Application of Extended Tanh Method to Generalized Burgers-type Equations

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Abstract

In this paper, we show that the extended tanh method can be applied readily to generate exact soliton solutions of generalized forms of Burgers-KdV, Burgers-EW, two-dimensional Burgers-KdV and two-dimensional Burgers-EW equations.

Keywords: Extended tanh method; generalized Burgers-type equations

1 Introduction

Nonlinear partial differential equations (PDEs) are widely used as models to describe physical phenomena in various fields of sciences such as fluid mechanics, solid state physics, plasma physics, plasma wave, chemical physics, condensed matter physics, optical fibers, biology, chemical kinematics, chemical physics and geochemistry.

Various powerful methods such as, Backlund transformation [40, 41], inverse scattering method [2], F-expansion method [27], homogeneous balance method [12], Hirota’s method [20] and Jacobi elliptic function method [17], have been developed to obtain exact soliton solutions of these equations.

Recently, tanh method [1, 5, 13, 24, 37, 47] which is a direct and beneficial algebraic method has been proposed to handling nonlinear equations. Fan [14, 15] extended this method after obtaining a new exact soliton solution that can not be handled by tanh method. The extended tanh method developed by Wazwaz [43, 44], provides a wider applicability for handling nonlinear wave equations [45, 46]. El-Wakil et al. [8, 9] modified the extended tanh method and obtained some new exact solutions for several types of nonlinear PDEs.

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The aim of this paper is to find the exact soliton solutions of some important generalized forms of Burgers-type equations such as the the generalized Burgers-KdV equation, the generalized Burgers-EW equation, the generalized two-dimensional Burgers-KdV equation and the generalized two-dimensional Burgers-EW equation, via extended tanh method, where the mathematical softwares facilitate the computational work.

2 The extended tanh method

The following nonlinear PDE
\[ F(u, u_t, u_x, u_y, u_{xx}, u_{yy}, \ldots) = 0, \quad (2.1) \]
can be converted to an ordinary differential equation
\[ G(v, v', v'', v''' \ldots) = 0, \quad (2.2) \]
using the transformation
\[ u(x, t) = v(\xi) \]
and the wave variable \( \xi = x - ct \). Eq.(2.2) is then integrated as long as all terms contain derivatives where integration constants are considered zeros. Using a new independent variable
\[ Y = \tanh(\mu \xi), \quad \xi = x - ct, \]
we have the following change of derivatives
\[
\frac{d}{d\xi} = \mu (1 - Y^2) \frac{d}{dY}, \\
\frac{d^2}{d\xi^2} = -2\mu^2 Y(1 - Y^2) \frac{d}{dY} + \mu^2 (1 - Y^2)^2 \frac{d^2}{dY^2}.
\]
The extended tanh method admits the use of finite expansion,
\[ v(\xi) = S(Y) = \sum_{k=0}^{n} a_k Y^k + \sum_{k=1}^{n} b_k Y^{-k}, \quad (2.3) \]
where \( n \) is a positive integer which will be determined by balancing the highest-order derivative term with the nonlinear terms in the resulting equation. If \( n \) is not an integer, then a transformation formula should be applied to overcome this difficulty. Expansion (2.3) reduces to the standard tanh method for \( b_k = 0, 1 \leq k \leq n \). By substituting Eq.(2.3) in Eq.(2.2), we have an algebraic system of equations in powers of \( Y \) that will lead to the determination of the parameters \( a_k \) \((k = 0, \ldots, n)\), \( b_k \) \((k = 1, \ldots, n)\), \( \mu \) and \( c \). Having these parameters, we obtain an exact solution \( u \) in a closed form.
3 The generalized Burgers-KdV equation

The KdV equation is the generic model for the study of weakly nonlinear long waves [31]. It arises in physical systems which involve a balance between nonlinearity and dispersion at leading-order [32]. For example, it describes surface waves of long wavelength and small amplitude on shallow water and internal waves in a shallow density-stratified fluid. The KdV equation is integrable by the inverse scattering transform [42].

The Burgers equation was first introduced by Bateman [3] and later treated by Burgers [4] after whom such an equation is widely referred to as Burgers’ equation. This equation plays a major role in the study of nonlinear waves since it is used as a mathematical model in turbulence problems, in the theory of shock waves, and in continuous stochastic processes [6].

In this paper, we consider the generalized Burgers-KdV equation of the form

\[ u_t + pu^m u_x + qu_{xx} - ru_{xxx} = 0. \]  

(3.4)

where \( p, q, r, \) and \( m \) are real constants. This equation incorporates the KdV equation \((m = 1, q = 0)\), modified KdV equation \((m = 2, q = 0)\), generalized KdV equation \((q = 0)\), Burgers equation \((m = 1, r = 0)\), modified Burgers equation \((m = 2, r = 0)\), generalized Burgers equation \((r = 0)\), and the modified Burgers-KdV equation \((m = 2)\), which are integrable. These equations are widely used in such fields as solid-states physics, plasma physics, fluid physics and quantum field theory [23, 25, 26, 33].

Many scientists are devoted to studying the exact solutions of these equations. Wazwaz and Triki surveyed bright soliton solutions of the generalized Burgers-KdV equation with time–dependent coefficients [34]. Wazwaz also implemented the standard tanh method for finding the travelling wave solutions of the generalized forms of Burgers and Burgers-KdV equations [39]. Lu et.al obtained solitary wave and periodic wave solutions for general types of KdV and Burgers-KdV equations by using extended Riccati equations [29]. Demiray presented a travelling wave solution to the Burgers-KdV equation by introducing a new potential function and by using the hyperbolic tangent method and an exponential rational function approach [7]. Soliman applied the Exp-function method to obtain generalized solitary solutions of the Burgers-KdV equation [38]. Hassan constructed the solitary wave solutions of the Burgers-KdV equation by using the special truncated expansion method [19].

In this section, we employ the extended tanh method to find soliton solutions of the generalized Burgers-KdV equation. Using the wave variable \( \xi = x - ct \) and \( u(x, t) = v(\xi) \), Eq.(3.4) changes to

\[ -cv' + pv^m v' + qv'' - rv''' = 0. \]  

(3.5)

By integrating Eq.(3.5) and neglecting the constant of integration, we have

\[-cv + \frac{p}{m+1} v^{m+1} + qv' - rv'' = 0. \]  

(3.6)

Now, we choose a particular change for the dependent variable \( v(\xi) = \phi^{2/m}(\xi) \) to reduce the power of the nonlinear term \( v^{m+1} \) in Eq.(3.6). After transformation and simplification we obtain

\[-c\phi^2 + \frac{p}{m+1} \phi^4 + \frac{2q}{m} \phi \phi' - \frac{2r}{m^2} \phi \phi'' - \frac{2r(2-m)}{m^2} (\phi')^2 = 0. \]  

(3.7)
Balancing $\phi\phi''$ with $\phi^4$ in Eq.(3.7) gives
\[ n = 1. \]
Therefore, we must set
\[ \phi(\xi) = S(Y) = a_0 + a_1 Y + \frac{b_1}{Y}. \] 
(3.8)
Substituting Eq.(3.8) in Eq.(3.7) and collecting the coefficients of $Y$, we obtain a system of algebraic equations for $a_0$, $a_1$, $b_1$, $c$ and $\mu$. Solving this system gives the following sets of solutions
(i) The first set
\[ a_0 = \pm \frac{q}{m+4} \sqrt{\frac{m^2+3m+2}{2r}}, \quad a_1 = \pm \frac{q}{m+4} \sqrt{\frac{m^2+3m+2}{2r}}, \]
\[ b_1 = 0, \quad \mu = \pm \frac{mq}{2r(m+4)}, \quad c = \frac{2q^2(m+2)}{r(m+4)^2}. \]
(ii) The second set
\[ a_0 = \pm \frac{q}{m+4} \sqrt{\frac{m^2+3m+2}{2r}}, \quad b_1 = \pm \frac{q}{m+4} \sqrt{\frac{m^2+3m+2}{2r}}, \]
\[ a_1 = 0, \quad \mu = \pm \frac{mq}{2r(m+4)}, \quad c = \frac{2q^2(m+2)}{r(m+4)^2}. \]
(iii) The third set
\[ a_0 = \pm \frac{2q}{m+4} \sqrt{\frac{m^2+3m+2}{8r}}, \quad a_1 = \pm \frac{q}{m+4} \sqrt{\frac{m^2+3m+2}{8r}}, \]
\[ b_1 = \pm \frac{q}{m+4} \sqrt{\frac{m^2+3m+2}{8r}}, \quad \mu = \pm \frac{mq}{4r(m+4)}, \quad c = \frac{2q^2(m+2)}{r(m+4)^2}. \]
Whereas $u(x,t) = v(\xi) = \phi^2/m(\xi)$, we have the following soliton solutions
\[ u_1 = \left( \pm \frac{q}{m+4} \sqrt{\frac{m^2+3m+2}{2r}} \left( 1 + \tanh \left( \pm \frac{mq}{2r(m+4)}(x - ct) \right) \right) \right)^{2/m}, \]
\[ u_2 = \left( \pm \frac{q}{m+4} \sqrt{\frac{m^2+3m+2}{2r}} \left( 1 + \coth \left( \pm \frac{mq}{2r(m+4)}(x - ct) \right) \right) \right)^{2/m}, \]
\[ u_3 = \left( \pm \frac{q}{m+4} \sqrt{\frac{m^2+3m+2}{8r}} \left( 2 + \tanh \left( \pm \frac{mq}{4r(m+4)}(x - ct) \right) + \right. \right. \]
\[ \left. \left. \coth \left( \pm \frac{mq}{4r(m+4)}(x - ct) \right) \right) \right)^{2/m}, \]
where $c = \frac{2q^2(m+2)}{r(m+4)^2}$. 
4 The generalized Burgers-EW equation

Now, we examine the generalized Burgers-EW equation of the form
\[ u_t + \alpha u^m u_x - \delta u_{xx} - \beta u_{xxt} = 0, \] (4.9)

which models the propagation of nonlinear and dispersive waves with certain dissipative effects. The analytical solutions of this equation is obtained for any order of the nonlinear terms and for any given value of the coefficients of the nonlinear, dispersive and dissipative terms. This equation reduces to the EW equation when \((m = 1, \delta = 0)\), modified EW equation \((m = 2, \delta = 0)\), Burgers equation \((m = 1, \beta = 0)\), modified Burgers equation \((m = 2, \beta = 0)\), generalized Burgers equation \((\beta = 0)\), and the modified Burgers-EW equation \((m = 2)\).

Many scientists are devoted to studying the exact solutions of these special kinds of the generalized Burgers-EW equation \([10,18,21]\).

In this section, we employ the extended tanh method to find soliton solutions of the generalized Burgers-EW equation. Using the wave variable \(\xi = x - ct\) and \(u(x,t) = v(\xi)\), Eq.(4.9) changes to
\[ -cv' + \alpha v^m v' - \delta v'' + \beta cv''' = 0. \] (4.10)

By integrating Eq.(4.10) and neglecting the constant of integration, we have
\[ -cv + \frac{\alpha}{m + 1} v^{m+1} - \delta v' + \beta cv'' = 0. \]

After transformation \(v(\xi) = \phi^{2/m}(\xi)\) and simplification, we obtain
\[ -c\phi^2 + \frac{\alpha}{m + 1} \phi^4 - \frac{2\delta}{m} \phi \phi' + \frac{2\beta c}{m^2} \phi'' + \frac{2\beta c(2 - m)}{m^2} (\phi')^2 = 0. \] (4.11)

Balancing \(\phi''\) with \(\phi^4\) in Eq.(4.11) gives
\[ n = 1. \]

Therefore, we must set
\[ \phi(\xi) = S(Y) = a_0 + a_1 Y + \frac{b_1}{Y}. \] (4.12)

Now by substituting Eq.(4.12) in Eq.(4.11) and collecting all terms with the same power in \(Y\) and putting to zero their coefficients, we get a system of algebraic equations among the unknowns \(a_0, a_1, b_1, \mu\) and \(c\). Solving this system gives the following sets of solutions

(i) The first set
\[ a_0 = \pm \sqrt{\frac{cm + c}{4\alpha}}, \quad a_1 = \pm \sqrt{\frac{cm + c}{4\alpha}}, \]
\[ b_1 = 0, \quad \mu = \frac{m}{2\sqrt{-2\beta m - 4\beta}}, \quad c = \frac{-2\delta(m + 2)}{(m + 4)\sqrt{-2\beta m - 4\beta}}. \]
(ii) The second set

\[ a_0 = \pm \sqrt{\frac{cm + c}{4\alpha}}, \quad a_1 = \pm \sqrt{\frac{cm + c}{4\alpha}}, \]

\[ b_1 = 0, \quad \mu = \frac{-m}{2\sqrt{-2\beta m - 4\beta}}, \quad c = \frac{2\delta(m+2)}{(m+4)\sqrt{-2\beta m - 4\beta}}. \]

(iii) The third set

\[ a_0 = \pm \sqrt{\frac{cm + c}{4\alpha}}, \quad b_1 = \pm \sqrt{\frac{cm + c}{4\alpha}}, \]

\[ a_1 = 0, \quad \mu = \frac{m}{2\sqrt{-2\beta m - 4\beta}}, \quad c = \frac{-2\delta(m+2)}{(m+4)\sqrt{-2\beta m - 4\beta}}. \]

(iv) The fourth set

\[ a_0 = \pm \sqrt{\frac{cm + c}{4\alpha}}, \quad b_1 = \pm \sqrt{\frac{cm + c}{4\alpha}}, \]

\[ a_1 = 0, \quad \mu = \frac{-m}{2\sqrt{-2\beta m - 4\beta}}, \quad c = \frac{2\delta(m+2)}{(m+4)\sqrt{-2\beta m - 4\beta}}. \]

(v) The fifth set

\[ a_0 = \pm \sqrt{\frac{cm + c}{4\alpha}}, \quad a_1 = \pm \sqrt{\frac{cm + c}{16\alpha}}, \]

\[ b_1 = \pm \sqrt{\frac{cm + c}{16\alpha}}, \quad \mu = \pm \frac{m}{4\sqrt{-2\beta m - 4\beta}}, \quad c = \pm \frac{2\delta(m+2)}{(m+4)\sqrt{-2\beta m - 4\beta}}. \]

This gives the following soliton solutions

\[ u_1 = \left( \pm \sqrt{\frac{cm + c}{4\alpha}} \left( 1 + \tanh \left( \frac{m}{2\sqrt{-2\beta m - 4\beta}}(x - ct) \right) \right) \right)^{2/m}, \]

\[ u_2 = \left( \pm \sqrt{\frac{cm + c}{4\alpha}} \left( 1 + \coth \left( \frac{m}{2\sqrt{-2\beta m - 4\beta}}(x - ct) \right) \right) \right)^{2/m}, \]

where \( c = \frac{-2\delta(m+2)}{(m+4)\sqrt{-2\beta m - 4\beta}}, \)

\[ u_3 = \left( \pm \sqrt{\frac{cm + c}{4\alpha}} \left( 1 + \tanh \left( \frac{-m}{2\sqrt{-2\beta m - 4\beta}}(x - ct) \right) \right) \right)^{2/m}, \]

\[ u_4 = \left( \pm \sqrt{\frac{cm + c}{4\alpha}} \left( 1 + \coth \left( \frac{-m}{2\sqrt{-2\beta m - 4\beta}}(x - ct) \right) \right) \right)^{2/m}. \]
where $c = \frac{2\delta(m+2)}{(m+4)\sqrt{-2\beta m-4\beta}}$ and

$$u_5 = \left( \pm \sqrt{\frac{cm + c}{16\alpha}} \left( 2 + \tanh \left( \pm \frac{m}{4\sqrt{-2\beta m-4\beta}}(x - ct) \right) \right) + \coth \left( \pm \frac{m}{4\sqrt{-2\beta m-4\beta}}(x - ct) \right) \right)^{2/m},$$

where $c = \pm \frac{2\delta(m+2)}{(m+4)\sqrt{-2\beta m-4\beta}}$.

5 The generalized two-dimensional Burgers-KdV equation

Consider the generalized two-dimensional Burgers-KdV equation

$$(u_t + u^m u_x + pu_{xxx} - qu_{xx})_x + ru_{yy} = 0. \tag{5.13}$$

where $p$, $q$, and $r$ are real constants. This equation is model equation for wide class of nonlinear wave models of fluid in an elastic tube, liquid with small bubbles and turbulence [14, 16, 28, 30, 36]. It has been considered to find out its exact solution by some analytic methods [11, 22, 35].

In this section, we employ the extended tanh method to find soliton solutions of the generalized two-dimensional Burgers-KdV equation. Using $u(x, y, t) = v(\xi)$, $\xi = x + y - ct$, Eq.(5.13) reduces to

$$(-cv' + v^m v' + pv''' - qv''')' + rv'' = 0 \tag{5.14}$$

By integrating Eq.(5.14) and considering each constant of integration to zero, we get

$$(r - c)v + \frac{1}{m + 1} v^{m+1} - qv' + pv'' = 0. \tag{5.15}$$

After transformation $v(\xi) = \phi^{2/m}(\xi)$ and simplification, we obtain

$$(r - c)\phi^2 + \frac{1}{m + 1} \phi^4 - \frac{2q}{m} \phi \phi' + \frac{2p}{m} \phi \phi''' + \frac{2p(2 - m)}{m^2} (\phi')^2 = 0. \tag{5.15}$$

Balancing $\phi \phi''$ with $\phi^4$ in Eq.(5.15) gives

$$n = 1.$$

Therefore, we must set

$$\phi(\xi) = S(Y) = a_0 + a_1 Y + \frac{b_1}{Y}. \tag{5.16}$$

Now by substituting Eq.(5.16) in Eq.(5.15) and collecting the coefficients of $Y$, we obtain a system of algebraic equations for unknowns $a_0$, $a_1$, $b_1$, $\mu$ and $c$. Solving this system gives the following sets of solutions
(i) The first set

\[ a_0 = \pm \frac{q}{m+4} \sqrt{-m^2-3m-2 \over 2p}, \quad a_1 = \pm \frac{q}{m+4} \sqrt{-m^2-3m-2 \over 2p}, \]

\[ b_1 = 0, \quad \mu = \pm \frac{mq}{2p(m+4)}, \quad c = \frac{rm^2+8rm^p-2q^2m-4q^2+16rp}{p(m+4)^2}. \]

(ii) The second set

\[ a_0 = \pm \frac{q}{m+4} \sqrt{-m^2-3m-2 \over 2p}, \quad b_1 = \pm \frac{q}{m+4} \sqrt{-m^2-3m-2 \over 2p}, \]

\[ a_1 = 0, \quad \mu = \pm \frac{mq}{2p(m+4)}, \quad c = \frac{rm^2+8rm^p-2q^2m-4q^2+16rp}{p(m+4)^2}. \]

(iii) The third set

\[ a_0 = \pm \frac{2q}{m+4} \sqrt{-m^2-3m-2 \over 8p}, \quad c = \frac{rm^2+8rm^p-2q^2m-4q^2+16rp}{p(m+4)^2}, \]

\[ a_1 = \pm \frac{q}{m+4} \sqrt{-m^2-3m-2 \over 8p}, \quad b_1 = \pm \frac{q}{m+4} \sqrt{-m^2-3m-2 \over 8p}, \quad \mu = \pm \frac{mq}{4p(m+4)}. \]

This in turn gives the following soliton solutions

\[ u_1 = \left( \pm \frac{q}{m+4} \sqrt{-m^2-3m-2 \over 2p} \left( 1 + \tanh \left( \pm \frac{mq}{2p(m+4)} (x + y - ct) \right) \right) \right)^{2/m}, \]

\[ u_2 = \left( \pm \frac{q}{m+4} \sqrt{-m^2-3m-2 \over 2p} \left( 1 + \coth \left( \pm \frac{mq}{2p(m+4)} (x + y - ct) \right) \right) \right)^{2/m}, \]

\[ u_3 = \left( \pm \frac{q}{m+4} \sqrt{-m^2-3m-2 \over 8p} \left( 2 + \tanh \left( \pm \frac{mq}{4p(m+4)} (x + y - ct) \right) \right) + \coth \left( \pm \frac{mq}{4p(m+4)} (x + y - ct) \right) \right)^{2/m}, \]

where \( c = \frac{rm^2+8rm^p-2q^2m-4q^2+16rp}{p(m+4)^2} \).

6 The generalized two-dimensional Burgers-EW equation

We now examine the generalized two-dimensional Burgers-EW equation

\[ (u_t + u^m u_x + pu_{xxt} - q u_{xx})_x + ru_{yy} = 0. \] (6.17)

The wave variable \( \xi = x + y - ct \) carries Eq.(6.17) into the ODE

\[ (-cv' + v^m v' - pcv^m - qv^m)' + rv'' = 0 \] (6.18)
By integrating Eq.(6.18) and considering each constant of integration to zero, we get

\[(r - c)v + \frac{1}{m + 1} v^{m+1} - qv' - pcv'' = 0\]

After transformation \(v(\xi) = \phi^{2/m}(\xi)\) and simplification, we obtain

\[(r - c)\phi^2 + \frac{1}{m + 1} \phi^4 - \frac{2q}{m} \phi\phi' - \frac{2pc}{m} \phi\phi'' - \frac{2pc(2 - m)}{m^2} (\phi')^2 = 0. \tag{6.19}\]

Balancing \(\phi\phi''\) with \(\phi^4\) in Eq.(6.19) gives

\[n = 1.\]

Therefore, we must set

\[\phi(\xi) = S(Y) = a_0 + a_1 Y + \frac{b_1}{Y}. \tag{6.20}\]

Now by substituting Eq.(6.20) in Eq.(6.19) and collecting the coefficients of \(Y\), we obtain a system of algebraic equations for unknowns \(a_0, a_1, b_1, \mu\) and \(c\). Solving this system gives the following sets of solutions

(i) The first set

\[a_0 = \pm \frac{1}{2} \sqrt{c(m + 1) - r(m + 1)}, \quad a_1 = \pm \frac{1}{2} \sqrt{c(m + 1) - r(m + 1)},\]

\[b_1 = 0, \quad \mu = \frac{4mpr + m^2 pr \pm \sqrt{m^2 p(8q^2(m+2)+pr^2(m+4)^2)}}{8pq(m+2)},\]

\[c = \frac{4pr + mpr \pm \sqrt{p(8q^2(m+2)+pr^2(m+4)^2)}}{2p(m+4)}.\]

(ii) The second set

\[a_0 = \pm \frac{1}{2} \sqrt{c(m + 1) - r(m + 1)}, \quad b_1 = \pm \frac{1}{2} \sqrt{c(m + 1) - r(m + 1)},\]

\[a_1 = 0, \quad \mu = \frac{4mpr + m^2 pr \pm \sqrt{m^2 p(8q^2(m+2)+pr^2(m+4)^2)}}{8pq(m+2)},\]

\[c = \frac{4pr + mpr \pm \sqrt{p(8q^2(m+2)+pr^2(m+4)^2)}}{2p(m+4)}.\]

(iii) The third set

\[a_0 = \pm \frac{1}{2} \sqrt{c(m + 1) - r(m + 1)}, \quad a_1 = \pm \frac{1}{4} \sqrt{c(m + 1) - r(m + 1)},\]

\[b_1 = \pm \frac{1}{4} \sqrt{c(m + 1) - r(m + 1)},\]

\[\mu = \frac{4mpr + m^2 pr \pm \sqrt{m^2 p(8q^2(m+2)+pr^2(m+4)^2)}}{16pq(m+2)},\]

\[c = \frac{4pr + mpr \pm \sqrt{p(8q^2(m+2)+pr^2(m+4)^2)}}{2p(m+4)}.\]
This in turn gives the following soliton solutions

\[ u_1 = \left( \pm \frac{1}{2} \sqrt{c(m+1) - r(m+1)} \left(1 + \tanh \left( \frac{4mpr + m^2pr \pm \sqrt{m^2p(8q^2(m+2) + pr^2(m+4)^2)}}{8pq(m+2)} (x + y - ct) \right) \right) \right)^{2/m}, \]

\[ u_2 = \left( \pm \frac{1}{2} \sqrt{c(m+1) - r(m+1)} \left(1 + \coth \left( \frac{4mpr + m^2pr \pm \sqrt{m^2p(8q^2(m+2) + pr^2(m+4)^2)}}{8pq(m+2)} (x + y - ct) \right) \right) \right)^{2/m}, \]

\[ u_3 = \left( \pm \frac{1}{2} \sqrt{c(m+1) - r(m+1)} \left(2 + \tanh \left( \frac{4mpr + m^2pr \pm \sqrt{m^2p(8q^2(m+2) + pr^2(m+4)^2)}}{16pq(m+2)} (x + y - ct) \right) \right) + \right. \]

\[ \left. \coth \left( \frac{4mpr + m^2pr \pm \sqrt{m^2p(8q^2(m+2) + pr^2(m+4)^2)}}{16pq(m+2)} (x + y - ct) \right) \right) \right)^{2/m}, \]

where \( c = \frac{4pr + mpr \pm \sqrt{p(8q^2(m+2) + pr^2(m+4)^2)}}{2p(m+4)} \).

### 7 Conclusion

In this work, the extended tanh method which is standard, direct and computerizable, has been successfully applied to find exact soliton solutions of some important nonlinear PDEs such as the generalized Burgers-KdV equation, the generalized Burgers-EW equation, the generalized two-dimensional Burgers-KdV equation and the generalized two-dimensional Burgers-EW equation. The tedious computations associated with the algebraic calculations are facilitated using symbolic computation softwares such as Maple and Mathematica.

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