Returns To Scale In PL-DEA models

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Abstract
In this paper we consider piecewise linear data envelopment analysis (PL-DEA) model which incorporates piecewise linear functions of factors. Then, we expand the Piecewise linear (PL) DEA models and define the PL-BCC, PL-BCC-CCR, PL-CCR-BCC models and we discuss the RTS methods for the DMUs in the presently available types of PL-DEA models. Also, the mentioned methods are compared and numerical example of these methods is provided for illustration.

Keywords: Data envelopment analysis, Returns to scale, Marginal value, Piecewise linear function.

1 Introduction

Data envelopment analysis (DEA) is a non parametric technique for measuring and evaluating the relative efficiencies of decision making unit (DMU) with multiple inputs and multiple outputs. The economic concept of returns to scale (RTS) have been studied widely nowadays. Among diverse models of DEA, radial models are the most well known ones which include CCR and BCC each of which implies a radial model under constant and variable returns to scale. First contributions on characterizing the RTS were made by Fare and Grosskopf [1] and Fare et al.[2]. Under constant returns to scale technology different approaches for estimating type of the RTS are proposed note that they can determine the type of RTS but not its magnitude [3], [4]. Under variable returns to scale the RTS type can be determined by the sign of the \( u_o \). Banker et al.[7] proposed method for estimating the non radial models, additive and multiplicative. Banker and Thrall [5] gave conditions for identifying the RTS for BCC model which is under variable returns to scale. Banker et al.[3] developed a number of modifications of the models by determining the maximum and minimum values of \( u_o \), which enables their method to determine the RTS without surveying all the alternative multiple solutions. It was showed that possible alternative optimal solutions only affected the estimation of the RTS on DMUs which should be classified as constant returns to scale. Färe et al.[2] proposed a method for evaluating the RTS of a DMU for which three Lp problems must be solved. The significant advantage of their method as compared to other method is that there is no need for exploring alternative optimal solutions. Also in Kerstens et al.[10] method, three LP problems must be solved that seems to be onerous and in regard of computational point of view is problematic.

In many situations, differences in values may not be reflected adequately by linear pricing. We deal with these situations where for certain outputs in input-oriented model, either a nonincreasing or nondecreasing set of multipliers

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for larger magnitude of the factors describes the weight function. Cook and Zhu examined such a situation[8], they verified that on the earlier work of Cook et al.[9] certain factors which were treated as behaving linearly, should be deemed as having a nonlinear impact on the efficiency. Lotfi et al. [6] proposed a modified PL-DEA model in order to determine the best possible target DMUs. In this paper we propose methods for estimating the issue of RTS on DMUs which have decreasing set of multipliers. This paper unfolds as follows: in section two we give a review of piecewise linear DEA and in section three we introduce new PL-DEA models. In section four we investigate for the RTS of DMUs which contains DMV variables. The last two sections provides numerical example and conclusion.

2 piecewise linear DEA background

Let us assume that we have a set of DMUs consisting of DMU, 1 ≤ j ≤ n. Every DMUj used m inputs to produce s outputs. All inputs and outputs are assumed to be nonnegative, but at least one input and one output are positive: xj = (x1j, . . . , xmj), xj ̸= 0 and yj = (y1j, . . . , ysj) , yj ̸= 0. In presence of those variables, which have nonlinear impact on efficiency, k segments are considered for the scale of the variable from the theory of piecewise linear programming. With this logic, the scale of variable should view as consisting of x and y ranges so the constraint (2a) is imposed.

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Then in stead of a single expression μryj, \( \sum_{k=1}^{K} \mu_k y^k_{rj} \) is used. It is necessary that \( \{\mu_k\}_{k=1}^{K} \) should form a decreasing sequence so the constraint (2a) is imposed. It is noteworthy that \( a_{r_k} \) and \( b_{r_k} \) would take on values > 1(< 1) for those variables have increasing and decreasing set of multipliers. The multiplier form of CCR model which contains variables with decreasing set of multipliers is as follows:

\[
Z_1 = \max \sum_{r \in R_1} u_r y_{0r} + \sum_{r \in R_2} \sum_{k=1}^{K_r} u_{r_k} y^k_{rj} - \sum_{r \in R_2} \sum_{k=1}^{K_r} \mu_k y^k_{rj}
\]

s.t.

\[
\sum_{i \in J} v_i x_{i0} = 1
\]

\[
\sum_{r \in R_1} u_r y_{rj} + \sum_{r \in R_2} \sum_{k=1}^{K_r} u_{r_k} y^k_{rj} - \sum_{i \in J} v_i y_{ij} \leq 0 \quad j \in J
\]

\[u_{r_k+1} a_{r_k} \leq u_{r_k} \leq u_{r_k}, \quad k = 1, \ldots, K_r, r \in R_2\]

\[u_r \geq 0, \quad \mu_k \geq 0, \quad v \geq 0, \quad u_{r_k} \geq 0, \quad j \in \{1, \ldots, n\}\]

where:

\[
y^k_{rj} = \begin{cases} 
0, & \text{if } k = 1 \\
0, & \text{if } k = 2, \ldots, k_j - 1 \\
L_k - y_{rj}, & \text{if } k = k_j \\
L_k - L_{k_j-1}, & \text{if } k > k_j 
\end{cases}
\]

R1 and R2, respectively, are used to denote the sets of regular and DMV/IMV outputs. \( k_r \) represents number of intervals \([L_{k_j-1}, L_{k_j}]\).
3 New PL-DEA models

Let us assume that we have a set of \( n \) DMUs consisting of DMU \( j, j = 1, \ldots, n \) with input-output vector \((x_j, y_j)\) in which \( x_j = (x_{1j}, \ldots, x_{mj}) \) and \( y_j = (y_{1j}, \ldots, y_{sj}) \).

With the presence of the variable with decreasing set of multipliers we introduce another PL-DEA model by adding the unrestricted \( u_o \) variable to the PL-CCR model, which we shall designate this new model as a multiplier form of PL-BCC model in the input orientation:

\[
Z_2 = \max \sum_{r \in R_1} u_r y_{r0} + \sum_{r \in R_2} \sum_{k=1}^{K_r} u_{rk} y_{rj}^k - \sum_{r \in R_2} \sum_{k=1}^{K_r} \mu_{rk} y_{rj}^k + u_0
\]

s.t.
\[
\sum_{r \in J} v_{rj} x_{r0} = 1
\]
\[
\sum_{r \in R_1} u_r y_{rj} + \sum_{r \in R_2} \sum_{k=1}^{K_r} u_{rk} y_{rj}^k - \sum_{i \in I} v_{ij} x_{ij} + u_0 \leq 0 \quad \forall j \in J
\]
\[
u \geq 0, \quad u_r \geq 0, \quad v \geq 0, \quad j = \{1, \ldots, n\}
\]
\( u_o \) unrestricted,  \( \mu_{rk} \geq 0 \).

**Theorem 3.1.** In optimal solution of model (3.4) \( \mu_{rk} = 0, k = 1, \ldots, K_r \).

*Proof.* Let \((u^*_r, v^*_j, \mu^*_r, u_o)\) be a optimal solution for model (3.4) Where \( y_{rj}^k \geq 0 \). By contradiction suppose that \( \mu^*_r > 0 \) thus we can have a feasible solution such as:

\((u_r = u^*_r, u_{rk} = u^*_r, \mu_{rk} = 0, u_o)\).

The objective function of this feasible solution is greater than that of optimal one. Therefore in optimal solution \( \mu^*_r = 0 \).

The following model is a multiplier form of PL-CCR-BCC model in the input-orientation:

\[
Z_3 = \max \sum_{r \in R_1} u_r y_{r0} + \sum_{r \in R_2} \sum_{k=1}^{K_r} u_{rk} y_{rj}^k - \sum_{r \in R_2} \sum_{k=1}^{K_r} \mu_{rk} y_{rj}^k - u_0
\]

s.t.
\[
\sum_{i \in I} v_{ij} x_{r0} = 1
\]
\[
\sum_{r \in R_1} \mu_r y_{rj} + \sum_{r \in R_2} \sum_{k=1}^{K_r} \mu_{rk} y_{rj}^k - \sum_{i \in I} v_{ij} x_{ij} - u_0 \leq 0 \quad \forall j \in J
\]
\[
u \geq 0, \quad u_r \geq 0, \quad \mu_r \geq 0, \quad j = \{1, \ldots, n\}
\]
\( u_o \geq 0, \quad \mu_{rk} \geq 0 \).

It is noteworthy that the above theorem holds true here, as well. By our previous explanations we introduce the
multiplier form of PL-BCC-CCR model as follows:

\[
Z_4 = \max \sum_{r \in R_1} u_r y_{r0} + \sum_{r \in R_2} \sum_{k=1}^{K_r} u_{r_k} y_{r_jk} - \sum_{r \in R_2} \sum_{k=1}^{K_r} v_{r_k} x_{r_jk} + u_0
\]

s.t.

\[
\sum_{r \in I} v_r x_{r0} = 1
\]

\[
\sum_{r \in R_1} u_r y_{rj} + \sum_{r \in R_2} \sum_{k=1}^{K_r} u_{r_k} y_{r_jk} - \sum_{r \in I} v_r x_{r_j} + u_0 \leq 0, \quad j \in J
\]

\[
u_{r_k} a_{r_k} \leq \mu_{r_k} \leq \nu_{r_k} a_{r_k}, \quad k = 1, \ldots, K_r, r \in R_2
\]

\[
u_{r_1} a_{r_1} r_{r_2j} \leq \sum_{k=1}^{K_r} \nu_{r_k} a_{r_k} r_{r_2j}, \quad j \in J, r_1 \in R_1, r_2 \in R_2
\]

\[
u_r \geq 0, \quad u_r \geq 0, \quad v \geq 0, J = \{1, \ldots, n\}
\]

\[u_0 \geq 0, \quad \mu_{r_j} \geq 0.
\]

the mentioned details and theorem hold true in this model as well. Here particular weight restrictions are imposed and they do not cause incorrect assessment of the relative efficiency since the only weight restrictions are imposed on output weights, and by imposing such homogeneous weight restrictions the following models do not suffer from being infeasible [11], [12]. And, Since the weight restrictions are imposed on output multipliers the efficiency score will be greater than zero.

4 PL-Returns to scale

Let us assume that we have a set of DMUs consisting of DMU$_j$, $j = 1, \ldots, n$. Every DMU$_j$ produces $s$ outputs $y_{rj}$ and uses $m$ inputs $x_{rj}$. All inputs and outputs are assumed to be nonnegative, but at least one input and one output are positive $x_j = (x_{1j}, \ldots, x_{mj})$, $x_j \neq 0$ and $y_j = (y_{1j}, \ldots, y_{sj})$, $y_j \neq 0$.

We use the objective function of the PL-CCR, PL-BCC, PL-BCC-CCR, PL-CCR-BCC for recognizing the RTS status for a DMU under assessment.

**Definition 4.1.** Let us assume that the DMUo under assessment is PL-BCC efficient, first solve the PL-CCR model, if DMUo is PL-CCR efficient then CRS will prevail on DMUo else i.e. $\theta_{PL-CCR} \neq 1$, solve the PL-BCC-CCR model to see whether DMUo is PL-BCC-CCR efficient or not, if $\theta_{PL-BCC-CCR} = 1$ then IRS will prevail on DMUo, else if DMUo is PL-CCR-BCC efficient then DTRS will prevail on DMUo.

Consider the multiplier form of PL-BCC model in input orientation, from the sign of $u_0^*$ we characterize the RTS status as follow:

**Theorem 4.1.** a) If $u_0^* = 0$ in any of the alternative optimal solutions then CRS prevail on DMUo.

b) If $u_0^* > 0$ for all alternative optimal solutions then IRS prevail on DMUo.

c) If $u_0^* < 0$ for all alternative optimal solution then DRS prevail on DMUo.

**Proof.** If $u_0^* = 0$ since we have $\theta_{PL-BCC} = 1$ hence $\theta_{PL-BCC} = \theta_{PL-CCR} = 1$ so CRS prevail on DMUo. If DMUo exhibits CRS so we have $\theta_{PL-BCC} = \theta_{PL-CCR} = 1$ and it implies that $u_0^* \neq 0$ can not hold true for any optimal solutions in PL-BCC model thus we get $u_0^* = 0$ and the proof (a) is completed. If $u_0^* > 0$ since we have $\theta_{PL-BCC} = 1$ therefore $\theta_{PL-BCC} = \theta_{PL-CCR}$ and $\theta_{PL-BCC} = \theta_{PL-CCR-CRR} = 1$ thus IRS prevail on DMUo. If DMUo exhibits IRS since we have $\theta_{PL-BCC-CRR} = 1$ and also we have $\theta_{PL-BCC} = 1$ and $\theta_{PL-CCR} \neq 1$ and these two imply that $u_0^* = 0$ can not hold true for all optimal solutions of PL-BCC model so by comparing the objective functions of PL-BCC and PL-BCC-CCR, $u_0^* > 0$ is satisfied, and (b) is completed. If $u_0^* < 0$ since we have $\theta_{PL-BCC} = 1$ therefore $\theta_{PL-BCC} = \theta_{PL-CCR}$ and $\theta_{PL-CCR-CRR} = 1$ thus DRS prevail on DMUo. If DMUo exhibits DRS since we $\theta_{PL-CCR-BCC} = 1$ and also we have $\theta_{PL-BCC} = 1$ and $\theta_{PL-CCR} \neq 1$ and these two imply that $u_0^* = 0$ can not hold true for all optimal solutions of PL-BCC model so, $u_0^* < 0$ is satisfied, and (c) is completed.

\[
\square
\]
The following corollary shows that for estimating the RTS there is no need to solve the above four models.

**Theorem 4.2.** If DMU\(_o\) is PL-BCC efficient:

1. IRS prevail on DMU\(_o\) if and only if
   \[ \theta^*_{PL-BCC-CCR} = \theta^*_{PL-CCR-BCC}. \]
2. DRS prevail on DMU\(_o\) if and only if
   \[ \theta^*_{PL-BCC-CCR} \geq \theta^*_{PL-CCR-BCC}. \]
3. CRS prevail on DMU\(_o\) if and only if
   \[ \theta^*_{PL-BCC-CCR} = \theta^*_{PL-CCR-BCC}. \]

So, from theorem 4.1 with help of the sign of \( u^*_o \) the RTS can be characterized. Estimating the RTS with this method involves generating all optimal solutions and it is computationally problematic. Another method for estimating the RTS in a easier way is to solve model (3.4) to see whether there exists alternative optimal solutions or not. In the case of having alternatives, instead of solving models (3.5) and (3.6) the following model can be solved, while we are given the existence of optimal solution \( u^*_o > 0 \).

\[
\min \quad \tilde{u}_o \quad \text{s.t.} \quad \sum_{r \in R_1} \mu_r y_{rj} + \sum_{r \in R_2, k=1}^{K_r} \mu_{r_k} y_{rj}^k - \sum_{i \in I} \nu_i x_{io} + \tilde{u}_o \leq 0 \quad j \in J
\]

\[
\mu_{r_{k+1}} a_{r_k} \leq \mu_{r_k} b_{r_k}, \quad k = 1, \ldots, K_r, r \in R_2
\]

\[
\sum_{r \in R_1} \mu_r y_{ro} + \sum_{r \in R_2, k=1}^{K_r} \mu_{r_k} y_{ro}^k + \tilde{u}_o = Z_2,
\]

\[
\mu_r \geq 0, \quad \mu_{r_k} \geq 0, \quad \nu \geq 0, \quad \tilde{u}_o \geq 0, \quad j \in \{1, \ldots, n\}
\]

The third equation helps us to make sure that we are confined to the efficiency frontier.

**Theorem 4.3.** Given the existence of the optimal solution \( u^*_o \) from PL-BCC model:

1. \( \tilde{u}_o = 0 \) if and only if CRS prevail on DMU\(_o\).
2. \( \tilde{u}_o > 0 \) if and only if IRS prevail on DMU\(_o\).

**Proof.** Having the optimal solution of model (4.7) we have

\[
\sum_{r \in R_1} \mu_r^* y_{ro} + \sum_{r \in R_2, k=1}^{K_r} \mu_{r_k}^* y_{ro}^k + \tilde{u}_o^* = 1
\]

\[
\sum_{i \in I} \nu_i^* x_{io} = 1
\]

so we have

\[
\sum_{r \in R_1} \mu_r^* y_{ro} + \sum_{r \in R_2, k=1}^{K_r} \mu_{r_k}^* y_{ro}^k + \tilde{u}_o^* = \sum_{i \in I} \nu_i^* x_{io}
\]

\[
\sum_{r \in R_1} \mu_r^* y_{ro} + \sum_{r \in R_2, k=1}^{K_r} \mu_{r_k}^* y_{ro}^k = \sum_{i \in I} \nu_i^* x_{io} - \tilde{u}_o^*.
\]
If \( \hat{u}_o^* = 0 \) then the first condition of theorem 4.1 is satisfied. If \( \hat{u}_o^* > 0 \) then the RTS at DMU\( o \) is increasing as it is set forth in the second condition of theorem 4.1.

As you have seen with this method examining all alternative optimal solutions has been avoided. Note that when we are given \( u_o^* \leq 0 \) as an optimal solution of the PL—BCC model, we alter the objective of model(4.7) to maximization and also \( \hat{u}_o \leq 0 \) is replaced with \( \hat{u}_o \geq 0 \) thus we get the following result;

a) The CTS of DMU\( o \) are CRS if and only if \( \hat{u}_o^* = 0 \).

b) The DTS of DMU\( o \) are DRS if and only if \( \hat{u}_o^* < 0 \).

t is noteworthy to say that as you have seen it is possible to estimate the RTS on each DMU without having to utilize the projection while given the existence of optimal solution.

Although with the above method exploring all optimal solutions has been avoided, this method involves solving two models.

We will show that for estimating the RTS there is no need to solve these two models. By solving the following model

\[
\begin{align*}
\min_{u_o, \sum_i y_i x_{o,i} = 1} u_o,
\end{align*}
\]

\[
\begin{align*}
\sum_{r \in R_1} \mu_r y_{ro} + \sum_{r \in R_2} \sum_{k=1}^{K_r} \mu_r^k y_{rko} - \sum_{i \in I} v_i x_{o,i} + u_0 & \leq 0 & j \in J
\end{align*}
\]

\[
\begin{align*}
\sum_{r \in R_1} \mu_r y_{ro} + \sum_{r \in R_2} \sum_{k=1}^{K_r} \mu_r^k y_{rko} + u_0 & = 1
\end{align*}
\]

we can say:

**Theorem 4.4.** a) If \( u_o^* > 0 \) then IRS prevail on DMU\( o \).
b) If \( u_o^* = 0 \) then CRS prevail on DMU\( o \).
c) If model is infeasible then \( u_o^* < 0 \) and DRS prevail DMU\( o \).

5 An application of PL-DEA models for estimating the RTS

In this section we consider twenty DMUs with four outputs and three inputs. Among these outputs, two outputs behave in nonlinear manner.In standard model and PL-DEA models we impose on the multipliers \( \mu_1, \mu_2, \mu_3 \text{and} \mu_4 \), the AR constraints

\[ 3 \mu_2 \leq \mu_1 \quad \text{and} \quad 3 \mu_3 \leq \mu_2 \]

and

\[ 5 \mu_1 \leq \mu_4 \leq \mu_1. \]

Also in piecewise models to invoke the requirement that each of the third and the fourth outputs should form a nonincreasing set of multipliers, we impose on the multipliers \( \mu_{41}, \mu_{42}, \mu_{43} \text{and} \mu_{31}, \mu_{32}, \mu_{33} \), respectively the constraints

\[ 4 \mu_{32} \leq \mu_{31} \leq 8 \mu_{32} \quad \text{and} \quad 4 \mu_{33} \leq \mu_{32} \leq 8 \mu_{33} \]

\[ 4 \mu_{42} \leq \mu_{41} \leq 8 \mu_{42} \quad \text{and} \quad 4 \mu_{43} \leq \mu_{42} \leq 8 \mu_{43}. \]
Here we take $k_r = 3$, and we use the ranges $[0, 200)$, $[200, 300)$, $[300, 500]$ for the third output and $[0, 800)$, $[800, 900)$, $[900, 1000]$ for the fourth output. By using pl-CCR-BCC and pl-BCC-CCR models the RTS of the following units can be characterized. The PL-BCC, PL-BCC-CCR and PL-CCR-BCC scores are displayed respectively in columns of Table 2. And the estimated RTS is displayed in the last column of Table 2. In the following the data are displayed in Table 1.

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<td>322</td>
<td>678</td>
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</table>
Table 2: Results

<table>
<thead>
<tr>
<th>DMU number</th>
<th>PL-BCC</th>
<th>PL-BCC-CCR</th>
<th>PL-CCR-BCC</th>
<th>RTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.86</td>
<td>0.86</td>
<td>0.7</td>
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<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>CRS</td>
</tr>
<tr>
<td>3</td>
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<td>0.79</td>
<td>0.98</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>0.98</td>
<td>0.81</td>
<td>0.98</td>
<td>-</td>
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<tr>
<td>5</td>
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<tr>
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<tr>
<td>7</td>
<td>1</td>
<td>0.76</td>
<td>1</td>
<td>DRS</td>
</tr>
<tr>
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<td>1</td>
<td>1</td>
<td>CRS</td>
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<tr>
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<td>0.76</td>
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<td>0.97</td>
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<td>1</td>
<td>1</td>
<td>CRS</td>
</tr>
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<td>0.76</td>
<td>-</td>
</tr>
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<td>0.97</td>
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<tr>
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<tr>
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<td>0.54</td>
<td>-</td>
</tr>
<tr>
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<td>0.76</td>
<td>0.76</td>
<td>0.64</td>
<td>-</td>
</tr>
<tr>
<td>20</td>
<td>0.98</td>
<td>0.78</td>
<td>0.98</td>
<td>-</td>
</tr>
</tbody>
</table>

As you see four units are PL-BCC efficient, and by comparing PL-BCC-CCR and PL-CCR-BCC scored the RTS of units 2, 7, 8 and 12 respectively CRS, DRS, CRS and CRS. The following table contains the RTS of the mentioned units, which are estimated with model (4.8).

Table 3: RTS status

<table>
<thead>
<tr>
<th>DMU number</th>
<th>2</th>
<th>7</th>
<th>8</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>RTS</td>
<td>CRS</td>
<td>DRS</td>
<td>CRS</td>
<td>CRS</td>
</tr>
</tbody>
</table>

6 Conclusion

The current study gives a method for characterizing the RTS in presence of variables which behave in nonlinear manner. To capture this idea special weight restrictions are imposed to the model. By adding such weight restrictions the status of returns to scale may change. Considering this discussion we introduced methods for estimating the RTS in presence of those variables for which a set of decreasing multipliers define the weight function.

References

   http://dx.doi.org/10.1007/978-94-015-7721-2


