banks). The fixed assets, deposits, loans and portfolio investment are measured in TND (Tunisian Dinar) and labour is measured in terms of number of staff. Imprecise used data are the innovation level as input and customer’s satisfaction as output. These data were obtained from two questionnaires. The first questionnaire is addressed to 120 potential customers of each bank. Every respondent describes his judgment about the innovation degree in his bank by the following linguistic terms; Not at all satisfied, unsatisfied, moderately satisfied, satisfied, and very satisfied. These linguistic expressions were converted into fuzzy numbers as (3, 4, 5), (6, 7, 8), (9, 10, 11), (12, 13, 14) and (15, 16, 17), respectively. In order to establish the imprecise value of the innovation level for each bank, we used the following aggregation function.
\[ \hat{y}_k = \left( y', y', y' \right) = \frac{1}{120} \sum_{j=1}^{120} \hat{y}_{jk} = \left( \frac{y'_{jk}}{120}, \frac{y'_{jk}}{120}, \frac{y'_{jk}}{120} \right), \quad k = 1, 2, \ldots, 14, \quad j = 1, 2, \ldots, 120 \]  

(4.4)

where \( \hat{y}_k \) is the value of the customers satisfaction of the \( k \)th bank, \( \hat{y}_{jk} \) is the value given by the \( j \)th respondent to the \( k \)th bank. The left spread and the right spread are given by \( c = y - y_j \) and \( d = y_{u} - y \), respectively. Table 1 gives the customer’s satisfaction data for each bank.

<table>
<thead>
<tr>
<th>Bank</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>BNA</td>
<td>(11, 13.5, 16)</td>
<td>(10, 12.5, 15)</td>
<td>(12, 13, 14)</td>
</tr>
<tr>
<td>STB</td>
<td>(10, 11, 12)</td>
<td>(11, 12, 13)</td>
<td>(11, 13.5, 16)</td>
</tr>
<tr>
<td>BIAT</td>
<td>(11, 13, 15)</td>
<td>(10, 13, 16)</td>
<td>(11, 11.5, 12)</td>
</tr>
<tr>
<td>UIB</td>
<td>(9, 10.5, 12)</td>
<td>(10, 11, 12)</td>
<td>(12, 13.5, 15)</td>
</tr>
<tr>
<td>BH</td>
<td>(11, 12, 13)</td>
<td>(9, 10.5, 12)</td>
<td>(9, 10.5, 12)</td>
</tr>
<tr>
<td>BS</td>
<td>(10, 13, 16)</td>
<td>(11, 12.5, 16)</td>
<td>(10, 12.5, 15)</td>
</tr>
<tr>
<td>BT</td>
<td>(9, 10.5, 12)</td>
<td>(9, 11.5, 14)</td>
<td>(11, 11.5, 12)</td>
</tr>
<tr>
<td>UBCI</td>
<td>(8, 10, 12)</td>
<td>(9, 9.5, 10)</td>
<td>(9, 10, 11)</td>
</tr>
<tr>
<td>ATB</td>
<td>(10, 12.5, 15)</td>
<td>(10, 12.5, 15)</td>
<td>(9, 11, 13)</td>
</tr>
<tr>
<td>AB</td>
<td>(11, 13.5, 16)</td>
<td>(10, 12.5, 15)</td>
<td>(10, 12.5, 15)</td>
</tr>
<tr>
<td>BFT</td>
<td>(6, 8.5, 11)</td>
<td>(6, 8, 10)</td>
<td>(7, 8, 9)</td>
</tr>
<tr>
<td>CB</td>
<td>(9, 11.5, 14)</td>
<td>(8, 10, 12)</td>
<td>(8, 10, 12)</td>
</tr>
<tr>
<td>BTS</td>
<td>(8, 11, 14)</td>
<td>(8, 10, 12)</td>
<td>(6, 8, 10)</td>
</tr>
<tr>
<td>ABCT</td>
<td>(5, 7, 9)</td>
<td>(6, 7, 8)</td>
<td>(8, 10, 12)</td>
</tr>
</tbody>
</table>

The second questionnaire was addressed to 60 technical person of each bank. Every respondent describes his judgment about the innovation degree in his bank by the following linguistic terms; very low, low, high and very high. These linguistic expressions were converted into fuzzy numbers as \( (5, 6, 7) \), \( (8, 10, 11) \), \( (12, 13, 14) \) and \( (15, 16, 17) \), respectively. In order to establish the imprecise value of the innovation level for each bank, we used the following aggregation function.

\[ \bar{x}_k = \left( x', x', x' \right) = \frac{1}{60} \sum_{j=1}^{60} \bar{x}_{jk} = \left( \frac{x'_{jk}}{60}, \frac{x'_{jk}}{60}, \frac{x'_{jk}}{60} \right), \quad k = 1, 2, \ldots, 14, \quad j = 1, 2, \ldots, 60 \]  

(4.5)

where \( \bar{x}_k \) is the value of the innovation level of the \( k \)th bank, \( \bar{x}_{jk} \) is the value given by the \( j \)th respondent to the \( k \)th bank. The left spread and the right spread are given by \( a = x - x_j \) and \( b = x_{u} - x \), respectively. Table 2 gives the innovation level data for each bank.

<table>
<thead>
<tr>
<th>Bank</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>BNA</td>
<td>(14, 15, 16)</td>
<td>(12, 12.5, 13)</td>
<td>(12, 13, 14)</td>
</tr>
<tr>
<td>STB</td>
<td>(12, 13, 14)</td>
<td>(11, 12, 13)</td>
<td>(11, 13.5, 16)</td>
</tr>
<tr>
<td>BIAT</td>
<td>(12, 13, 14)</td>
<td>(11, 13.5, 16)</td>
<td>(12, 13.5, 15)</td>
</tr>
<tr>
<td>UIB</td>
<td>(9, 10.5, 12)</td>
<td>(11, 11.5, 12)</td>
<td>(12, 13.5, 15)</td>
</tr>
<tr>
<td>BH</td>
<td>(12, 14, 16)</td>
<td>(12, 14, 16)</td>
<td>(9, 10.5, 12)</td>
</tr>
<tr>
<td>BS</td>
<td>(12, 14, 16)</td>
<td>(11, 12.5, 16)</td>
<td>(10, 12.5, 15)</td>
</tr>
<tr>
<td>BT</td>
<td>(12, 13, 14)</td>
<td>(10, 12, 14)</td>
<td>(11, 11.5, 12)</td>
</tr>
<tr>
<td>UBCI</td>
<td>(10, 11, 12)</td>
<td>(11, 10.5, 12)</td>
<td>(12, 12.5, 13)</td>
</tr>
<tr>
<td>ATB</td>
<td>(13, 14, 15)</td>
<td>(12, 13.5, 15)</td>
<td>(11, 12, 14)</td>
</tr>
<tr>
<td>AB</td>
<td>(12, 14, 16)</td>
<td>(10, 12.5, 15)</td>
<td>(11, 12.5, 16)</td>
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<tr>
<td>BFT</td>
<td>(12, 13, 14)</td>
<td>(11, 12, 14)</td>
<td>(8, 10, 12)</td>
</tr>
<tr>
<td>CB</td>
<td>(10, 12, 14)</td>
<td>(10, 11, 12)</td>
<td>(10, 10.5, 11)</td>
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<tr>
<td>BTS</td>
<td>(10, 12, 14)</td>
<td>(8, 10, 12)</td>
<td>(10, 12, 14)</td>
</tr>
<tr>
<td>ABCT</td>
<td>(11, 12, 13)</td>
<td>(10, 10.5, 11)</td>
<td>(10, 10.5, 11)</td>
</tr>
</tbody>
</table>
Used models

In order to obtain the efficiency scores, we used the BRONF approach proposed by Tlig and Rebai [23] and the possibility approach developed by Lertworasirikul et al. [22]. The BRONF approach is a technique based on relations between fuzzy numbers.

With the BRONF approach, to solve the primal of FCCR, the objective function is considered as fuzzy constraint. This constraint was discovered into crisp function with the use of a weighting function. The fuzzy normalization equality was transformed into two crisp equalities by the use of a goal programming problem. Then the resulting model takes the form of crisp linear programming problem and can be solved by standard linear programming software. The crisp equivalent problem of the FCCR is given by:

\[
\begin{align*}
\text{max } & \sum_{r=1}^{m} \lambda_r \left( y_{r0} - c_{r0} (1-h) \right) + \sum_{r=1}^{m} \lambda_r \left( y_{r0} + d_{r0} (1-h) \right), \\
\text{s.t. } & \sum_{i=1}^{n} u_i \left( x_{i0} - a_{i0} (1-h) \right) - \rho_1 = 1, \\
& \sum_{i=1}^{n} u_i \left( x_{i0} + a_{i0} (1-h) \right) - \rho_2 = 1, \\
& \sum_{i=1}^{n} u_i \left( x_{ij} - a_{ij} (1-h) \right) \geq \sum_{r=1}^{m} v_r \left( y_{r0} - c_{r0} (1-h) \right) \\
& \sum_{i=1}^{n} u_i \left( x_{ij} + b_{ij} (1-h) \right) \geq \sum_{r=1}^{m} v_r \left( y_{r0} + d_{r0} (1-h) \right) \\
& u \geq 0, v \geq 0, h \in [0,1]
\end{align*}
\]

Where \( \rho_1 \) and \( \rho_2 \) represents respectively the difference between the positive and the negative deviations in the used goal programming problem.

The possibility approach developed by Lertworasirikul [19] adopts the concept of chance-constrained programming (CCP) to solve fuzzy DEA models. CCP deals with uncertainty by specifying the desired levels of confidence with which the constraints hold. Using the concepts of CCP and possibility of fuzzy events, the equivalent problem of FCCR model with triangular fuzzy data becomes the following crisp possibility CCCR (CPCCR) model:

\[
\begin{align*}
\text{max } & f \quad (1-h) \sum_{r=1}^{m} v_r y_{r0} \geq f, \\
\text{s.t. } & (1-h) \sum_{i=1}^{n} u_i x_{i0} \geq 1, \\
& (1-h) \sum_{i=1}^{n} u_i x_{i0} \leq 1, \\
& (1-h) \left( \sum_{i=1}^{n} u_i x_{ij} + \sum_{r=1}^{m} v_r y_{rj} \right) \leq 0, \quad j = 1, \ldots, n \\
& v \geq 0, \\
& u \geq 0.
\end{align*}
\]

Where \( f \) is the efficiency score of the decision unit maker under evaluation and \( h \) is a possibility level.
4 Results

We used LINGO software to solve models (4.6) and (4.7). As seen in table (3), in 2011, the mean efficiency score obtained by the BRONF approach was 0.879 at the possibility level 0.25. This means that Tunisian banks could reduce their level of inputs by 12.1% to be efficiency. The efficient frontier is constructed by the large banks (BNA, STB and BIAT) which are public banks. The medium-sized banks, especially the UIB and BT are inefficient, but they are not far from the frontier. Indeed, they have a mean efficiency score above 0.790 for all possibility levels.

Table 3: Efficiency scores obtained by the BRONF approach

<table>
<thead>
<tr>
<th>Year</th>
<th>h</th>
<th>BNA</th>
<th>STB</th>
<th>BIAT</th>
<th>UIB</th>
<th>BH</th>
<th>BS</th>
<th>BT</th>
</tr>
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<tbody>
<tr>
<td>2011</td>
<td>0.00</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.878</td>
<td>0.797</td>
<td>0.799</td>
<td>0.785</td>
</tr>
<tr>
<td></td>
<td>0.25</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.871</td>
<td>0.774</td>
<td>0.791</td>
<td>0.723</td>
</tr>
<tr>
<td></td>
<td>0.50</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.863</td>
<td>0.763</td>
<td>0.796</td>
<td>0.723</td>
</tr>
<tr>
<td></td>
<td>0.75</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.854</td>
<td>0.762</td>
<td>0.786</td>
<td>0.722</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.846</td>
<td>0.760</td>
<td>0.785</td>
<td>0.721</td>
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<td></td>
<td>0.00</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.879</td>
<td>0.801</td>
<td>0.799</td>
<td>0.786</td>
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<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.873</td>
<td>0.794</td>
<td>0.798</td>
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<tr>
<td>2012</td>
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<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
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<td>0.783</td>
<td>0.797</td>
<td>0.749</td>
</tr>
<tr>
<td></td>
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<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
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<td>0.765</td>
<td>0.789</td>
<td>0.732</td>
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<td></td>
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<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
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<td>1.000</td>
<td>0.893</td>
<td>0.849</td>
<td>0.812</td>
<td>0.786</td>
</tr>
<tr>
<td>2013</td>
<td>0.50</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>0.896</td>
<td>0.833</td>
<td>0.799</td>
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</tr>
<tr>
<td></td>
<td>0.75</td>
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<td>1.000</td>
<td>0.888</td>
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<td>0.787</td>
<td>0.773</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
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<td>1.000</td>
<td>0.886</td>
<td>0.830</td>
<td>0.786</td>
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</tr>
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</table>

The average efficiencies vary between 84.6% and 89.1%. The highest scores are obtained in 2013 and the lowest are registered in 2011 which is the year of Tunisian revolution. The results indicate that large banks (STB, BIAT and BNA) are more efficient than small banks (BFT, BC,…) and medium-sized banks(BT,BH, AB). This superiority efficiency score can be explained by better resource management and better organization. In addition, these banks make big part of their capital in new technologies and diversified their marketing strategies in order to satisfy their customers.

With the possibility approach, the average efficiency score for all banks, at the possibility level 0.5, was about 0.859 in 2011, 0.983 in 2012 and 0.864 in 2013. Over the entire period, the score was around 0862. So to be efficient, the Tunisian commercial banks must conserve their resources of 13.8%, while keeping the same level of production. Efficiency scores of banks at all possibility levels are shown in table 4.
the fact that fuzzy data have linear membership functions. The efficiency scores obtained by the possibility and the BRONF approach are very similar and this can be explained by the fact that fuzzy data have linear membership functions.

In this study, we propose to use fuzzy data to evaluate the efficiency of Tunisian banks. However, we can sometimes take into account no financial data (fuzzy data) to evaluate the efficiency of banks more accurately and realistically.

The efficiency scores obtained by the Possibility approach are lower than those given by the BRONF approach. At all possibility level, both approaches provide deterministic efficiency values. With the BRONF approach, the efficiency value is a decreasing function of the possibility level. This property is not verified with the possibility approach.

### 5 Conclusion

DEA is widely applied to evaluate the performance of the banking sector since it is capable to calculate the efficiency scores with multi-output and multi-input. Most previous studies used the conventional DEA with only crisp data to evaluate the performance of Tunisian banks. However, we can sometimes take into account no financial data (fuzzy data) to evaluate the efficiency of banks more accurately and realistically.

In this study, we propose an empirical study that consists to assessing the performance of commercial banks in Tunisia during the period 2011-2013. The empirical results show that large banks are the most efficient since it is capable to calculate the efficiency scores with multi-output and multi-input. Most previous studies used the conventional DEA with only crisp data to evaluate the performance of Tunisian banks. However, we can sometimes take into account no financial data (fuzzy data) to evaluate the efficiency of banks more accurately and realistically.

A comparison between tables (3) and (4.4) allows us to make the following notes:

(i) Banks identified as efficient by the BRONF approach are also efficient using the possibility approach.

(ii) The efficiency scores provided by the possibility approach are lower than those given by the BRONF approach.

(iii) At all possibility level, both approaches provide deterministic efficiency values.

(iv) With the BRONF approach, the efficiency value is a decreasing function of the possibility level. This property is not verified with the possibility approach.

### 5 Conclusion

DEA is widely applied to evaluate the performance of the banking sector since it is capable to calculate the efficiency scores with multi-output and multi-input. Most previous studies used the conventional DEA with only crisp data to evaluate the performance of Tunisian banks. However, we can sometimes take into account no financial data (fuzzy data) to evaluate the efficiency of banks more accurately and realistically.

In this study, we propose an empirical study that consists to assessing the performance of commercial banks in Tunisia during the period 2011-2013. The empirical results show that large banks are the most efficient because they spend much of their total budget for investment in new technologies. In addition the efficiency scores obtained by the possibility and the BRONF approach are very similar and this can be explained by the fact that fuzzy data have linear membership functions.
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