Evaluating performance supply chain by a new non-radial network DEA model with fuzzy data

Mohsen Rostamy-Malkhalifeh*
Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran

Elahe Mollaeian
Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran

ABSTRACT

Data Envelopment Analysis (DEA) is a non-parametric technique for evaluating the efficiency of Decision Making Units (DMUs) with multiple inputs and outputs. Evaluating performance supply chain is one of the uses of DEA. But hence, the traditional DEA models treat with each DMU as a “black box”, thus, the performance measurement may be not effective. So, there are necessities for network DEA models. The primary condition for the use of DEA models is that the data are exact. But in the real world, we are often conformed to vague and uncertain data and performance evaluation by usual methods in the presence such data may lead errors in decision-making process, so for making applied decision and more adaptive to real world, it is undeniable need for fuzzy logic to evaluate the efficiency of unit. In this paper, at first, a new non-radial network DEA model for evaluating performance supply chain is introduced, by considering intermediate production. Its optimal solution can separate inefficient and strong efficient DMUs, and finally we solve this model when the all data are fuzzy numbers.

Keywords: Data Envelopment Analysis (DEA); Fuzzy Number; Supply Chain; Network DEA.

1. Introduction

Data Envelopment Analysis (DEA) was suggested by Charnes et al. [1], is a mathematic technique for evaluating the relative efficiency of a set Decision Making Units (DMUs), with the common set of inputs and outputs. The assumption of DEA was based on the exact data, But in the real word, data presented by natural languages including good, bad,… . Usually data are imprecise and performance evaluation by usual methods in the presence of inaccurate data may lead to errors in decision making process and conventional DEA cannot easily measure the performance, so for making sensible
decision and more adaptive to real word, it is required to use fuzzy logic as a tool to achieve the final objective. Bellman and Zadeh proposed the concept of decision making in fuzzy environment [12]. Several approaches have been developed to deal with fuzzy data in DEA, Sengupta, who was the first person researches about using fuzzy set theory in the DEA and applied principle of fuzzy set theory to introduce fuzziness in the objective function and the right-hand-said vector of the conventional DEA model [10]. Chiang and Shiang, developed a method which is able to provide fuzzy efficiency measures for DMUs, with fuzzy observations [4]. Guo, Tanaka used the ranking method and introduced a bi-level programming model [6].

Evaluation performance is an important issue in supply chain management. But in regard to supply chain has complex structure, must be its scale of performance so that consider to its internal structure. But classical DEA models make no assumption concerning the internal operations of a DMU and measure the perfect performance of a DMU, in other hand, teats each DMU as a “black box”. And only consider the initial inputs to product final outputs. So to estimate the efficiency of complex network system, several authors proposed Network DEA models, and now there are a number of Network DEA models for evaluating performance of supply chain. Fure and Grosskopf proposed a Network DEA model for measuring efficiency for DMUs with multiple production stage [5]. Seiford and Zhu [9] and Chen and Zhu [3], provide two approaches in modeling efficiency as a two-stage process. Liang et al. identified the efficiency of supply chain and its members through one DEA model [7].

Yan and Chen [2] proposed the Network DEA models for measuring efficiency of supply chain, that base of its work was a radial network DEA model under three mechanisms: Centralized, Decentralized and Mixed. Then we have introduced a model in regard to advantages of non-radial models. The implement process of this model is according to non-radial SBM model that works base of Slacks. The rest of this paper is organized as follows, Section 2 introduce a new non-radial Network DEA model for measuring the efficiency of the supply chain and its members. In section 3, preliminary definition of fuzzy logic is expressed. Then, the model presented in Section 2, is evaluated with fuzzy data. Numerical example is given in section 4, and finally in section 5 we present the conclusion of this paper.

2. Propose model

In this section we propose a new non-radial network DEA model to evaluate performance of supply chain. Such as benefits of our model on previous models could mention, that, the our model can identify strong efficient DMUs of inefficient DMUs by solving one problem, while the radial models need to solve two problems for separating inefficient DMUs or strong efficient DMUs.

Consider a two stage supplier-manufacturer chain as following Figure 1. Where S and M characterize the supplier and the manufacturer, respectively. X is the input vector of supplier (S) and Y1, Y2 are its output vectors which are also input vectors to the manufacturer stage Z1 and Z2 corresponding to manufacturer (M1) and manufacturer (M2), respectively[2]. Consider n same supply chains called Decision Making Units (DMUs) in DEA literatures. Assume the Decentralized mechanism, that ever sections of supply chain are control under unique decision maker. When the jth supply chain or DMU, called DMUo in short, is under evaluation, the supply chain efficiency is calculated by our model as follow:
\[
\min P_{\text{decentral}} = 1 - \frac{1}{m} \sum_{i=1}^{m} \frac{s_i^{-}}{x_{i0}} \left( 1 + \frac{1}{a} \sum_{k=1}^{a} \frac{b_{k}^{1+}}{z_{o}^{1+}} \right) \left( 1 + \frac{1}{a} \sum_{k=1}^{a} \frac{b_{k}^{2+}}{z_{o}^{2+}} \right)
\]

s.t
\[
\sum_{j=1}^{n} \lambda^{1} \ x_{ij} + s^{-}_{i} = x_{io} \\
\sum_{j=1}^{n} \lambda^{2} \ y_{ij}^{1} \geq \sum_{j=1}^{n} \lambda^{3} \ y_{ij}^{2} \\
\sum_{j=1}^{n} \lambda^{2} \ y_{ij}^{2} \geq \sum_{j=1}^{n} \lambda^{1} \ y_{ij}^{1} \\
\sum_{j=1}^{n} \lambda^{2} \ z_{kj}^{1} - b_{k}^{1+} = z_{ko}^{1} \\
\sum_{j=1}^{n} \lambda^{2} \ y_{ij}^{1} \leq y_{io}^{1} \\
\sum_{j=1}^{n} \lambda^{1} \ z_{kj}^{2} - b_{k}^{2+} = z_{ko}^{2} \\
\sum_{j=1}^{n} \lambda^{1} \ y_{ij}^{2} \leq y_{io}^{2} \\
\lambda^{1}, \lambda^{2}, \lambda^{3} \geq 0, j = 1, \ldots, n
\]

The first, the fourth and the sixth inequalities in constraint set represent the initial input \(X\) at given final outputs. The second, the third, the fifth and seventh inequalities in above constraint set are corresponding to intermediate products. Furthermore, note that according to the second and third constrains, inputs of manufacturer’s parts mustn’t be more than amount of output of supplier’s parts.

**Theorem 1.** DMU\(_o\) is the Parato-efficient in non-radial model if and only if \(P^{*} = 1\).

**Proof** Suppose \(P^{*} = 1\), then;

\[
1 - \frac{1}{m} \sum_{i=1}^{m} \frac{s_i^{-}}{x_{i0}} = \left( 1 + \frac{1}{a} \sum_{k=1}^{a} \frac{b_{k}^{1+}}{z_{o}^{1+}} \right) \left( 1 + \frac{1}{a} \sum_{k=1}^{a} \frac{b_{k}^{2+}}{z_{o}^{2+}} \right)
\]

\[
\Rightarrow \frac{1}{a} \sum_{k=1}^{a} \frac{b_{k}^{2+}}{z_{o}^{2+}} + \frac{1}{a} \sum_{k=1}^{a} \frac{b_{k}^{1+}}{z_{o}^{1+}} + \left( \frac{1}{a} \sum_{k=1}^{a} \frac{b_{k}^{1+}}{z_{o}^{1+}} \right) \left( \frac{1}{a} \sum_{k=1}^{a} \frac{b_{k}^{2+}}{z_{o}^{2+}} \right) + \frac{1}{m} \sum_{i=1}^{m} \frac{s_i^{-}}{x_{i0}} = 0
\]

\[
\Rightarrow s_i^{-} = b_i^{1+} = b_i^{2+} = 0
\]

Its inverse is true.

**Corollary.** Above theorem states that all inefficiency consider in the computation of \(P\). In this model, weak efficiency DMUs does not show and gets to powerful efficiency through solving a model.

**Theorem 2.**
We indicate that \(\theta^{*}_{\text{decentral}}\) is the optimal solution of radial model [1]

\[
P^{*}_{\text{decentral}} \leq \theta^{*}_{\text{decentral}}
\]
**Proof** suppose \((\lambda^1*, \lambda^2*, \lambda^3*, s^-, b^1*, b^2*, \theta*)\) is the optimal solution for decentral control model [1]. Replacing this optimal solution in constraints of this model:

\[
\begin{align*}
\sum_{j=1}^{n} \lambda^1_{j} x_{ij} + s_{i}^- &= \theta^*_{decentral} x_{io} \\
\sum_{j=1}^{n} \lambda^2_{j} y_{rj}^1 &\geq \sum_{j=1}^{n} \lambda^2_{j} y_{rj}^1 \\
\sum_{j=1}^{n} \lambda^3_{j} y_{rj}^2 &\geq \sum_{j=1}^{n} \lambda^3_{j} y_{rj}^2 \\
\sum_{j=1}^{n} \lambda^2_{j} z_{kj}^1 - b_{1}^* &= z_{k}^1 \\
\sum_{j=1}^{n} \lambda^3_{j} z_{kj}^2 - b_{2}^* &= z_{k}^2 \\
\sum_{j=1}^{n} \lambda^2_{j} y_{rj}^1 &\leq y_{r}^1 \\
\sum_{j=1}^{n} \lambda^3_{j} y_{rj}^1 &\leq y_{r}^2 
\end{align*}
\]

It can write:

\[
\begin{align*}
x_{io} &= \sum_{j=1}^{n} \lambda^1_{j} x_{ij} + s_{i}^- + (1 - \theta^*) x_{io} \\
z_{k}^1 &= \sum_{j=1}^{n} \lambda^2_{j} z_{kj}^1 - b_{1}^* \\
z_{k}^2 &= \sum_{j=1}^{n} \lambda^3_{j} z_{kj}^2 - b_{2}^* 
\end{align*}
\]

And we replace;

\[
\begin{align*}
\lambda^1 &= \lambda^1*, \quad \lambda^2 = \lambda^2*, \quad \lambda^3 = \lambda^3* \\
s^- &= s^- + (1 - \theta) x_{io} \\
b^1 &= b^1*, \quad b^2 = b^2*. 
\end{align*}
\]

And \((\lambda^1, \lambda^2, \lambda^3, s^-, b^1, b^2)\) is introduced a feasible solution for non-radial model. Now we get the value of objective function via replacing feasible solution:

\[
P_{decentral} = \frac{1 - \frac{1}{m} \left\{ \sum_{i=1}^{m} s_{i}^- + (1 - \theta^*) \right\}}{\left(1 + \frac{1}{a} \sum_{k=1}^{a} b_{1}^* \right) \left(1 + \frac{1}{a} \sum_{k=1}^{a} b_{2}^* \right)}
\]
Therefore we have:
\[ P_{decentral} \leq \theta^*_{decentral} \]

And because the problem is minimal and we have through replacing feasible solution; \( p \leq \theta^* \), then we will have:
\[ P^*_{decentral} \leq \theta^*_{decentral} \]

3. Preliminary

In the real life, problems there may exist uncertainly about the parameters. In such a situation the parameters of linear programming problems may be represented a fuzzy number. Nowadays, there are a number of researches about fuzzy logic and some methods for solving DEA models with fuzzy data. In this section, we give some basic concept of fuzzy numbers. Then we consider the model that is introduced in section 2 with fuzzy data.

3.1 Definition

In this section, we give some basic concepts of fuzzy numbers which are needed in the rest of paper.

3.1.1 Definition A fuzzy number \( \tilde{a} = (a_1, a_2, \alpha_1, \alpha_2) \) is said to be a trapezoidal fuzzy number, if its membership function is given by function:

\[
\mu_{\tilde{a}}(x) = \begin{cases} 
\frac{x-(a_1-a_2)}{a_1} & a_1 - \alpha_1 \leq x \leq a_1, \\
1 & a_1 \leq x \leq a_2, \\
\frac{(a_2+a_3)-x}{a_2} & a_2 \leq x \leq a_2 + \alpha_2, \\
0 & \text{else}
\end{cases}
\]

The \( \alpha \)-cut set of \( \tilde{a} \), denoted by \( \tilde{a}_\alpha \), is:
\[ \tilde{a}_\alpha = \{ x, \mu_{\tilde{a}}(x) \geq \alpha \}. \]

3.1.2 Definition For any arbitrary fuzzy number \( \tilde{a} \),\[12\]
\[ R(\tilde{a}) = \frac{1}{2} \int_0^1 (\inf \tilde{a}_\alpha + \sup \tilde{a}_\alpha) \, d\alpha. \]

This reduces to
\[ R(\tilde{a}) = \frac{1}{2} (a_1 + a_2) + \frac{1}{4} (a_2 - a_1). \]
3.1.3 Definition Suppose $\tilde{a}$ and $\tilde{b}$ are fuzzy numbers so we have:

$$\tilde{a} \geq \tilde{b} \quad \text{if and only if} \quad R(\tilde{a}) \geq R(\tilde{b}).$$


3.2 Fuzzy DEA model

Consider the set of supply chain, such as n, DMUs, including DMUj : j=1,…,n. with fuzzy inputs, outputs and intermediate products.

\[ X_{ij} = (X_{1ij}, X_{2ij}, X_{3ij}, X_{4ij}), \quad \tilde{Y}_{rj} = (Y_{11rj}, Y_{12rj}, Y_{13rj}, Y_{14rj}), \quad \tilde{Z}_{k} = (Z_{21k}, Z_{22k}, Z_{23k}, Z_{24k}) \]

are inputs, intermediate products and outputs respectively. And \( \tilde{s}_i = (s_{1i}, s_{2i}, s_{3i}, s_{4i}) \), \( \tilde{b}_k = (b_{1k}, b_{2k}, b_{3k}, b_{4k}) \) and all of them have trapezoidal property. Now consider model (1) with fuzzy data as follows:

\[
\begin{align*}
\min P_{\text{Decentral}} = & -\frac{1}{m} \sum_{i=1}^{m} \frac{\tilde{S}_i}{\tilde{X}_{io}} \\
\text{s.t.} & \sum_{j=1}^{n} \lambda_{ij}^1 \tilde{x}_{ij} + \tilde{s}_i = \tilde{X}_{io} \\
& \sum_{j=1}^{n} \lambda_{ij}^1 \tilde{y}_{rj}^1 \geq \sum_{j=1}^{n} \lambda_{ij}^2 \tilde{y}_{rj}^2 \\
& \sum_{j=1}^{n} \lambda_{ij}^1 \tilde{y}_{rj}^1 \geq \sum_{j=1}^{n} \lambda_{ij}^2 \tilde{y}_{rj}^2 \\
& \sum_{j=1}^{n} \lambda_{ij}^2 \tilde{z}_{kj} - \tilde{b}_k^+ = \tilde{z}_{ko} \\
& \sum_{j=1}^{n} \lambda_{ij}^2 \tilde{z}_{kj} - \tilde{b}_k^+ = \tilde{z}_{ko} \\
& \sum_{j=1}^{n} \lambda_{ij}^2 \tilde{y}_{rj}^1 \leq \tilde{y}_{r0} \\
& \sum_{j=1}^{n} \lambda_{ij}^2 \tilde{y}_{rj}^1 \leq \tilde{y}_{r0} \\
& \lambda_{ij}^1, \lambda_{ij}^2, \lambda_{ij}^3 \geq 0, j = 1, \ldots, n
\end{align*}
\]

Model (2) deals with a case that inputs and outputs are fuzzy. In that manner, all inputs, outputs, middle productions and slack variables are fuzzy numbers. There isn’t possible to solve this model with current methods and must be used from de-fuzzy methods that it is possible with the following way.
By using ranking function, we have:

\[
\sum_{j=1}^{n} \lambda_j^1 \left[ \frac{1}{2} \left( x_{ij} + x_{2ij} \right) + \frac{1}{4} \left( x_{4ij} - x_{3ij} \right) \right] + \frac{1}{2} \left( s_{1l} + s_{2l} \right) + \frac{1}{4} \left( s_{4l} - s_{3l} \right) = \left[ \frac{1}{2} \left( x_{11o} + x_{21o} \right) + \frac{1}{4} \left( x_{41o} - x_{31o} \right) \right].
\]

The rest of the problem’s constraints are changing with the same way.

Now consider the following changes of variable:

\[
\tilde{x}_{ij}^r = \left[ \frac{1}{2} \left( x_{ij} + x_{2ij} \right) + \frac{1}{4} \left( x_{4ij} - x_{3ij} \right) \right], \quad \tilde{s}_i^r = \left[ \frac{1}{2} \left( s_{1l} + s_{2l} \right) + \frac{1}{4} \left( s_{4l} - s_{3l} \right) \right], \quad \tilde{x}_{1o}^r = \left[ \frac{1}{2} \left( x_{11o} + x_{21o} \right) + \frac{1}{4} \left( x_{41o} - x_{31o} \right) \right].
\]

We apply such this variables changing for all the problem’s variables and model (2) can be converted to a form as follows:

\[
\min P_{\text{decentral}} = \frac{1 - \frac{1}{m} \sum_{i=1}^{m} \tilde{s}_i^r}{\left( 1 + \frac{1}{a} \sum_{k=1}^{a} \frac{b_k^{i+}}{z^o} \right) \left( 1 + \frac{1}{a} \sum_{k=1}^{a} \frac{b_k^{i+}}{z^o} \right)}
\]

s.t

\[
\sum_{j=1}^{n} \lambda_j^1 \tilde{x}_{ij}^r + \tilde{s}_i^r = \tilde{x}_{io}^r \quad (3)
\]

In the end, model (2) is conversed to model (3) with using Yager ranking function and applying variable changing. All inputs, outputs, middle productions and slack variables are crisp and we can easily evaluate the performance of supply chain with a non-radial network DEA model. Current methods can be used for solving this model. Note that according to [8] these two models’ optimal solution is equivalent.
4. Numerical example

Assume a two-stage supply chains according to Figure 1. We show the supply chain's with trapezoidal fuzzy data in Table 1 that include of 5 DMUs. We have shown in Table 2 the data by using ranking function are changed in to crisp data and the results of solving model $p_{Deccental}$ for the estimation of supply chain's efficiency in final column. We can see DMU$_1$ is parato-efficient and other DMUs are inefficient.

Table 1

<table>
<thead>
<tr>
<th></th>
<th>$x_1$</th>
<th>$y_1$</th>
<th>$y_2$</th>
<th>$z_1$</th>
<th>$z_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DMU1</td>
<td>(4, 8, 1/3, 1/3)</td>
<td>(4,6,2,14)</td>
<td>(1/2,3/2, 1/4, 1/4)</td>
<td>(10,24, 1/5,1/5)</td>
<td>(3/2 , 5/2 , 1 , 5)</td>
</tr>
<tr>
<td>DMU2</td>
<td>(3.5,2,6)</td>
<td>(1.5,1/2, 9/2)</td>
<td>(2.3,1,7)</td>
<td>(2.5,1,3)</td>
<td>(1.5,1/2,1/2)</td>
</tr>
<tr>
<td>DMU3</td>
<td>(3, 1/2, 1/2)</td>
<td>(2,4,1,5)</td>
<td>(1/3,5/3,1/8,1/8)</td>
<td>(7,9,1/6,1/6)</td>
<td>(3/2 , 2 , 1, 2)</td>
</tr>
<tr>
<td>DMU4</td>
<td>(3, 7,1/3, 1/3)</td>
<td>(1/3,5/3,1/9, 1/9)</td>
<td>(2,4,1,8, 1/8)</td>
<td>(1,3, 1/2, 1/2)</td>
<td>(3, 5, 2,10)</td>
</tr>
<tr>
<td>DMU5</td>
<td>(9, 15, 1/5,1/5)</td>
<td>(3,5,2,18)</td>
<td>(1,3, 1/9, 1/9)</td>
<td>(6,10,1/2,1/2)</td>
<td>(2,4, 1,5)</td>
</tr>
</tbody>
</table>

Table 2

<table>
<thead>
<tr>
<th></th>
<th>$x_1$</th>
<th>$y_1$</th>
<th>$y_2$</th>
<th>$z_1$</th>
<th>$z_2$</th>
<th>$p$</th>
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<tbody>
<tr>
<td>DMU1</td>
<td>6</td>
<td>8</td>
<td>1</td>
<td>17</td>
<td>3</td>
<td>1.00</td>
</tr>
<tr>
<td>DMU2</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>0.20</td>
</tr>
<tr>
<td>DMU3</td>
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<td>4</td>
<td>1</td>
<td>8</td>
<td>2</td>
<td>0.35</td>
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<td>1</td>
<td>3</td>
<td>2</td>
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<td>8</td>
<td>2</td>
<td>8</td>
<td>4</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Figure 1. Supplier-Manufacturer supply chain

5. Conclusion

The manager must have exact and sufficient information of organization's performance. But in some cases encounter with inaccurate data so a fuzzy DEA model is used. In this paper is considered the state that ever section of a supply chain control by a unique decision maker and by incorporating the advantages of non-radial model, introduced a new non-radial network DEA model. This model can
categorize efficient and inefficient DMUs, through one problem solving certainly, while the previous models need to solve two problems. Also we compared the optimal solution of our model with a radial model. Then the fuzzy version is shown for cases dealing with fuzzy data and it has become a definitive model.

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