A new decision-making model for solving multi-objective large-scale programming problems with a block angular structure

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Abstract
This paper presents a model based on a novel compromised solution method to solve the multi-objective large-scale nonlinear programming (MOLSNLP) problems with block angular structure. In this method, an aggregating function that is developed from TOPSIS and VIKOR is proposed based on the particular measure of “closeness” to the “ideal” solution. The decomposition algorithm is utilized to reduce the large-dimensional objective space into a two-dimensional space. Furthermore, two independent solution methods are proposed to solve each nonlinear sub problem respectively. In the last step of proposed method, Single objective nonlinear programming problem is solved to find the final solution. Finally, to justify the proposed method, an illustrative example is provided. Then, the sensitivity analysis is described.

Keywords: TOPSIS, VIKOR, Multiple criteria decision making (MCDM), Multi Objective decision making (MODM), multi objective large-scale nonlinear programming (MOLS NLP), block angular structure.

1 Introduction

Decision making is a practical tool to rank the alternatives under various situations. Decision making problems is applied in many fields of engineering such as industrial engineering. Decision making methods is divided in into two fields of decision making problems. First of them is the Multi-Objective Decision Making (MODM) and second of them is Multi-Attribute Decision Making (MADM). MADM is applied to rank the alternatives relative versus to criteria but MODM studies decision making problems with continuous space. Moreover, there are many decision making problems with multi objective during decision making so they may conflict with each other [1], [2]. The complexity of many decision making problems is associated to number of decision making variables. In other words, as the number of variables increases, the complexity of problem increases.
The complexity of problems demonstrates the various factors in the objective functions and constraints during decision making in these problems. The computational complexity of nonlinear decision making problems increases sharply. Further, in large scale problems, it becomes difficult to obtain efficient solutions for these problems in a less time and efficient manner. Fortunately, most of large scales programming problems usually have some special structures that can be solved more easily. Block angular structure is one familiar structure of these special structures [3], [4], [1], [5]. Using decomposition methods, The block angular structure problems can be solved by a decomposition method. In other words, the original block angular structure problem can be decomposed in to smaller dimension sub problems. Dantzig-wolf and Benderz are two types of decomposition methods [3]. A Dantzig-wolf decomposing algorithm as a decomposition method is first introduced for large scale linear optimization problems with block angular structure [3], [6]. Developing this method, dantzig- wolf is applied on large-scale nonlinear programming problems [7], [5]. Recently, some compromise MCDM methods are applied to aggregate the multi objective programming problems in to single objective to solve the MOLSNLP problems. First, TOPSIS method is extended to solve multi-objective dynamics programming problems [8]. Moreover, TOPSIS is applied to propose an effective method to solve the inter-company comparison process and multi-person multi-criteria decision making problems in fuzzy environment problem [9], [10]. An extended TOPSIS method is introduced for solving MODM problems [11]. TOPSIS is extended for solving some problems with special structure such as MOLSNLP problems with block angular structure [1]. An interactive TOPSIS method is proposed by Abo-Sinna and Abou-El-Enien to solve large scale multiple objective programming problems with fuzzy parameters [12].

VIKOR is another compromise MCDM method that is extended by Heydari for solving MOLSNLP problems [5]. The VIKOR method was proposed as a compromised approach to rank the alternatives versus some criteria. The VIKOR is applied to find suitable solution based on the average distance and maximum distance from ideal solutions [13], [14]. Moreover, this method is applied to decision about multi-response process. The VIKOR method also extends to preference alternatives with fuzzy parameter by many researchers. The fuzzy sets and VIKOR method is integrated to fuzzy VIKOR for solving the fuzzy MCDM programming problems [15].

This paper presents a model based on a novel compromised solution method to solve the multi-objective large-scale nonlinear programming (MOLSNLP) problems with block angular structure. In this method, an aggregating function that is developed from TOPSIS and VIKOR is proposed based on the particular measure of “closeness” to the “ideal” solution. The decomposition algorithm is utilized to reduce the large-dimensional objective space into a two-dimensional space. In the last step of proposed method, Single objective nonlinear programming problem is solved to find the final solution. Finally, to justify the proposed method, an illustrative example is provided. Then, the sensitivity analysis is described.

The remaining of this paper is organized as follows. The problem formulation is presented in the next section. In this section, the decomposed problem is proposed to decompose original problem. In section 3, the new compromised Solution method is introduced to solve MOLSNLP is introduced. In section 4, an example is proposed to illustrate the process of proposed method step by step. Then, the result is described for each sub problem. The last section is devoted to conclusion.
2 Problem statement

In this paper, the MOLSLP problem with the block angular structure is constructed as follows.

\[
P:
\text{Max (Min) } f_i(X)
\]
\[
\text{Max (Min) } f_2(X)
\]
\[
\ldots
\]
\[
\text{Max (Min) } f_q(X)
\]

\[
S. t. \quad FS = \left\{ \begin{array}{ll}
g_m(x1) & \leq B_1 \\
g_m(x2) & \leq B_2 \\
\ldots & \\
g_m(xN) & \leq B_N \\
H_i(X) = \sum_{j=1}^{N} h_{ij}(X_j) & \leq B \\
\end{array} \right.
\]

\[
g_m(x_i); i = 1, 2, \ldots, s_1 \text{ are the inequality constraint functions and } H_i(X) \text{ are the common constraints functions on } R^N \text{ which can be constrained as: } H_i(X) \quad i = 1, 2, \ldots
\]

Where

Model parameters:

- \( L \) The number of objective functions
- \( q \) The number of sub problems
- \( N \) The number of variables
- \( N_i \) The set of variables of the ith sub problem, \( i = 1, 2, \ldots, q \)
- \( p_i \) Ith sub problem
- \( R \) The set of all real numbers
- \( W \) The number of common constraints on \( R^N \)
- \( S_i \) Maximum amount of index for the constraints for the ith variable
- \( B \) A \( w \)-dimensional column vector of right-hand sides of the common constraints whose elements are constants
- \( B_{ij} \) An \( S_i \)-dimensional column vector of independent constraints right-hand sides whose elements are fuzzy parameters for the ith sub problem, \( i = 1, 2, \ldots, q \).

Where \( X = (x_1, x_2, \ldots, x_N) \) is the \( N \)-dimensional decision vector. \( f_i(X), i = 1, 2, \ldots, L \) are the objective functions. It is assumed that the objective functions have an additively separable form. Using Dantzig-Wolfe decomposition algorithm, the fuzzy MOLSLP problem can be decomposed into \( q \) sub-problems. The ith sub-problem for \( i = 1, \ldots, q \) is defined as:
As shown in problem (3.3), the $i$th sub problem consists of $L$ objective functions. Moreover, where $h_{ij}$ is the function of $j$th variable in $i$th common constraint and $c$ is the coefficient of the objective function and $B$ is the coefficient of the right-hand side of constraints in problem (3.3).

3 The new compromised Solution method for MOLSLP

Suppose that the problem $P$ is a convex programming problem. The Dantzig-wolf decomposition method is successfully applied to decompose the original problem into the $q$ independent sub problems. In other words, the $N$-dimensional problem space is reduced to a $N_i$-dimensional space by applying the Dantzig-Wolfe decomposition algorithm. Then the TOPSIS and VIKOR methods are applied as a compromised method to aggregate the objectives of each sub problem. To obtain compromise solution of original problem, the individual positive ideal solution (PIS) and negative ideal solution (NIS) are calculated for each objective. Applying PIS and NIS, the bi- objective problems are constructed for $j$th sub problem. Using Zimmerman method, the final single objective programming problem is constructed. Afterwards, the final single objective problem is solved to obtain final optimal solution. The proposed method has the following steps:

**Step 1.**
Decompose the original problem in to $q$ sub problems by applying the Dantzig-wolf decomposition method for objective functions and constraints to reduce the dimension of primal problem. The $i$th sub problem can be proposed as:

$$
\begin{align*}
\text{Max } (\text{Min}) f_1(X) &= \sum_{j \in N_i} f_1(X_j) \\
\text{Max } (\text{Min}) f_2(X, U_2) &= \sum_{j \in N_i} f_2(X_j) \\
&\vdots \\
\text{Max } (\text{Min}) f_L(X, U_L) &= \sum_{j \in N_i} f_L(X_j) \\
\text{S. t. } FS_i &= \left\{ \begin{array}{l}
\sum_{j \in N_i} g_m(X_j) \leq B_m \\
H_i(X) = \sum_{j=1}^{N} h_{ij}(X_j) &\leq B \\
m = s_{j-1} + 1, \ldots, s_j \\
i = 1, 2, \ldots, w
\end{array} \right.
\end{align*}
$$

(2.2)
Step 2.
Calculate the positive ideal solution (PIS) and the negative ideal solution (NIS) of each objective function. Note that the values of PIS and NIS are calculated through solving the multi-objective problem as a single objective using, each time, only one objective.

PIS: $f_{bj}^+ = \{\text{Max (Min)} f_{bj}(X_j) (f_{cj}(X_j), \forall b (\forall c))\}$  \hspace{1cm} (3.4)

NIS: $f_{bj}^- = \{\text{Min (Max)} f_{bj}(X_j) (f_{cj}(X_j), \forall b (\forall c))\}$  \hspace{1cm} (3.5)

$f_{bj}(X_j)$ Benefit objective for maximization

$f_{cj}(X_j)$ Cost objective for maximization

Step 3.
Applying PIS and NIS from the results of step 2, Construct the functions of $S_i^{PIS}$, $R_i^{PIS}$, $S_i^{NIS}$ and $R_i^{NIS}$ as a maximum "group utility" for the "majority" and a minimum of an individual regret for the "opponent". As shown follow:

$S_i^{PIS} = \sum_{j \in B_i} w_j \left( \frac{f_{ij}^+-f_{ij}}{f_{ij}^-f_{ij}} \right) + \sum_{j \notin B_i} w_j \left( \frac{f_{ij}^+-f_{ij}^-}{f_{ij}^-f_{ij}} \right)$  \hspace{1cm} (3.6)

$R_i^{PIS} = \max_i \left( \frac{f_{ij}^+-f_{ij}}{f_{ij}^-f_{ij}} \right)$  \hspace{1cm} (3.7)

$S_i^{NIS} = \sum_{j \in B_i} w_j \left( \frac{f_{ij}^-f_{ij}^-}{f_{ij}^-f_{ij}} \right) + \sum_{j \notin B_i} w_j \left( \frac{f_{ij}^-f_{ij}}{f_{ij}^-f_{ij}^-} \right)$  \hspace{1cm} (3.8)

$R_i^{NIS} = \min_i \left( \frac{f_{ij}^-f_{ij}}{f_{ij}^-f_{ij}^-} \right)$  \hspace{1cm} (3.9)

Applying VIKOR method, calculate the minimum $S_i^{NIS}$ and $R_i^{PIS}$ among all objective functions. Furthermore calculate the maximum $S_i^{PIS}$ and $R_i^{NIS}$ among all objective functions.
Step 4.
Construct the two objective functions problem as follow:

\[
Q_{i}^{PIS} = v \left( \sum_{j \in B_{i}} w_{j} \left( \frac{S_{PIS}^{i} - S_{PIS}^{*}}{S_{PIS}^{i} - S_{PIS}^{j}} \right) \right) + (1 - v) \left( \frac{R_{PIS}^{i} - R_{PIS}^{*}}{R_{PIS}^{i} - R_{PIS}^{j}} \right) 
\]

(3.10)

\[
Q_{i}^{NIS} = v \left( \sum_{j \in B_{i}} w_{j} \left( \frac{S_{NIS}^{i} - S_{NIS}^{*}}{S_{NIS}^{i} - S_{NIS}^{j}} \right) \right) + (1 - v) \left( \frac{R_{NIS}^{i} - R_{NIS}^{*}}{R_{NIS}^{i} - R_{NIS}^{j}} \right) 
\]

(3.11)

Then Construct the two objective functions programming problem as follow:

Min \( Q_{i}^{PIS} \)

Max \( Q_{i}^{NIS} \)

\( X \in F S_{i} \)

(3.12)

Step 5.
Construct the two membership functions for \( Q^{PIS} \) and \( Q^{NIS} \), respectively. As shown in Fig. 1, Fig. 2.

The membership function for the negative (or \( Q^{PIS} \)) objective can be defined as:

\[
\mu_{1}(x) = \frac{(Q_{i}^{PIS}) - (Q_{i}^{PIS})^{*}}{(Q_{i}^{PIS}) - (Q_{i}^{PIS})^{*}}
\]

(3.13)

![Figure 1: The membership function of \( \mu_{1}(x) \)](image)

The membership function for the positive (or \( Q^{NIS} \)) objective can be defined as:

\[
\mu_{2}(x) = \frac{(Q_{i}^{NIS})^{*} - (Q_{i}^{NIS})}{(Q_{i}^{NIS})^{*} - (Q_{i}^{NIS})^{*}}
\]

(3.14)

![Figure 2: The membership function of \( \mu_{2}(x) \)](image)
Applying Zimmermann method, Construct the final single objective problem for each sub problem instead of problem (3.13) based on the membership functions. This method is proposed by Bellman and Zadeh and extended by Zimmermann [16], [17]. Then solve it to obtain the final optimal solution. The problem (45) is equivalent to the form of following problem as:

\[
\begin{align*}
\max \lambda \\
\frac{(Q_i^{PIS}) - (Q_i^{NIS})}{(Q_i^{PIS}) - (Q_i^{NIS})} \geq \lambda \\
\frac{(Q_i^{NIS}) - (Q_i^{NIS})}{(Q_i^{NIS}) - (Q_i^{NIS})} \geq \lambda \\
0 \leq \lambda \leq 1, \ X \in FS_i
\end{align*}
\]

The final compromised solution and satisfactory level are obtained by solving problem (3.15).

4 Illustrative numerical example

In this section, we use an illustrative example to demonstrate our proposed approach. This example has three objective functions. The objective functions and constrains are proposed as convex problem on \( R^3 \) Moreover the weights of objective functions are same for all sub problems. The programming example is proposed as:

\[
P: \min f_1(x) = 2(x_1 - 1)^2 + 3x_2^2 + 3(x_3 + 1)^2 \\
max f_2(x) = 4x_1 + 2x_2 + 3x_3^2 \\
max f_3(x) = 2x_1^2 + 3x_2 + 2x_3^2
\]

Subject to:
\[
\begin{align*}
x_1 - x_2 + 2x_3 & \leq 6 \\
2x_1 - 2x_2 + 4x_3 & \leq 7 \\
3x_1 - 3x_2 + 6x_3 & \leq 8 \\
x_1^2 + x_2 + x_3 & \leq 10 \\
2x_1^2 + 3x_2 + 2x_3 & \leq 11 \\
3x_1^2 + 5x_2 + 3x_3 & \leq 12 \\
0 & \leq x_1 \leq 1.6667 \\
0 & \leq x_2 \leq 2 \\
0 & \leq x_3 \leq 1.3333
\end{align*}
\]

Then Step by Step solution of the problem is given below.

Step 1.
Decompose the original programming problem into three sub problems based on dantzig-wolf decomposition method. The decomposed sub problems \( P_1, P_2 \) and \( P_3 \) are proposed as:
Step 2.

Applying TOPSIS method, calculate the individual PIS and NIS of each objective function for sub problems $P_1$, $P_2$ and $P_3$. The obtained PIS, NIS of sub problem $P_1$ are shown in Tables 1, 2.

### Table 1: PIS payoff table of $(P_1)$

<table>
<thead>
<tr>
<th></th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>min $f_1$</td>
<td>0&quot;</td>
<td>4</td>
<td>2</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>max $f_2$</td>
<td>0.8890</td>
<td>6.6667*</td>
<td>5.5558</td>
<td>1.6667</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>max $f_3$</td>
<td>0.8890</td>
<td>6.6667</td>
<td>5.5556*</td>
<td>1.6667</td>
<td>0</td>
</tr>
</tbody>
</table>

### Table 2: NIS payoff table of $(P_1)$

<table>
<thead>
<tr>
<th></th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>max $f_1$</td>
<td>2*</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>min $f_2$</td>
<td>2</td>
<td>0*</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>min $f_3$</td>
<td>2</td>
<td>0</td>
<td>0*</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

PIS: $f^* = (f_1^*, f_2^*, f_3^*) = (0, 6.6667, 5.5556)$

NIS: $f^- = (f_1^-, f_2^-, f_3^-) = (2, 0, 0)$

Step 3.

Applying PIS and NIS from the results of step 2, Construct the functions of $S^{PIS}$, $R^{PIS}$ as shorter distance from the PIS and $S^{NIS}$, $R^{NIS}$ as farther distance from NIS for each sub problem. The values $d^{PIS}$ and $d^{NIS}$ for problem $d_1$ are calculated as follows:

$$S^{PIS}_1 = \frac{1}{3} \left( \frac{2(x_1-1)^2-0.0000}{2.0000-0.0000} \right) + \frac{1}{3} \left( \frac{6.6667-4x_1}{6.6667-0.0000} \right) + \frac{1}{3} \left( \frac{5.5556-2x_1^2}{5.5556-0.0000} \right)$$

$$R^{PIS}_1 = \frac{1}{3} \left( \frac{5.5556-2x_1^2}{5.5556-0.0000} \right)$$
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\[ S_1^{NIS} = \frac{1}{3} \left( \frac{2}{2.0000-0.0000} \right) + \frac{1}{3} \left( \frac{4x_1-0.0000}{6.6667-0.0000} \right) + \frac{1}{3} \left( \frac{2x_1^2-0.0000}{5.5556-0.0000} \right) \]  
(4.22)

\[ R_1^{NIS} = \frac{1}{3} \left( \frac{2x_1^2-0.0000}{5.5556-0.0000} \right) \]  
(4.23)

\[ S_{PIS}^* = -0.4085 \quad R_{PIS}^* = -0.2733 \]

\[ S_{PIS}^- = 0.4443 \quad R_{PIS}^- = 0.5999 \]

\[ S_{NIS}^* = 0.2552 \quad R_{NIS}^* = 0.3333 \]

\[ S_{NIS}^- = 0.0000 \quad R_{NIS}^- = 0.0000 \]

**Step 4.**
The calculated values of \( Q_1^{PIS} \) and \( Q_1^{NIS} \) are proposed as:

\[ Q_1^{PIS} = -0.5048x_1^2 + 1.6981x_1 - 1.9554 \]  
(4.24)

\[ Q_1^{NIS} = -0.3051x_1^2 + 0.5082x_1 + 0.2395 \]  
(4.25)

**Step 5.**
Applying \( Q_1^{PIS} \) and \( Q_1^{NIS} \) from the results of step 4, the two memberships function for \( Q_1^{PIS} \) and \( Q_1^{NIS} \) can be defined as:

\[ \mu_1(x) = 0.3535x_1^2 + 1.1892x_1 + 1 \]  
(4.26)

\[ \mu_2(x) = -0.2137x_1^2 + 0.3559x_1 + 0.2476 \]  
(4.27)

Final solution is obtained by solving the single problem (4.28) as:

max \( \lambda \)

\[ 0.3535x_1^2 + 1.1892x_1 + 1 \geq \lambda \]

\[ -0.2137x_1^2 + 0.3559x_1 + 0.2476 \geq \lambda \]

\[ 0 \leq \lambda \leq 1, \ X \in FS_i \]

\[ \lambda^* = 0.3959 \quad x_1^* = 0.8327 \]

Where \( \lambda^* \) the maximum is satisfactory level and \( x_1^* \) is the final solution which obtained from sub problem 1.

We want to obtain the ideal compromised solution. In other words, the objective function \( Q_1^{PIS} \) should be minimized whereas the function \( Q_1^{NIS} \) should be maximized. As shown in Fig. 4. Point \( x_1^* = 0.8327 \) is optimum compromised solution. Moreover, the maximum level of \( \lambda \) is \( \lambda^* = 0.3959 \).
using the proposed method, we solve the second sub problem similar to first sub problem. The individual PIS and NIS of each objective function for sub problems $P_2$ as shown in Tables 3, 4.

Table 3: PIS payoff table of $(P_2)$

<table>
<thead>
<tr>
<th>$P_2$</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>min $f_1$</td>
<td>0$^*$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>max $f_2$</td>
<td>12</td>
<td>4$^*$</td>
<td>6</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>max $f_3$</td>
<td>12</td>
<td>4</td>
<td>6$^*$</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

PIS: $f_{2z}^* = (f_1^*, f_2^*, f_3^*) = (0.0000, 4.0000, 6.0000)$

Table 4: NIS payoff table of $(P_2)$

<table>
<thead>
<tr>
<th>$P_2$</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>max $f_1$</td>
<td>12$^*$</td>
<td>4</td>
<td>6</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>min $f_2$</td>
<td>0</td>
<td>0$^*$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>min $f_3$</td>
<td>0</td>
<td>0</td>
<td>0$^*$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

NIS: $f_2^{-\infty} = (f_1^{-\infty}, f_2^{-\infty}, f_3^{-\infty}) = (12.0000, 0.0000, 0.0000)$

Now calculate the amount of $Q^{PIS}$ as shorter distance from the PIS and $Q^{NIS}$ as farther distance from NIS for second sub problem as:

The membership functions for $Q_2^{PIS}$ and $Q_2^{NIS}$ is proposed respectively in Eqs (4.29), (4.30).

\[
Q_2^{PIS} = -0.0510x_2^2 + 0.6019x_2 \\
Q_2^{NIS} = 0.0035x_2^2 - 0.5071x_2 + 1
\]  

Now, the amounts of membership function for $Q_1^{PIS}$ and $Q_1^{NIS}$ are proposed as:

\[
\mu_1(x) = 0.0320x_2^2 - 0.3780x_2 - 1.7711 \\
\mu_2(x) = 0.0320x_2^2 - 0.3780x_2 - 1.7711
\]
Solving final single objective programming problem, the compromised solution for second sub problem is obtained.

\[
\begin{align*}
\text{max } & \lambda \\
-0.7334x_2 + 0.6266 & \geq \lambda \\
-0.5001x_2 + 1 & \geq \lambda \\
0 & \leq \lambda \leq 1, \ X \in FS_2 \\
\lambda^* & = 0.5236 \quad x_2^* = 0.9457
\end{align*}
\] (4.33)

In this sub problem, \(Q_2^{PIS}\) and \(Q_2^{NIS}\) are optimized as shown in Fig 4. The maximum level of \(\lambda\) is occurred at \(\lambda^* = 0.5236\). In other words, the more value of \(x_2\) in this problem is better. But considering the constraint, the optimum compromised solution is \(x_2^* = 0.9457\).

![Figure 4: The values of function \(Q_{ij}\) for problem \(P_2\)](image)

Similar to sub problems \(P_1, P_2\), the problem \(P_3\) is solved as follow:

Similar to sub problems \(P_1, P_2\), the values of PIS and NIS for sub problem \(P_3\), are proposed in Tables 5, 6.

<table>
<thead>
<tr>
<th>Table 5: PIS payoff table of ((P_3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f_1)</td>
</tr>
<tr>
<td>(P_3)</td>
</tr>
<tr>
<td>(\max f_2)</td>
</tr>
<tr>
<td>(\max f_3)</td>
</tr>
</tbody>
</table>

PIS: \(f_3^* = (f_1^*, f_2^*, f_3^*) = (3.0000, 5.3333, 3.5556)\)

<table>
<thead>
<tr>
<th>Table 6: NIS payoff table of ((P_3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f_1)</td>
</tr>
<tr>
<td>(P_3)</td>
</tr>
<tr>
<td>(\min f_2)</td>
</tr>
<tr>
<td>(\min f_3)</td>
</tr>
</tbody>
</table>

NIS: \(f_3^- = (f_1^-, f_2^-, f_3^-) = (16.3333, 0.0000, 0.0000)\)
Applying Eqs (3.11), (3.12), we compute the values $Q_3^{PIS}$ and $Q_3^{NIS}$ as:

\begin{align*}
Q_3^{PIS} &= -0.2438x_3^2 + 0.0750x_3 - 2.7777 \
Q_3^{NIS} &= -2.6601x_3^2 - 0.0776x_3 + 1.7068
\end{align*}

Using Eqs (3.13), (3.14), $\mu_1(x)$ and $\mu_2(x)$ can be obtained as follows:

\begin{align*}
\mu_1(x) &= 42.0345x_3^2 - 12.9310x_3 + 1 \
\mu_2(x) &= -4.6158x_3^2 - 0.1347x_3 + 1
\end{align*}

After using the proposed method, the resulting solution and the maximum satisfactory level is obtained for sub problem 3 as:

\begin{align*}
\max \lambda \\
x_3 + 3.3333 \geq \lambda \\
0.6551x_3 + 0.3448 \geq \lambda \\
0 \leq \lambda \leq 1, X \in FS_3 \\
\lambda^* = 0.6157 \quad x_3^* = 0.2753
\end{align*}

The maximum satisfactory level ($\lambda^* = 0.6157$) is achieved for the compromised solution $x_3^* = 0.2753$.

As shown in Fig 5, the best final solution of third sub problem is $x_3^* = 0.2753$. Moreover, the maximum satisfactory level is $\lambda^* = 0.6157$

![Figure 5: The values of function $Q_{ij}$ for problem $P_3$](image)

In addition, the proposed method is applied for each sub problem independently. Therefore, this method allows utilizing the TOPSIS and VIKOR to obtain a compromise solution for each sub problems. Moreover, the satisfactory level of each objective is determined.

5 Conclusion

In this paper a new method was proposed based on TOPSIS and VIKOR to solve the MOLSLP problems with block angular structure. Using Dantzig-wolf decomposition method, the original problem is decomposed from $N$-dimension problem into some single space sub problems. To obtain compromise solution of original problem, the individual positive ideal solution (PIS) and negative ideal solution (NIS) were calculated for each sub problem. The TOPSIS and VIKOR methods are applied to transfer multi objective programming problem into single objective. The concept of membership function is introduced and applied to aggregate the objective functions in each sub problem. Therefore this method is applied in large number of issues to deal with the real world
problems. The final solutions are obtained from each sub problem respectively. Finally, to justify the proposed method, an illustrative example was provided. The objective functions and constraints may be proposed as a fuzzy nonlinear programming problem. Moreover, the programming problem can be proposed as a no convex problem. These subjects give a new opportunity for further research.

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